

Worksheet 8

Q1. For the function

$$f(x, y) = x^3 - 3xy^2 + 2x - y,$$

find the first-order partial derivatives f_x and f_y and evaluate them at $(1, -1)$.

Q2. For the function

$$f(x, y) = \sqrt{x^2 + 4y^2 + 1},$$

evaluate $f_x(3, 2)$ and $f_y(3, 2)$.

Q3. For the function

$$f(x, y) = \frac{\sin x + y^2}{x^2 + y^2 + 1},$$

find $f_x(0, 1)$ and $f_y(0, 1)$.

Q4. For the function

$$f(x, y) = \frac{e^{xy}}{1 + x^2},$$

evaluate $f_x(0, 5)$ and $f_y(0, 5)$.

Q5. For the function

$$f(x, y) = \frac{x^3y^2}{x^2 + y^2 + 4},$$

find $f_x(2, 1)$ and $f_y(2, 1)$.

Q6. For the function

$$f(x, y) = e^{x^2y + y^3},$$

compute the second-order partial derivatives f_{xx} , f_{yy} , and f_{xy} , and evaluate all of them at $(0, 1)$.

Q7. For the function

$$f(x, y) = \frac{x^2 + 3y}{x + y^2 + 2},$$

compute the second-order partial derivatives f_{xx} , f_{yy} , and f_{xy} , and evaluate them at $(1, 0)$.

Q8. Consider the function

$$f(x, y) = x^2y^3 \sin(xy).$$

- (a) Compute f_{xy} and f_{yx} .
- (b) Show that $f_{xy} = f_{yx}$ for all (x, y) , justifying the use of Clairaut's Theorem.

Q9. For the function

$$f(x, y) = (x^2 + y^2) \ln(x^2 + y^2 + 1),$$

- (a) Compute f_{xy} and f_{yx} .
- (b) Show that the mixed partial derivatives are equal everywhere.
- (c) Explain why continuity and differentiability guarantee $f_{xy} = f_{yx}$ for this function.