Answer Key: Problem Set 1

QTM 200: Applied Regression Analysis

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Instructions

- Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in `R`, please include the code you used to get your answers. Please also include the `.R` file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.
- Your homework should be submitted electronically on the course GitHub page in `.pdf` form.
- This problem set is due at the beginning of class on Wednesday, January 28, 2020. No late assignments will be accepted.
- Total available points for this homework is 100.

Question 1 (25 points)

A private school counselor was curious about the average of IQ of the students in her school and took a random sample of 25 students' IQ scores. The following is the data set:

Find a 90% confidence interval for the student IQ in the school assuming the population of IQ from which our random sample has been selected is normally distributed.

First, let's calculate the t-score for the 90% confidence interval with degrees of freedom equal to 24 (remember that df $= n - 1$ for the t-distribution, and we're using the t-distribution because we have a small sample size). In the 90% confidence interval, the lower tail is equal to 0.05 ($(1 - \alpha)/2 = (1 - 0.90)/2 = 0.05$).

```r
# load data as vector
y <- c(105, 69, 86, 100, 82, 111, 104, 110, 87, 108, 87, 90, 94, 113, 112, 98,
     80, 97, 95, 111, 114, 89, 95, 126, 98)
```

```r
# capture the number of observations
n <- length(y)

# Calculate the 90% confidence interval for the student IQ
# Step 1: get t-score
t <- qt(0.05, n-1, lower.tail = F)
```

Second, let's calculate the mean ($\bar{y}$), the sample standard deviation S, and then $\hat{\sigma}_{\bar{y}} = \dfrac{S}{\sqrt{n}}$. This allows us to calculate our 90% confidence interval for the student IQ as $\bar{y} \pm T \times \hat{\sigma}_{\bar{y}}$, which equals $98.44 \pm 1.71 \times \frac{13.09}{5} = [93.969, 102.92]$.

```r
lower_CI <- mean(y)-(t*(sd(y)/sqrt(n)))
upper_CI <- mean(y)+(t*(sd(y)/sqrt(n)))

# print CIs with mean
c(lower_CI, mean(y), upper_CI)
```

We can interpret this result by saying that if we took 100 samples, the true population mean of the student IQ in the school is within the interval in 90 of those samples.

Question 2 (25 points)

A private school counselor was curious whether the average of IQ of the students in her school is higher than the average IQ score 100 among all the schools in the country. She took a random sample of 25 students' IQ scores. The following is the data set:

Conduct a test with 0.05 significance level assuming the population of IQ from which our random sample has been selected is normally distributed.

First, let's set up our null hypothesis: we want to know whether the mean of the sample ($\bar{y}$ or $\hat{\mu}$) is greater than the theoretical mean (μ_0). So, using proof by contradiction, $H_0 : \hat{\mu} \leq \mu_0$. Next, let's compute the standard error and our test statistic to get a p-value. Remember, since this is a one-sided test, we don't want both tails, so `lower.tail=F`.

```r
# Step 1: Calculate the standard error
SE <- sd(y)/sqrt(n)
# Step 2: Calculate the test statistic for this hypothesis testing of mean
t <- (mean(y) - 100)/SE
# Get the p-value from t-distribution
pvalue <- pt(t, n-1, lower.tail = F)
```

We can see that the p-value (≈ 0.72) is not equal to or below the $\alpha = 0.05$ threshold, so we would say that we do not find sufficient evidence to reject the null hypothesis that the average IQ of the students in this school is less than or equal to the population average IQ score ($\mu_0 \leq 100$). This makes sense, it's unlikely that we would have enough evidence to suggest that the average in the sample was larger than the population mean given that it is in fact lower.

We can also check our answer by using the `t.test` function in `R`. Note that if you only run this function and do not describe the steps of conducting a hypothesis test, that is not enough for full credit.

```r
# Or another way to do this hypothesis testing is to use the function t.test directly
t.test(y, mu = 100, conf.level = 0.95, alternative = "greater")
```

Question 3 (50 points)

Researchers are curious about what affects the education expenditure on public education. The following is availabe variables in a data set about the education expenditure.

