1 Preamble: Basic Concepts

1.1 Signal Detection Theory

Signal Detection Theory (SDT) is a framework used to analyze decision-making in the presence of uncertainty. It helps separate the sensitivity of the decision-maker from their bias.

1.2 Decision Variable

A decision variable (V_d) represents the internal evidence used to make a decision. It's often modeled as a random variable due to noise in sensory processes.

1.3 Criterion

A criterion (C_d) is a threshold value used to make decisions. When the decision variable exceeds the criterion, one decision is made; otherwise, the alternative decision is chosen.

1.4 Confidence

Confidence represents the degree of certainty in a decision. It's often modeled as a function of the distance between the decision variable and the criterion.

1.5 Gaussian Distribution

Many models assume that decision variables follow a Gaussian (normal) distribution due to the accumulation of many small, independent factors.

1.6 Meta-uncertainty

Meta-uncertainty refers to uncertainty about one's own uncertainty. It captures variability in how people assess their own decision-making processes.

A Appendix: Advanced Concepts

A.1 Log-likelihood

Log-likelihood is a statistical measure used in model fitting. It quantifies how well a model's predictions match observed data, with higher values indicating better fit.

A.2 Interior Point Algorithm

The interior point algorithm is an optimization method used to find the best-fitting parameters for a model. It works by approaching the optimal solution from within the feasible region.

A.3 2-AFC and 2-IFC Tasks

- 2-AFC (Two-Alternative Forced Choice): A task where participants must choose between two alternatives.
- 2-IFC (Two-Interval Forced Choice): A task where stimuli are presented in two separate time intervals, and participants must choose which interval contained the target.

A.4 Gaussian Mixture Models

A Gaussian mixture model is a probabilistic model that assumes all data points are generated from a mixture of a finite number of Gaussian distributions with unknown parameters.

A.5 Log-normal Distribution

A log-normal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. It's often used to model variables that are always positive and have a skewed distribution.

A.6 Cumulative Normal Distribution

The cumulative normal distribution, denoted as $\Phi(x)$, gives the probability that a normally distributed random variable with mean 0 and variance 1 is less than or equal to x.

B Hierarchical Decision-Making Model

B.1 Model Overview

The model describes choice-confidence data in binary decision-making tasks as arising from a hierarchical process:

- First stage: Conventional signal detection theory
- Decision variable (V_d): One-dimensional, compared to fixed criterion (C_d)
- V_d is subject to zero-mean Gaussian noise
- $V_d \sim \mathcal{N}(\mu_d, \sigma_d)$

B.2 Confidence Variable

The decision variable is converted into a signed confidence variable:

$$V_c = \frac{V_d - C_d}{\hat{\sigma_d}} \tag{1}$$

where $\hat{\sigma_d}$ is the subject's estimate of σ_d.

B.3 Confidence Distribution

If $\hat{\sigma_d}$ were constant, V_c would follow:

$$V_c \sim \mathcal{N}(\mu'_c, \sigma'_c) \tag{2}$$

where $\mu'_c = (\mu_d - C_d)/\hat{\sigma_d}$ and $\sigma'_c = \sigma_d/\hat{\sigma_d}$

B.4 Confidence Report

The confidence report results from comparing V_c with fixed criterion magnitude C_c, mirrored across 0 to create two signed criteria.

B.5 Probability Calculations

For a 'confident' judgment given a 'category A' decision:

$$P(C = 1|D = 0) = \Phi(-C_c) \tag{3}$$

Other probabilities:

$$P(C = 0|D = 0) = \Phi(0) - \Phi(-C_c) \tag{4}$$
$$P(C = 0|D = 1) = \Phi(C_c) - \Phi(0) \tag{5}$$
$$P(C = 1|D = 1) = 1 - \Phi(C_c) \tag{6}$$

where $\Phi(.)$ is the cumulative normal distribution with mean μ'_c and standard deviation σ'_c.

B.6 CASANDRE Model

Key feature: σ_d is not constant, but a random variable:

$$\sigma_d \sim \text{LogNormal}(\sigma_d, \sigma_m) \tag{7}$$

Consequently, the signed confidence variable is a mixture of normal distributions, with mixing weights determined by σ_m.

B.7 Probability Estimation

To obtain probabilities under this mixture:

1. Sample σ_d in steps of constant cumulative density (using Matlab's 'logninv')

2. Compute probability of each response option under each sample's normal distribution (using Matlab's 'normcdf')

3. Average probabilities across all samples

B.8 Implementation Details

- 100 samples used, sampling σ_d at cumulative densities: 0.5%, 1.5%, 2.5%,... 99.5%
- σ_d fixed to 1, scaling the relation between stimulus value and decision variable mean
- σ_m reported as coefficient of variation (σ_m/σ_d)

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C Hierarchical Decision-Making Model

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D Model Parameterization, Simulations and Fitting

D.1 CASANDRE Model Parameters

The CASANDRE model's predicted probability of each response option is fully specified by five parameters:

- μ_d: Mean of the decision variable
- σ_d: Standard deviation of the decision variable
- C_d: Decision criterion
- σ_m: Level of meta-uncertainty
- C_c: Confidence criterion

D.2 Model Identifiability

To make the model identifiable:

- μ_d is assumed identical to the true stimulus value
- σ_d is constant for a given level of stimulus reliability
- C_d, σ_m, and C_c are constant across multiple conditions
- σ_m is limited to a minimum value of 0.1

D.3 Additional Parameter for Fitting

When fitting data, an additional parameter λ is used to account for stimulus-independent lapses, assumed to be uniformly distributed across all response options.

D.4 Fitting Procedure

- Model fit on a subject-by-subject basis
- Compute log-likelihood of given set of model parameters across all choice-confidence reports
- Use iterative procedure (interior point algorithm, Matlab function 'fmincon') to identify the most likely set of parameter values

E Task Designs and Model Variations

E.1 2-AFC Categorization with Binary Confidence

- Simplest design
- Five free parameters: λ, σ_d, C_d, σ_m, and C_c
- Used to capture data across 20 experimental conditions

E.2 2-AFC Categorization with Multi-level Confidence Rating

- Multiple confidence criteria used (one less than the number of confidence levels)
- Example: Seven free parameters for four-point confidence rating scale
- Variations:
  - 17 free parameters: λ, σ_d (6), C_d (6), σ_m, C_c (3)
  - 12 free parameters: λ, σ_d (4), C_d, σ_m, C_c (5)
  - 10 free parameters: σ_d (3), C_d, σ_m, C_c (5)

E.3 2-AFC Categorization with Different Spread

- Two category distributions with same mean but different spread
- Confidence estimate based on distance between decision variable and nearest decision criterion
- Example: 23 free parameters: λ, σ_d (6), C_d (12), σ_m, C_c (3)
- Variation: 22 free parameters with attention levels

E.4 2-IFC Categorization with Confidence Report

- Subject shown two stimulus intervals, judges which contained the 'signal' stimulus
- Decision based on comparison of evidence from each stimulus interval
- One-dimensional decision variable V_d reflects outcome of this comparison
- Modeled as difference operation
- Difference of two Gaussian distributions is itself Gaussian:
  - Mean equal to the difference of the means
  - Standard deviation equal to $\sqrt{\sigma_1^2 + \sigma_2^2}$
- C_d reflects an interval bias (preference for 'interval 1' choices) when different from zero