1 Preamble: Basic Concepts

1.1 Signal Detection Theory

Signal Detection Theory (SDT) is a framework used to analyze decision-making in the presence of uncertainty. It helps separate the sensitivity of the decision-maker from their bias.

1.2 Decision Variable

A decision variable (V_d) represents the internal evidence used to make a decision. It's often modeled as a random variable due to noise in sensory processes.

1.3 Criterion

A criterion (C_d) is a threshold value used to make decisions. When the decision variable exceeds the criterion, one decision is made; otherwise, the alternative decision is chosen.

1.4 Confidence

Confidence represents the degree of certainty in a decision. It's often modeled as a function of the distance between the decision variable and the criterion.

1.5 Gaussian Distribution

Many models assume that decision variables follow a Gaussian (normal) distribution due to the accumulation of many small, independent factors.

1.6 Meta-uncertainty

Meta-uncertainty refers to uncertainty about one's own uncertainty. It captures variability in how people assess their own decision-making processes.

2 Hierarchical Decision-Making Model

2.1 Model Overview

The model describes choice-confidence data in binary decision-making tasks as arising from a hierarchical process:

- First stage: Conventional signal detection theory
- Decision variable (V_d): One-dimensional, compared to fixed criterion (C_d)
- V_d is subject to zero-mean Gaussian noise
- $V_d \sim \mathcal{N}(\mu_d, \sigma_d)$

2.2 Confidence Variable

The decision variable is converted into a signed confidence variable:

$$V_c = \frac{V_d - C_d}{\hat{\sigma_d}} \tag{1}$$

where $\hat{\sigma_d}$ is the subject's estimate of σ_d.

2.3 Confidence Distribution

If $\hat{\sigma_d}$ were constant, V_c would follow:

$$V_c \sim \mathcal{N}(\mu_c', \sigma_c') \tag{2}$$

where $\mu_c' = (\mu_d - C_d)/\hat{\sigma_d}$ and $\sigma_c' = \sigma_d/\hat{\sigma_d}$

2.4 Confidence Report

The confidence report results from comparing V_c with fixed criterion magnitude C_c, mirrored across 0 to create two signed criteria.

2.5 Probability Calculations

For a 'confident' judgment given a 'category A' decision:

$$P(C = 1|D = 0) = \Phi(-C_c) \tag{3}$$

Other probabilities:

$$P(C = 0|D = 0) = \Phi(0) - \Phi(-C_c) \tag{4}$$
$$P(C = 0|D = 1) = \Phi(C_c) - \Phi(0) \tag{5}$$
$$P(C = 1|D = 1) = 1 - \Phi(C_c) \tag{6}$$

where $\Phi(.)$ is the cumulative normal distribution with mean μ_c' and standard deviation σ_c'.

2.6 CASANDRE Model

Key feature: σ_d is not constant, but a random variable:

$$\sigma_d \sim \text{LogNormal}(\sigma_d, \sigma_m) \tag{7}$$

Consequently, the signed confidence variable is a mixture of normal distributions, with mixing weights determined by σ_m.

2.7 Probability Estimation

To obtain probabilities under this mixture:

1. Sample σ_d in steps of constant cumulative density (using Matlab's 'logn-inv')
2. Compute probability of each response option under each sample's normal distribution (using Matlab's 'normcdf')
3. Average probabilities across all samples

2.8 Implementation Details

- 100 samples used, sampling σ_d at cumulative densities: 0.5%, 1.5%, 2.5%,... 99.5%
- σ_d fixed to 1, scaling the relation between stimulus value and decision variable mean
- σ_m reported as coefficient of variation (σ_m/σ_d)

article amsmath

3 Model Parameterization, Simulations and Fitting

3.1 CASANDRE Model Parameters

The CASANDRE model's predicted probability of each response option is fully specified by five parameters:

- μ_d: Mean of the decision variable
- σ_d: Standard deviation of the decision variable
- C_d: Decision criterion
- σ_m: Level of meta-uncertainty
- C_c: Confidence criterion

3.2 Model Identifiability

To make the model identifiable:

- μ_d is assumed identical to the true stimulus value
- σ_d is constant for a given level of stimulus reliability
- C_d, σ_m, and C_c are constant across multiple conditions
- σ_m is limited to a minimum value of 0.1

3.3 Additional Parameter for Fitting

When fitting data, an additional parameter λ is used to account for stimulus-independent lapses, assumed to be uniformly distributed across all response options.

3.4 Fitting Procedure

- Model fit on a subject-by-subject basis
- Compute log-likelihood of given set of model parameters across all choice-confidence reports
- Use iterative procedure (interior point algorithm, Matlab function 'fmincon') to identify the most likely set of parameter values

4 Task Designs and Model Variations

4.1 2-AFC Categorization with Binary Confidence

- Simplest design
- Five free parameters: λ, σ_d, C_d, σ_m, and C_c
- Used to capture data across 20 experimental conditions

4.2 2-AFC Categorization with Multi-level Confidence Rating

- Multiple confidence criteria used (one less than the number of confidence levels)
- Example: Seven free parameters for four-point confidence rating scale
- Variations:
  - 17 free parameters: λ, σ_d (6), C_d (6), σ_m, C_c (3)
  - 12 free parameters: λ, σ_d (4), C_d, σ_m, C_c (5)
  - 10 free parameters: σ_d (3), C_d, σ_m, C_c (5)

4.3 2-AFC Categorization with Different Spread

- Two category distributions with same mean but different spread
- Confidence estimate based on distance between decision variable and nearest decision criterion
- Example: 23 free parameters: λ, σ_d (6), C_d (12), σ_m, C_c (3)
- Variation: 22 free parameters with attention levels

4.4 2-IFC Categorization with Confidence Report

- Subject shown two stimulus intervals, judges which contained the 'signal' stimulus
- Decision based on comparison of evidence from each stimulus interval
- One-dimensional decision variable V_d reflects outcome of this comparison
- Modeled as difference operation
- Difference of two Gaussian distributions is itself Gaussian:
  - Mean equal to the difference of the means
  - Standard deviation equal to $\sqrt{\sigma_1^2 + \sigma_2^2}$
- C_d reflects an interval bias (preference for 'interval 1' choices) when different from zero

A Appendix: Advanced Concepts

A.1 Log-likelihood

Log-likelihood is a statistical measure used in model fitting. It quantifies how well a model's predictions match observed data, with higher values indicating better fit.

A.2 Interior Point Algorithm

The interior point algorithm is an optimization method used to find the best-fitting parameters for a model. It works by approaching the optimal solution from within the feasible region.

A.3 2-AFC and 2-IFC Tasks

- 2-AFC (Two-Alternative Forced Choice): A task where participants must choose between two alternatives.
- 2-IFC (Two-Interval Forced Choice): A task where stimuli are presented in two separate time intervals, and participants must choose which interval contained the target.

A.4 Gaussian Mixture Models

A Gaussian mixture model is a probabilistic model that assumes all data points are generated from a mixture of a finite number of Gaussian distributions with unknown parameters.

A.5 Log-normal Distribution

A log-normal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. It's often used to model variables that are always positive and have a skewed distribution.

A.6 Cumulative Normal Distribution

The cumulative normal distribution, denoted as $\Phi(x)$, gives the probability that a normally distributed random variable with mean 0 and variance 1 is less than or equal to x.