State	*50 states in US*
Y	*per capita expenditure on public education*
X1	*per capita personal income*
X2	*Number of residents per thousand under 18 years of age*
X3	*Number of people per thousand residing in urban areas*
Region	*1=Northeast, 2= North Central, 3= South, 4=West*

(a) *Explore the* ***expenditure*** *data set and import data into* ***R****.*

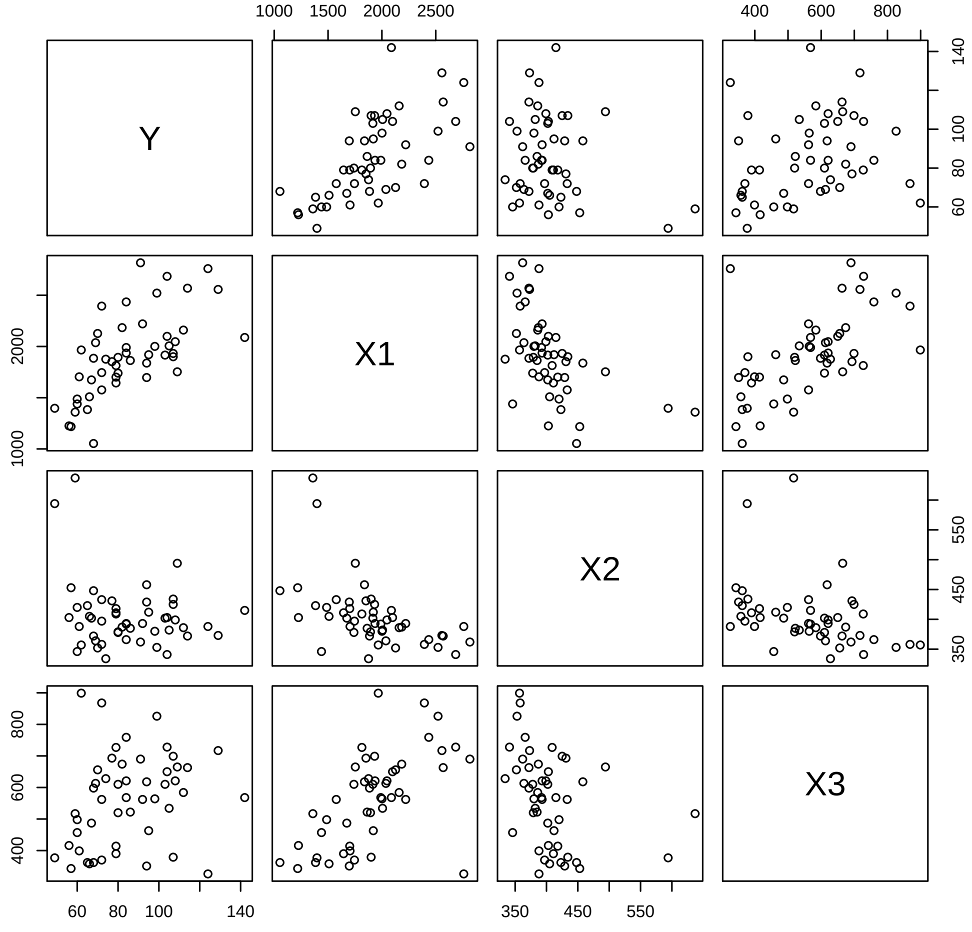
```r
# read in expenditure data
expenditure <- read.table("expenditure.txt", header=T)
# inspect data through summary
summary(expenditure)
```

```
STATE          Y                 X1              X2               X3
AK     : 1   Min.   : 49.00   Min.   :1053   Min.   :334.0   Min.   :326.0
AL     : 1   1st Qu.: 68.25   1st Qu.:1698   1st Qu.:374.2   1st Qu.:426.2
AR     : 1   Median : 81.00   Median :1897   Median :395.0   Median :568.0
AZ     : 1   Mean   : 85.04   Mean   :1912   Mean   :404.7   Mean   :561.7
CA     : 1   3rd Qu.:102.00   3rd Qu.:2096   3rd Qu.:419.5   3rd Qu.:661.2
CO     : 1   Max.   :142.00   Max.   :2817   Max.   :637.0   Max.   :899.0
```

