(4) $\displaystyle\int \underbrace{x}_{u}\,\underbrace{\ln^2(x)}_{v}\,dx = ⊛$

$$\boxed{\int u\cdot v\,dx = Pu\cdot v - \int Pu\cdot v'}$$

$P[x] = \frac{x^2}{2}$

$v' = 2\ln^{1}(x)\cdot\frac{1}{x}$

$$⊛ = \frac{x^2}{2}\cdot\ln^2(x) - \int \frac{x^{\cancel{2}}}{\cancel{2}}\cdot\cancel{2}\cdot\ln(x)\cdot\frac{1}{\cancel{x}}\,dx =$$

$$= \frac{x^2}{2}\ln^2(x) - \int \underbrace{x}_{u}\,\underbrace{\ln(x)}_{v}\,dx = \frac{x^2}{2}\ln^2(x) - \left[\frac{x^2}{2}\cdot\ln(x) - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx\right] =$$

$P[x] = \frac{x^2}{2}$

$v' = \frac{1}{x}$

$$= \frac{x^2}{2}\ln^2(x) - \left(\frac{x^2}{2}\cdot\ln(x) - \frac{1}{2}\int x\,dx\right) =$$

$$= \frac{x^2}{2}\ln^2(x) - \frac{x^2}{2}\ln(x) + \frac{1}{2}\cdot\frac{x^2}{2} + C.\ C\in\mathbb{R}$$

(5) $\int \frac{x^2+3}{x^2-3x+2}\,dx$ (*)

$$\begin{array}{r|l} x^2 \quad\quad 3 & x^2-3x+2 \\ \cline{2-2} -x^2+3x-2 & 1 \\ \hline 3x+1 & \end{array}$$

$$\frac{x^2+3}{x^2-3x+2} = 1 + \frac{3x+1}{x^2-3x+2} = 1 + \frac{3x+1}{(x-1)(x-2)}$$

$$x^2-3x+2=0 \Leftrightarrow x = \frac{-(-3)\pm\sqrt{9-8}}{2} \Leftrightarrow$$

$$x = \frac{3\pm\sqrt{1}}{2} \begin{cases} x = \frac{3+1}{2} = 2 \\ x = \frac{3-1}{2} = 1 \end{cases}$$

$$\frac{3x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \Leftrightarrow 3x+1 = A(x-2)+B(x-1)$$

$$\Leftrightarrow 3x+1 = Ax-2A+Bx-B \Leftrightarrow 3x+1 = x(A+B)-2A-B$$

$$\begin{cases} A+B=3 \\ -2A-B=1 \end{cases} \begin{cases} B=3-A \\ -2A-3+A=1 \end{cases} \begin{cases} \boxed{B=7} \\ -A=4 \Leftrightarrow \boxed{A=-4} \end{cases}$$

$$* \int 1\,dx + \int \frac{-4}{x-1}\,dx + \int \frac{7}{x-2}\,dx =$$

$$x - 4\ln|x-1| + 7\ln|x-2| + C, \quad C \in \mathbb{R}$$

(7) $\int_{-\infty}^{1} \frac{\operatorname{arctg}(x)}{1+x^2}\,dx = \lim_{t\to-\infty}\int_{t}^{1} \frac{\operatorname{arctg}(x)}{1+x^2}\,dx$ (*)

