Proposta de Resolução - Teste 2 - Matemática I - 2016/2017

**1.**

**(a)**

$$\int \frac{dx}{(1+x^2)\operatorname{arctg}(x)} = \int \underbrace{\frac{1}{(1+x^2)}}_{u'}\left[\underbrace{\operatorname{arctg}(x)}_{u}\right]^{-1} dx = \ln|\operatorname{arctg}(x)| + C, C \in \mathbb{R}$$

**(b)**

$$\int \frac{\sqrt[3]{1+\ln(x)}}{x}\,dx = \int \underbrace{\frac{1}{x}}_{u'}\left[\underbrace{1+\ln(x)}_{u}\right]^{1/3} dx = \frac{[1+\ln(x)]^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{3}{4}[1+\ln(x)]\sqrt[3]{1+\ln(x)} + C, C \in \mathbb{R}$$

**2.**

Cálculo Auxiliar:

$u = x \to P(u) = \dfrac{x^2}{2}$

$v = \operatorname{arcsen}(x^2) \longrightarrow v' = \dfrac{2x}{\sqrt{1-x^4}}$

$$\int x \operatorname{arcsen}(x^2)\,dx = \frac{x^2}{2}\operatorname{arcsen}(x^2) - \int \frac{x^2}{2}\frac{2x}{\sqrt{1-x^4}}dx = \frac{x^2}{2}\operatorname{arcsen}(x^2) - \int \frac{x^3}{\sqrt{1-x^4}}dx =$$

$$= \frac{x^2}{2}\operatorname{arcsen}(x^2) - \int x^3\,(1-x^4)^{-\frac{1}{2}}dx = \frac{x^2}{2}\operatorname{arcsen}(x^2) + \frac{1}{4}\int \underbrace{-4x^3}_{u'}\Big(\underbrace{1-x^4}_{u}\Big)^{-\frac{1}{2}} dx =$$

$$= \frac{x^2}{2}\operatorname{arcsen}(x^2) + \frac{1}{4}\frac{(1-x^4)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^2}{2}\operatorname{arcsen}(x^2) + \frac{1}{2}\sqrt{1-x^4} + C, C \in \mathbb{R}$$

**3.**

$$\int \frac{x^5+x^4-8}{x^3-4x}dx \underbrace{=}_{\substack{divisão\\ polinómios}} \int \left(x^2+x+4+\frac{4x^2+16x-8}{x^3-4x}\right)dx$$

$$\underbrace{=}_{\substack{coeficientes\\ indeterminados}} \int \left(x^2+x+4+\frac{2}{x}+\frac{5}{x-2}-\frac{3}{x+2}\right)dx =$$

$$= \frac{x^3}{3}+\frac{x^2}{2}+4x+2\ln|x|+5\ln|x-2|-3\ln|x+2|+C, C \in \mathbb{R}$$

Cálculo Auxiliar Coeficientes Indeterminados:

$$\frac{4x^2+16x-8}{x^3-4x} = \frac{4x^2+16x-8}{x(x-2)(x+2)} = \frac{A}{x}+\frac{B}{x-2}+\frac{C}{x+2}$$

$$\Leftrightarrow 4x^2+16x-8 = Ax^2-4A+Bx^2+2Bx+Cx^2-2Cx$$

$$\Leftrightarrow 4x^2+16x-8 = (A+B+C)x^2+(2B-2C)x-4A$$

$$\begin{cases} A+B+C=4 \\ 2B-2C=16 \\ -4A=-8 \end{cases} \Leftrightarrow \begin{cases} B=2-C \\ -4C=12 \\ A=2 \end{cases} \Leftrightarrow \begin{cases} B=5 \\ C=-3 \\ A=2 \end{cases}$$

**4.**

Cálculo Auxiliar Mudança Variável:
$t = \ln(x) \Leftrightarrow x = e^t \longrightarrow dx = e^t dt$

$$\int \frac{\ln(x) - 8}{x[\ln^3(x) - 2\ln^2(x) + \ln(x)]} dx = \int \frac{\mathrm{t} - 8}{t^3 - 2t^2 + t} dt$$

$$\underbrace{=}_{\substack{coeficientes \\ indeterminados}} \int \left(-\frac{8}{t} + \frac{8}{t-1} - \frac{7}{(t-1)^2}\right) dt = -8\ln|\mathrm{t}| + 8\ln|\mathrm{t} - 1| - 7\frac{(t-1)^{-2+1}}{-2+1} + C$$

$$= -8\ln|\ln(x)| + 8\ln|\ln(x) - 1| + \frac{7}{\ln(x) - 1} + C, C \in \mathbb{R}$$

