$$\int \frac{e^{2x}}{(e^x-1)(e^{2x}+1)}\,dx$$

$$t = e^x$$
$$x = \ln(t)$$
$$1\,dx = \frac{1}{t}\,dt$$

$$\int \frac{t^{\cancel{2}}}{(t-1)(t^2+1)} \cdot \frac{1}{\cancel{t}}\,dt = \int \frac{t}{(t-1)(t^2+1)}\,dt = *$$

$$(t-1)(t^2+1) = 0 \Leftrightarrow t-1=0 \vee t^2+1=0$$

$$\boxed{t=1} \qquad t^2=-1$$

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$$\frac{t}{(t-1)(t^2+1)} = \frac{A}{t-1} + \frac{Bt+C}{t^2+1} \Leftrightarrow$$

$$t = A(t^2+1) + (Bt+C)(t-1) \Leftrightarrow$$

$$t = At^2 + A + Bt^2 - Bt + Ct - C \Leftrightarrow$$

$$t = t^2(A+B) + t(-B+C) + A - C$$

$$\begin{cases} A+B=0 \\ -B+C=1 \\ A-C=0 \end{cases} \quad \begin{cases} B=-A \\ A+A=1 \\ C=A \end{cases} \quad \begin{cases} B=-\frac{1}{2} \\ A=\frac{1}{2} \\ C=\frac{1}{2} \end{cases}$$

$$\frac{1}{2}\int \frac{1}{t-1}\,dt + \frac{1}{2}\int \frac{-t+1}{t^2+1}\,dt =$$

$$= \frac{1}{2}\int \frac{1}{t-1}\,dt - \frac{1}{2}\cdot\frac{1}{2}\int \frac{2t}{t^2+1}\,dt + \frac{1}{2}\int \frac{1}{t^2+1}\,dt =$$

$$= \frac{1}{2}\ln|t-1| - \frac{1}{4}\ln|t^2+1| + \frac{1}{2}\operatorname{arctg}(t) + C$$

$$= \frac{1}{2}\ln|e^x-1| - \frac{1}{4}\ln|e^{2x}+1| + \frac{1}{2}\operatorname{arctg}(e^x) + C, \quad C \in \mathbb{R}$$