MSc Data Science & Artificial Intelligence - AIEDA323

Biomedical Signal Processing

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Introduction

Course organization

- 4 lectures
- 4 computer lab sessions
- Course material sur Moodle: **Biomedical Signal Processing - AIEDA323**
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Resources

- Reference books
  - [RAN02] Rangayyan, *Biomedical Signal Analysis*, IEEE Press, 2002
  - [OPP89] Oppenheim, Schafer, *Discrete-time Signal Processing*, Prentice-Hall, 1989
  - [HAY96] Hayes, *Statistical Digital Signal Processing and Modeling*, John Wiley, 1996
- Course slides
- Lab guide

Evaluation

- Written reports of computer labs

Goals and organization

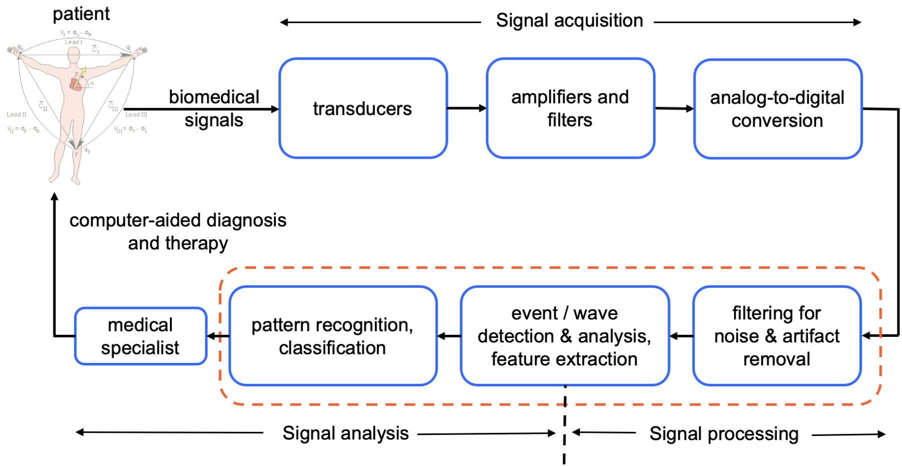
Course objectives

- Understand the genesis of **biomedical signals** — a.k.a. *biosignals* — issued from
  - different organs (heart, brain, muscles, nerves) and pathophysiological processes
  - different modalities (electric / magnetic / acoustic, invasive / noninvasive, etc.)
- Recognize the need for **biomedical signal processing (BSP)** to
  - remove noise and artifacts from observed records
  - extract useful information for monitoring, diagnosis, prognosis, therapy selection
  - model physiological phenomena
- Get acquainted with basic and advanced **BSP techniques** for
  - spectral (frequency-domain) characterization and filtering
  - optimal spatial filtering
  - blind source separation
- **Implement and apply** BSP techniques on real signals using scientific computing software
- Acknowledge that BSP is a key ingredient of **explainable AI for health**

Syllabus

1. Introduction to biosignals and BSP (O. Meste, 1 lecture + 1 lab)
2. Spectral analysis (1 lecture + 1 lab)
3. Optimal spatial filtering (1 lecture + 1 lab)
4. Blind source separation (1 lecture + 1 lab)

Biomedical signal processing and analysis workflow

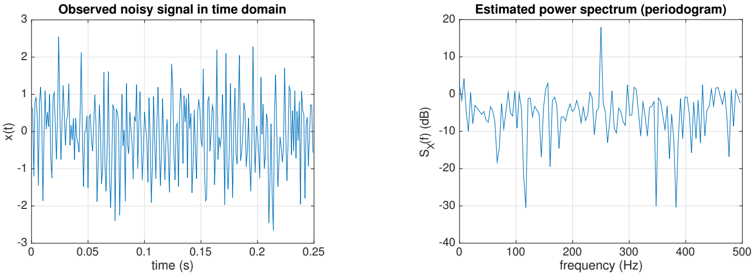


Spectral analysis and filtering

Spectral analysis — motivating examples (1/2)

Finding harmonic structure

Spectral analysis can often reveal repetitive or periodic components that are otherwise hidden in the time-domain representation of noisy signals.



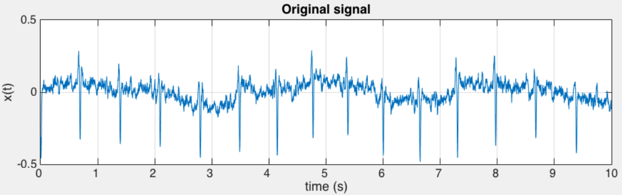
- [Left plot] Noisy data in time domain seem to lack 'interesting' components.
- [Right plot] PSD reveals periodic component at $f_0 = 250$ Hz.

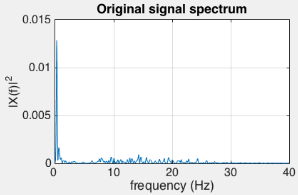
Spectral analysis — motivating examples (2/2)

Artifact cancellation in biomedical data

- Electrocardiogram (ECG) records are often corrupted by noise and artifacts.
- Spectral estimation allows the identification of corrupted frequency bands.
- Optimal frequency filters can be designed for artifact cancellation and signal enhancement.