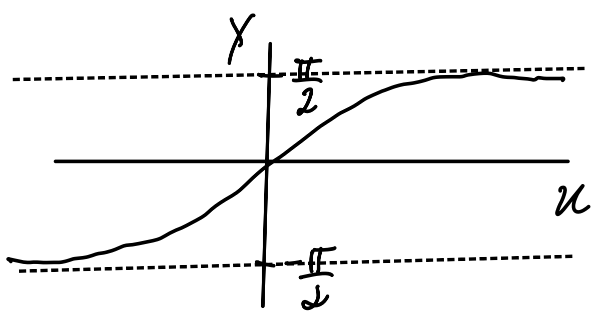
$1 \to a$

$$\int \underbrace{\frac{1}{1+x^2}}_{u'} \cdot \underbrace{\operatorname{arctg}^{1}(x)}_{u}\,dx$$

$$\int u' u^{a}\,du = \frac{u^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$* \lim_{t\to-\infty}\left[\frac{\operatorname{arctg}^{1+1}(x)}{1+1}\right]_{t}^{1} = \lim_{t\to-\infty}\left[\frac{\operatorname{arctg}^{2}(x)}{2}\right]_{t}^{1} =$$

$$\lim_{t\to-\infty}\left(\frac{\operatorname{arctg}^{2}(1)}{2} - \frac{\operatorname{arctg}^{2}(t)}{2}\right) = \frac{\left(\frac{\pi}{4}\right)^2}{2} - \frac{\left(-\frac{\pi}{2}\right)^2}{2} =$$



$$= \frac{\frac{\pi^2}{16} - \frac{\pi^2}{4}}{2} = $$

$$= \frac{\frac{\pi^2 - 4\pi^2}{16}}{2} = \frac{\frac{-3\pi^2}{16}}{2} =$$

$= \frac{-3\pi^2}{32}$, logo convergente.

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$(u \cdot v)' = u' \cdot v + v' \cdot u$$

(1) $f(x,y,z) = \frac{x}{y} \cdot \cos(z) = \frac{x\cos(z)}{y} = x \cdot \frac{\cos(z)}{y}$

$$\Delta f \approx \left|\frac{\partial f}{\partial x}(\bar{x},\bar{y},\bar{z})\right| \cdot \Delta x + \left|\frac{\partial f}{\partial y}(\bar{x},\bar{y},\bar{z})\right| \cdot \Delta y + \left|\frac{\partial f}{\partial z}(\bar{x},\bar{y},\bar{z})\right| \cdot \Delta z$$

$\frac{\partial f}{\partial x} = \frac{\cos(z)}{y}$ $\qquad$ $(ax)' = a$

ou

$$\frac{\partial f}{\partial x} = \frac{\cos(z) \cdot y - 0 \cdot x\cos(z)}{y^2} = \frac{\cos(z) \cdot \cancel{y}}{y^{\cancel{2}}} = \frac{\cos(z)}{y}$$

$$\frac{\partial f}{\partial y} = \frac{0 \cdot y - 1 \cdot x\cos(z)}{\underset{u'}{y^2}} = \frac{-x\cos(z)}{y^2}$$

$$\frac{\partial f}{\partial z} = \frac{(0 \cdot \cos(z) + (-\text{sen}(z) \cdot x))\, y + 0 \cdot x\cos(z)}{y^2} =$$

$$= -\frac{\text{sen}(z) \cdot x\cancel{y}}{y^{\cancel{2}}} = -\frac{x\,\text{sen}(z)}{y}$$

$$\bar{x} = -13.520 \times 10^{-2} = -0.13520 \times 10^{2} \times 10^{-2}$$
$$= -0.13520 \times 10^{0} \quad 5\ a.s$$

$$\bar{y} = 0.0056 = 0.56 \times 10^{-2} \quad 2\ a.s$$

$$\bar{z} = 0.0000456 \times 10^{2} = 0.456 \times 10^{-4} \times 10^{2}$$
$$= 0.456 \times 10^{-2} \quad 3\ a.s$$

$$\Delta \bar{x} \leq 0.5 \times 10^{-t+a} \leq 0.5 \times 10^{-5+0} \leq 0.5 \times 10^{-5}$$

$$\Delta \bar{y} \leq 0.5 \times 10^{-2-2} \leq 0.5 \times 10^{-4}$$

$$\Delta \bar{z} \leq 0.5 \times 10^{-3-2} \leq 0.5 \times 10^{-5}$$