(b) *Please plot the relationships among* Y, X1, X2, *and* X3*? What are the correlations among them (you just need to describe the graph and the relationships among them)?*

```r
# create a matrix scatter plot to
# visualize the relationship among Y, X1, X2 and X3
# so not the first column of expenditure
pdf(”answer_key/plot_3b.pdf”)
pairs(expenditure[,2:5], main = ””)
dev.off()
```

Figure 1: Scatterplot of relationship between Y, X_1, X_2, and X_3.



The correlation (r) between Y and X_1 is 0.649, which indicates a moderate correlation and is consistent with the two subplots in the top-left of Figure 1. However, the correlation between Y and $X2$, and Y and $X3$, are weak (-0.21, 0.25).

(c) *Please plot the relationship between* Y *and* Region*? On average, which region has the highest per capita expenditure on public education?*

```
# generate boxplot with comparisons for different values of Region
pdf(”answer_key/plot_3c.pdf”)
boxplot(expenditure$Y~expenditure$Region, xlab=”Region”, ylab=”Y”, main=”
    ”)
dev.off()
```

Figure 2: Boxplot of Y by *Region*.

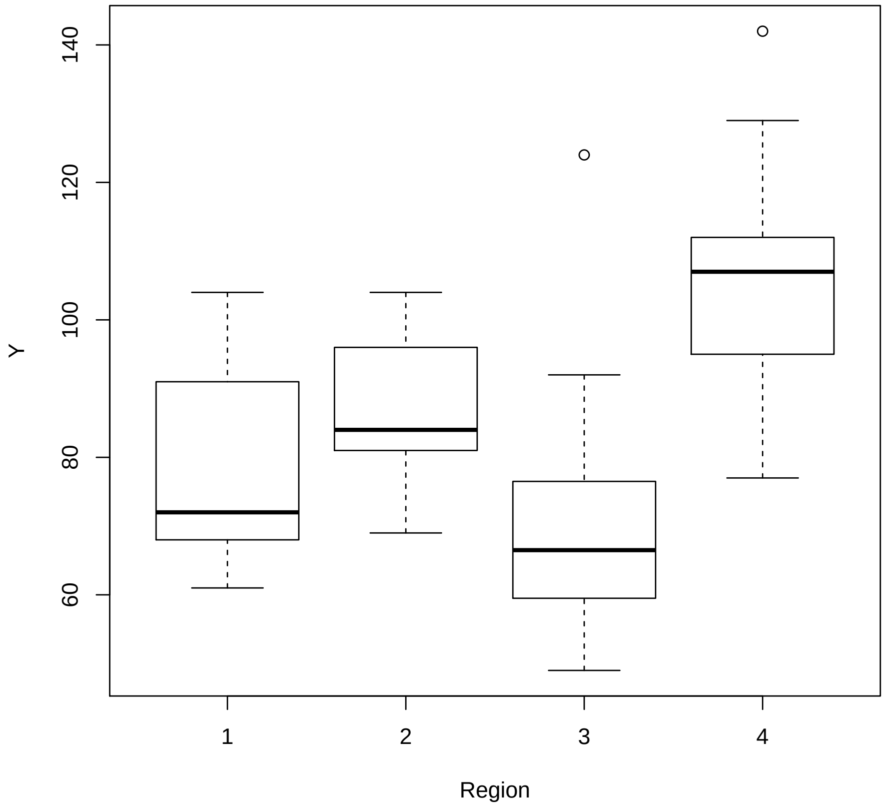


Figure 2 displays the box-plot of Y by *Region* side-by-side, which is appropriate because *Region* is a categorical variable and Y is a quantitative variable. The above code generates a side-by-side box-plot for the variables Y and *Region*. From Figure 2, we can see that Region 4 has the highest per capita expenditure on public education.