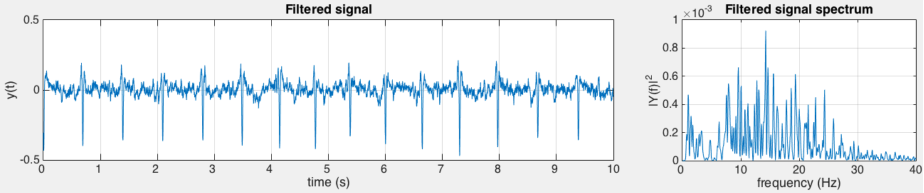
Original signal: ECG record corrupted by baseline wandering and high-frequency noise



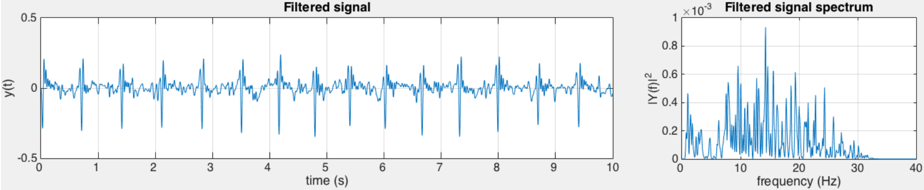


Spectral analysis — motivating examples (2/2, cont'd)

Highpass filtering ($f_c = 0.5$ Hz) $\rightarrow$ baseline wandering removal



Lowpass filtering ($f_c = 30$ Hz) $\rightarrow$ high-frequency noise suppression



Spectral analysis (or estimation)

Definition

From a finite record of a stationary data sequence, estimate how the signal power is distributed over frequency — i.e., find its **power spectral density (PSD)**.

Useful in many application domains

- **Medicine**: physiological data analysis (electrocardiogram, electroencephalogram, ...).
- **Mechanics**: vibration monitoring, fault detection.
- **Astronomy, finance**: hidden periodicity finding.
- **Speech and audio processing**: speech recognition, audio compression, music recognition.
- **Seismology**: earthquake analysis, focus localization, tremor prediction.
- **Control systems**: dynamic behavior analysis, controller synthesis.

Spectral analysis is a fundamental tool

for the electrical engineer and the data scientist.

Continuous-time (analog) signals

Most signals — including biosignals — can be mathematically described as a function of continuous time, i.e., they are **analog signals**:

$$x(t), \quad t \in \mathbb{R}.$$

Continuous-time Fourier transform (FT)

$$X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

$$x(t) = \mathcal{F}^{-1}\{X(f)\} = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df$$

where j represents the imaginary unit: $j^2 = -1$.

- $x(t)$ and $X(f)$ are called *FT pairs*:

$$x(t) \xleftrightarrow{\mathcal{F}} X(f)$$

- The FT is a key tool in signal processing. It provides an alternative representation of the signal, emphasizing its periodic (harmonic) components.

From continuous time to discrete time

To be processed by digital means, analog signals must be transformed into **digital signals** through time sampling and quantization.

- **Time sampling**:

$$x[n] = x(nT_s), \quad n \in \mathbb{Z}$$

T_s: sampling period (seconds); $F_s \stackrel{\text{def}}{=} \frac{1}{T_s}$: sampling frequency (samples/second or Hz)

- **Quantization**: signal values are restricted to a discrete set.

A discrete-time signal $x[n]$, $n \in \mathbb{Z}$, is also called a *sequence*.

Example 1: Regular sampling of $x(t) = \cos(2\pi F_0 t)$ at sampling rate $F_s = 1/T_s$.

$$x[n] = x(nT_s) \underset{\substack{\uparrow \\ t=nT_s}}{=} \cos(2\pi F_0 n T_s) = \cos(\omega_0 n), \quad \omega_0 \stackrel{\text{def}}{=} \frac{2\pi F_0}{F_s}$$

ω_0: radians/sample

Some usual sequences

- **Unit sample sequence**

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Any sequence $x[n]$ can be expressed as the *convolution*

$$x[n] = x[n] * \delta[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{1}$$

- **Unit step sequence**

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

- **Exponential sequence**

$$x[n] = Ae^{\alpha n}$$

Noting $A = |A|e^{j\phi}$ and $\alpha = |\alpha|e^{j\omega_0}$ $\Rightarrow$ $x[n] = |A||\alpha|^n e^{j(\omega_0 n + \phi)}$

- If $|\alpha| = 1$: **complex exponential sequence**

$$x[n] = |A|e^{j(\omega_0 n+\phi)} = |A|\cos(\omega_0 n + \phi) + j|A|\sin(\omega_0 n + \phi)$$

$$|A| : \text{amplitude} \qquad \omega_0 : \text{frequency} \qquad \phi : \text{phase}$$

- $\mathrm{Re}\{x[n]\} = |A|\cos(\omega_0 n + \phi)$: **discrete sinusoid**

Some usual sequences (cont'd)

→ *Exercise 1:* Prove that discrete-time sinusoids are periodic in frequency, i.e., sinusoids with frequency $\omega_0 + 2\pi k$ are identical sequences for any integer $k \in \mathbb{Z}$.

