① $z = 3\cos(a) + ab^2$

$\bar{a} = 0.52$

$\bar{b} = 0.000162 \times 10^2$

$$\Delta_{\bar{z}} = \left|\frac{\partial z}{\partial a}(\bar{a},\bar{b})\right| \Delta\bar{a} + \left|\frac{\partial z}{\partial b}(\bar{a},\bar{b})\right| \Delta\bar{b}$$

$\frac{\partial z}{\partial a} = b^2 - 3\,\mathrm{sen}(a)$ $\qquad$ $\frac{\partial z}{\partial a}(\bar{a},\bar{b}) = -1.490377$

$\frac{\partial z}{\partial b} = 2ab$ $\qquad$ $\frac{\partial z}{\partial b}(\bar{a},\bar{b}) = 0.016848$

$\bar{a} = 0.52 \times 10^0$

$\Delta\bar{a} \leq 0.5 \times 10^{-t+\alpha} \leq 0.5 \times 10^{-2+0} \leq 0.5 \times 10^{-2}$

$\bar{b} = 0.162 \times 10^{-1}$

$\Delta\bar{b} \leq 0.5 \times 10^{-t+\alpha} \leq 0.5 \times 10^{-3-1} \leq 0.5 \times 10^{-4}$

$\Delta_{\bar{z}} \cong 1.490377 \times 0.5 \times 10^{-2} + 0.016848 \times 0.5 \times 10^{-4}$

$\Delta_{\bar{z}} \cong 0.007453$

$$r_{\bar{z}} = \frac{\Delta \bar{z}}{|3\cos(\bar{a}) + \bar{a}\bar{5}^2|} = \frac{0.007453}{2.603594} \cong 0.002862$$

$$\cong 2.862 \times 10^{-3}$$

$$\leq 5 \times 10^{-3}$$

3 a.s.

$z = 3\cos(\bar{a}) + \bar{a}\bar{5}^2 = 2.603594$

$z = 2.60$

(2) $x^2 + e^x = 2$ $\quad [0,1]$

$f(x) = x^2 + e^x - 2$

$f(0) = -1$ $\quad f(0) \cdot f(1) < 0$ ✓

$f(1) \cong 1.72$

f é uma função contínua em $\mathbb{R}$, pois é a adição de funções contínuas (função quadrática, função exponencial e função constante), e portanto é contínua em $[0,1]$. ✓

Logo podemos concluir que a equação tem uma solução no intervalo $[0,1]$.

b) $\frac{b-a}{2^K} \leq 10^{-5} \Leftrightarrow \frac{1}{2^K} \leq 10^{-5} \Leftrightarrow 2^{-K} \leq 10^{-5} \Leftrightarrow$

$-K \leq \log_2 10^{-5} \Leftrightarrow K \geq 5 \log_2 10 \approx 16{,}6$

$K \geq 17$

Têm de ser efetuados 17 iterações.

3

$$\int \underbrace{sen(x)}_{u} \underbrace{tg^2(x)}_{v} dx = -\cos(x) . tg^2(x) + 2\int \cos(x) . tg(x) . sec^2(x) dx =$$

$$= -\cos(x) tg^2(x) + 2\int \cancel{\cos(x)} . tg(x) . \frac{1}{\cos^{\cancel{2}}(x)} dx =$$

$$= -\cos(x) tg^2(x) + 2\int tg(x) sec(x) dx =$$

$$= -\cos(x) tg^2(x) + 2 sec(x) + C, C \in \mathbb{R}$$

C.A.

$P[sen(x)] = -\cos(x)$ | $\frac{1}{\cos(x)} = sec(x)$

$v' = 2 tg(x) . sec^2(x)$ | $\frac{1}{\cos^2(x)} = sec^2(x)$

④ $$\int \frac{1}{x^{1/2}+x^{1/3}}\,dx = \qquad x = t^6 \iff t = x^{1/6}$$
$$dx = 6t^5\,dt$$

$$= \int \frac{1}{(t^6)^{1/2}+(t^6)^{1/3}}\,6t^5\,dt = 6\int \frac{t^5}{t^{6/2}+t^{6/3}}\,dt =$$

$$= 6\int \frac{t^5}{t^3+t^2}\,dt = 6\int t^2-t+1+\frac{-t^2}{t^3+t^2}\,dt =$$

$$= 6\int t^2-t+1\,dt - 6\int \frac{\cancel{t^2}}{\cancel{t^2}(t+1)}\,dt =$$

$$= 6\int t^2\,dt - 6\int t\,dt + \int 6\,dt - 6\int \frac{1}{t+1}\,dt =$$

$$= 2t^3-3t^2+6t-6\ln|t+1| + C,\ C\in\mathbb{R} =$$

$$= 2(x^{1/6})^3-3(x^{1/6})^2+6x^{1/6}-6\ln|x^{1/6}+1|+C,\ C\in\mathbb{R} =$$

$$= 2x^{1/2}-3x^{1/3}+6x^{1/6}-6\ln|x^{1/6}+1|+C,\ C\in\mathbb{R}$$

C.A.

$$
\begin{array}{l|l}
t^5 & t^3 + t^2 \\ \cline{2-2}
-t^5 - t^4 & t^2 - t + 1 \\ \cline{1-1}
\quad -t^4 & \\
\quad t^4 + t^3 & \\ \cline{1-1}
\qquad t^3 & \\
\quad -t^3 - t^2 & \\ \cline{1-1}
\qquad -t^2 &
\end{array}
$$

5

$$A = \int_{-1}^{-1/2} 2 - e^x \, dx + \int_{-1/2}^{0} -2x + 1 - e^x \, dx =$$

$$= \left[2x - e^x\right]_{-1}^{-1/2} + \left[-x^2 + x - e^x\right]_{-1/2}^{0} =$$

$$= \left(2\cdot\left(-\tfrac{1}{2}\right) - \cancel{e^{-1/2}}\right) - \left(2\times(-1) - e^{-1}\right) + \left(0 + 0 - e^0\right) -$$

$$\left(-\left(-\tfrac{1}{2}\right)^2 - \tfrac{1}{2} - \cancel{e^{-1/2}}\right) = -\cancel{1} + \cancel{2} + e^{-1} - \cancel{e^0} + \tfrac{1}{4} + \tfrac{1}{2} =$$

$$= \tfrac{1}{e} + \tfrac{3}{4}$$

C.A.

$$\begin{cases} y = 2 \\ y = -2x+1 \end{cases} \Leftrightarrow \Big\} \; 2 = -2x+1 \Leftrightarrow \boxed{x = -\frac{1}{2}}$$

6) $\displaystyle\sum_{n=2}^{+\infty} \left(\frac{1}{2}\right)^{n-1}$

$$\frac{a_{n+1}}{a_n} = \frac{\left(\frac{1}{2}\right)^{n+1-1}}{\left(\frac{1}{2}\right)^{n-1}} = \frac{\left(\frac{1}{2}\right)^{n}}{\left(\frac{1}{2}\right)^{n-1}} = \left(\frac{1}{2}\right)^{n-n+1} = \frac{1}{2}$$

$|r| = \frac{1}{2} < 1$, série convergente

$$S = \frac{u_1}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$