Cálculo Auxiliar Coeficientes Indeterminados:

$$\frac{\mathrm{t} - 8}{t^3 - 2t^2 + t} = \frac{\mathrm{t} - 8}{t(t-1)^2} = \frac{A}{t} + \frac{B}{t-1} + \frac{C}{(t-1)^2}$$

$$\Leftrightarrow t - 8 = At^2 - 2At + A + Bt^2 - Bt + Ct$$

$$\Leftrightarrow t - 8 = (A + B)t^2 - (2A + B - C)t + A$$

$$\begin{cases} A + B = 0 \\ -2A - B + C = 1 \\ A = -8 \end{cases} \Leftrightarrow \begin{cases} B = -A \\ C = 1 + 2A + B \\ A = -8 \end{cases} \Leftrightarrow \begin{cases} B = 8 \\ C = -7 \\ A = -8 \end{cases}$$

**5.**

$$\mathrm{A} = \int_0^{\frac{\pi}{4}} [\cos(x) - \mathrm{sen}(x)]\, dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} [\mathrm{sen}(x) - \cos(x)]\, dx =$$

$$= [\mathrm{sen}(x) + \cos(x)]_0^{\frac{\pi}{4}} + [-\cos(x) - \mathrm{sen}(x)]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} =$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0 + 1) + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = 3\sqrt{2} - 1\ u.a.$$

**6.**

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 2x + 2} = \lim_{t \to -\infty} \int_t^0 \frac{dx}{x^2 + 2x + 1 + 1} + \lim_{t \to +\infty} \int_0^t \frac{dx}{x^2 + 2x + 1 + 1} =$$

$$= \lim_{t \to -\infty} \int_t^0 \overbrace{\frac{1}{1 + (x+1)^2}}^{u'/(1+u^2)} dx + \lim_{t \to +\infty} \int_0^t \frac{1}{1 + (x+1)^2} dx$$

$$= \lim_{t \to -\infty} [\mathrm{arctg}(x+1)]_t^0 + \lim_{t \to +\infty} [\mathrm{arctg}(x+1)]_0^t$$

$$= \lim_{t \to -\infty} (\mathrm{arctg}(1) - \mathrm{arctg}(t+1)) + \lim_{t \to +\infty} (\mathrm{arctg}(t+1) - \mathrm{arctg}(1)) = \frac{\pi}{4} - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \frac{\pi}{4} = \pi$$

**7.**

$$f(x,y) = xye^{\frac{x}{y}}$$

$$\frac{\partial f(x,y)}{\partial x} = \left[\underbrace{xy}_{u}\underbrace{e^{\frac{x}{y}}}_{v}\right]' = (xy)' \cdot e^{\frac{x}{y}} + xy\left(e^{\frac{x}{y}}\right)' = ye^{\frac{x}{y}} + xy\frac{1}{y}e^{\frac{x}{y}} = ye^{\frac{x}{y}} + xe^{\frac{x}{y}}$$

<u>Cálculo Auxiliar (derivar em ordem a *x*):</u>

$$\left(e^{\frac{x}{y}}\right)' = \left(\frac{x}{y}\right)' e^{\frac{x}{y}} = \frac{1}{y}e^{\frac{x}{y}}$$

$$\frac{\partial f(x,y)}{\partial y} = \left[\underbrace{xy}_{u}\underbrace{e^{\frac{x}{y}}}_{v}\right]' = (xy)' \cdot e^{\frac{x}{y}} + xy\left(e^{\frac{x}{y}}\right)' = xe^{\frac{x}{y}} - xy\frac{x}{y^2}e^{\frac{x}{y}} = xe^{\frac{x}{y}} - \frac{x^2}{y}e^{\frac{x}{y}}$$

<u>Cálculo Auxiliar (derivar em ordem a *y*):</u>

$$\left(e^{\frac{x}{y}}\right)' = \left(\frac{x}{y}\right)' e^{\frac{x}{y}} = (xy^{-1})' e^{\frac{x}{y}} = x(-1)y^{-2}e^{\frac{x}{y}} = -\frac{x}{y^2}e^{\frac{x}{y}}$$

$$x\frac{\partial f(x,y)}{\partial x} + y\frac{\partial f(x,y)}{\partial y} = x\left(ye^{\frac{x}{y}} + xe^{\frac{x}{y}}\right) + y\left(xe^{\frac{x}{y}} - \frac{x^2}{y}e^{\frac{x}{y}}\right) =$$

$$= e^{\frac{x}{y}}(xy + x^2 + yx - x^2) = 2xye^{\frac{x}{y}} = 2f(x,y) \text{ c.q.m.}$$