$$\begin{vmatrix} 1 & 0 & -1 & 0 \\ 1 & \alpha & \alpha^2+\beta & \alpha\beta \\ 0 & 1 & \alpha & \beta \\ 1 & \alpha & \alpha^2+\beta & \alpha+\alpha\beta \end{vmatrix} \quad \begin{matrix} N_{L_1 = -L_1+L_2} \\ L_4 = -L_1+L_4 \end{matrix} \quad \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & \alpha & \alpha^2+\beta+1 & \alpha\beta \\ 0 & 1 & \alpha & \beta \\ 0 & \alpha & \alpha^2+\beta+1 & \alpha+\alpha\beta \end{vmatrix}$$

$$N_{L_2 \leftrightarrow L_3} - \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & \alpha & \beta \\ 0 & \alpha & \alpha^2+\beta+1 & \alpha\beta \\ 0 & \alpha & \alpha^2+\beta+1 & \alpha+\alpha\beta \end{vmatrix} \quad N_{L_3 = -\alpha L_2+L_3} - \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & \alpha & \beta \\ 0 & 0 & \beta+1 & 0 \\ 0 & 0 & \beta+1 & \alpha \end{vmatrix}$$

$$N_{L_4 = -L_3+L_4} - \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & \alpha & \beta \\ 0 & 0 & \beta+1 & 0 \\ 0 & 0 & 0 & \alpha \end{vmatrix} = -\alpha(\beta+1)$$