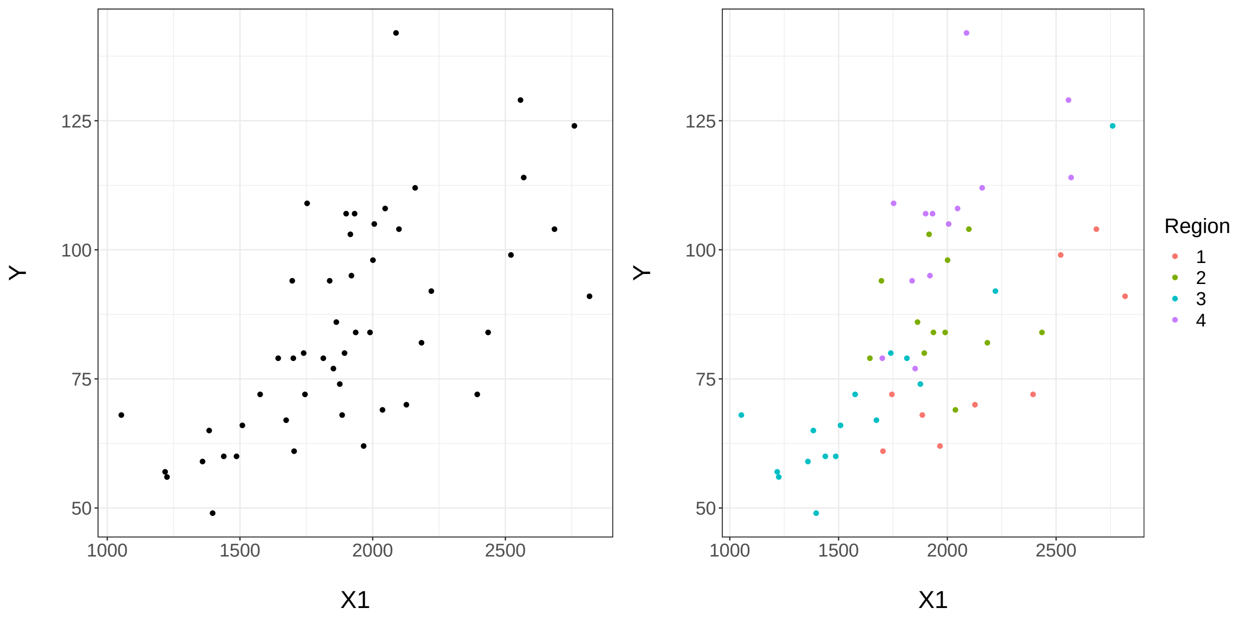
(d) *Please plot the relationship between* Y *and* X1*? Describe this graph and the relationship. Reproduce the above graph including one more variable* Region *and display different regions with different types of symbols and colors.*

```
# create scatterplot of Y and X1
# basic and then differentiate color by region
pdf(”answer_key/plot_3d1.pdf”)
ggplot(expenditure, aes(x=X1, y=Y)) +
  geom_point() + labs(y=”Y\n”, x=”\nX1”) +
```

```
expenditure$Region <- as.factor(expenditure$Region)

pdf(”answer_key/plot_3d2.pdf”)
ggplot(expenditure, aes(x=X1, y=Y, colour=Region)) +
  geom_point()  +labs(y=”Y\n”, x=”\nX1”) +
  theme_bw() +
```

Figure 3: Relationship between X_1 and Y.



(a) Base scatter plot.

(b) Differentiating between region by color and shape.

We're using a scatter plot because both of these variables are quantitative, and we can see from Figure 3 that there is a moderate positive linear correlation between X_1 and Y (which we noted in the above question). However, we can see in the right plot of Figure 3 that certain regions have much steeper (higher) or flatter (lower) correlations between Y and X_1, which suggests that the effect of X_1 of Y differs by region.