→ *Exercise 2:* Using MATLAB `stem` command, plot in the same figure the discrete sinusoids $\cos(\omega_0 n)$ and $\cos((\omega_0 + 2\pi)n)$, with $\omega_0 = \pi/5$ rad/sample, for $0 \leq n \leq 20$.

Frequency-domain representation of discrete-time signals

Discrete-time Fourier transform (DTFT)

$$X(e^{j\omega}) = \mathcal{F}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}, \quad \omega \in \mathbb{R}$$

$$x[n] = \mathcal{F}^{-1}\{X(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega, \quad n \in \mathbb{Z}$$

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\omega})$$

Properties

- $X(e^{j\omega})$ is a complex-valued function of real-valued argument ω:

$$X(e^{j\omega}) = |X(e^{j\omega})|e^{j\sphericalangle X(e^{j\omega})}$$

$$|X(e^{j\omega})| : \text{magnitude} \qquad\qquad \sphericalangle X(e^{j\omega}) : \text{phase}$$

- $X(e^{j\omega})$ is 2π-periodic: $X(e^{j(\omega+2\pi k)}) = X(e^{j\omega})$, $k \in \mathbb{Z}$ (corollary of [Exercise 1])

  $\rightarrow$ it suffices to specify the DTFT over an interval of length 2π rad/sample, typically $[-\pi, \pi]$

- *Sufficient condition for existence*: $x[n]$ absolutely summable

$$\sum_{n=-\infty}^{\infty} |x[n]| < +\infty \qquad (2)$$

DTFT properties (cont'd)

- *Hermitian symmetry*

$$\text{If } x[n] \in \mathbb{R} \quad \Rightarrow \quad X(e^{j\omega}) = X^*(e^{-j\omega}) \quad \begin{cases} |X(e^{j\omega})| = |X(e^{-j\omega})| \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$$

- *Linearity*:

$$\mathcal{F}\left\{\sum_{p=1}^{P} a_p x_p[n]\right\} = \sum_{p=1}^{P} a_p X_p(e^{j\omega}), \quad \text{with } X_p(e^{j\omega}) = \mathcal{F}\{x_p[n]\}$$

- *Time shift*:

$$\mathcal{F}\{x[n-n_0]\} = X(e^{j\omega})e^{-j\omega n_0}$$

- *Modulation*:

$$\mathcal{F}\{x[n]e^{j\omega_0 n}\} = X(e^{j(\omega-\omega_0)})$$

- *Convolution*:

$$\mathcal{F}\{x[n] * y[n]\} = X(e^{j\omega})Y(e^{j\omega})$$

- *Product*:

$$\mathcal{F}\{x[n]y[n]\} = X(e^{j\omega}) * Y(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$$

- *Parseval's theorem*:

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \int_{-1/2}^{1/2} |X(e^{j2\pi f})|^2 df \qquad (3)$$

Some discrete-time Fourier transform pairs

$x[n]$	$X(\omega), \quad \|\omega\| < \pi$	$X(f), \quad \|f\| < 1/2$
1	$2\pi\delta(\omega)$	$\delta(f)$
$e^{j\omega_0 n}$	$2\pi\delta(\omega - \omega_0)$	$\delta(f - f_0)$
$\cos(\omega_0 n)$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
$\delta[n]$	1	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$	$e^{-j2\pi f n_0}$
$a^n u[n], \ \|a\| < 1$	$\frac{1}{1-ae^{-j\omega}}$	$\frac{1}{1-ae^{-j2\pi f}}$

Remark

- $\omega = 2\pi f \ \Rightarrow \ \delta(\omega) = \frac{1}{2\pi}\delta(f)$

$\rightarrow$ *Exercise 3:* Prove the last row of the above table.

Some discrete-time Fourier transform pairs (cont'd)

→ *Exercise 4:* Compute the DTFT of the length-N *rectangular window*

$$w_N[n] \stackrel{\text{def}}{=} \begin{cases} 1, & 0 \leq n \leq (N-1) \\ 0, & \text{elsewhere} \end{cases}$$

Hint: $1 - e^{-a} = e^{-a/2}(e^{a/2} - e^{-a/2})$, for any $a \in \mathbb{C}$.

→ *Exercise 5:* Using MATLAB commands `linspace`, `abs` and `plot`, plot the DTFT magnitude (absolute value) of the 64-point rectangular window $w_{64}[n]$ in 1000 equispaced points of the interval $[-\pi, \pi]$ rad/sample. Compare the theoretical expression found in the previous exercise and the output of the `freqz` command. Label the plot axes with `xlabel` and `ylabel`.