EEG/MEG source localisation

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Application of ML to MRI, electrophysiology and brain computer interfaces

Theoretical results / Practice

- Ill-posed problem
  - Non existence.
  - Non uniqueness $\rightarrow$ Silent sources.
  - Non continuity.
- Several cases where uniqueness can be proved.
  - Linear combination of isolated dipoles.
  - Surfacic distribution (up to a constant).
- This is with continuous measurements.
  In practice, we only have a finite number of them.

Measurement model

$$\mathrm{M} = \boldsymbol{\Sigma}\, \mathbf{G}(\mathbf{r}_{\mathrm{i}})\ \mathbf{J}_{\mathrm{i}} + \boldsymbol{\varepsilon}$$

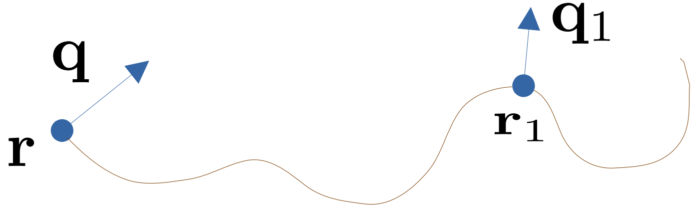
$$\mathrm{M} = \mathbf{G}\,\mathbf{J} + \boldsymbol{\varepsilon}$$

Source models (J)

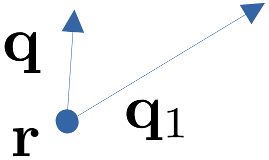
- Continuous vs isolated dipoles.
  We can model continuous distributions over a surface or a volume or just keep a finite number of single dipoles.
- Decrease number of parameters (often needed).
  - Known location
  - Cortical patches.
  - Constrain moments.

Source models (constraining moments)

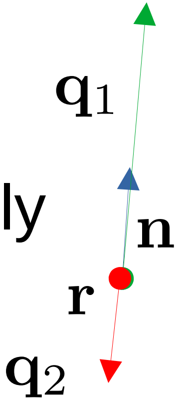
- Moving dipole: position and moment can change (6 parameters $\mathbf{r}$, $\mathbf{q}$).



- Rotation dipole: position is fixed only the moment can change (3 parameters).



- Fixed dipole: position and moment direction are fixed, only the strength of the moment can change (1 parameter).



$$\mathbf{q} = \lambda \mathbf{n}$$

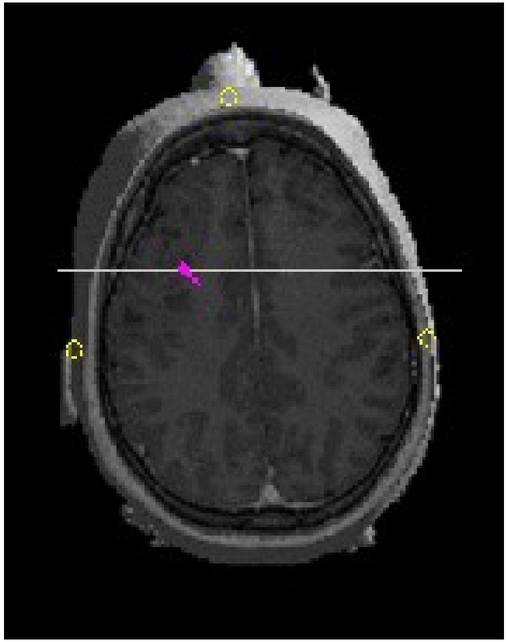
Dipole fit

- Find the dipole(s) position(s) and moment(s) that best fit the measurements.

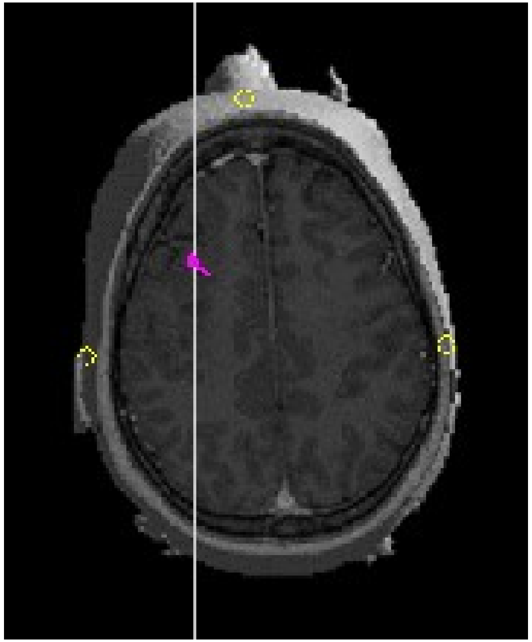
$$\mathbf{J}_{\mathrm{sol}} = \underset{\text{dipole(s) } \mathbf{J}}{\text{minimize}} \; \|\mathbf{M} - \mathbf{G}\,\mathbf{J}\|_F^2$$

- Works when the number of (isolated) dipoles is low.
- Non-linear problem (in position), linear (in moment) → Solved by gradient descent.

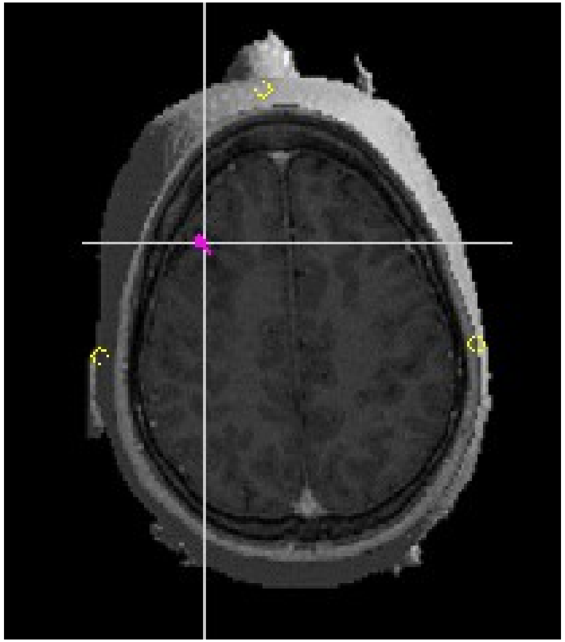
Dipole fit (example of solution)



$\sigma_{\mathrm{scalp}}/\sigma_{\mathrm{skull}} = 20$



$\sigma_{\mathrm{scalp}}/\sigma_{\mathrm{skull}} = 40$



$\sigma_{\mathrm{scalp}}/\sigma_{\mathrm{skull}} = 80$

Dipole fit

Advantages

- Very simple method.
- No assumption on dipole positions.

Drawbacks

- Depends on initialization.
- More complex when the number of dipole increases.
- Choice of the right number of dipoles ?
- Local minima.

Imaging method

- Opposite view of dipole fit.
- Place dipole everywhere and evaluate their strengths.
- Very often used with “Fixed dipole paradigm”.
- Add regularization to remove “spurious” solutions.

Imaging method

Data attachment

$$C_\lambda(\mathbf{J}) = \|\mathbf{M} - \mathbf{G}\,\mathbf{J}\|^2 + \lambda\|\mathbf{J}\|^2 \ .$$

$$\mathbf{J}_{\mathrm{sol}} = \mathbf{J}_\lambda = \underset{\mathbf{J}}{\text{minimize}}\ \mathrm{C}_\lambda(\mathbf{J})$$

Smoothness / regularisation.

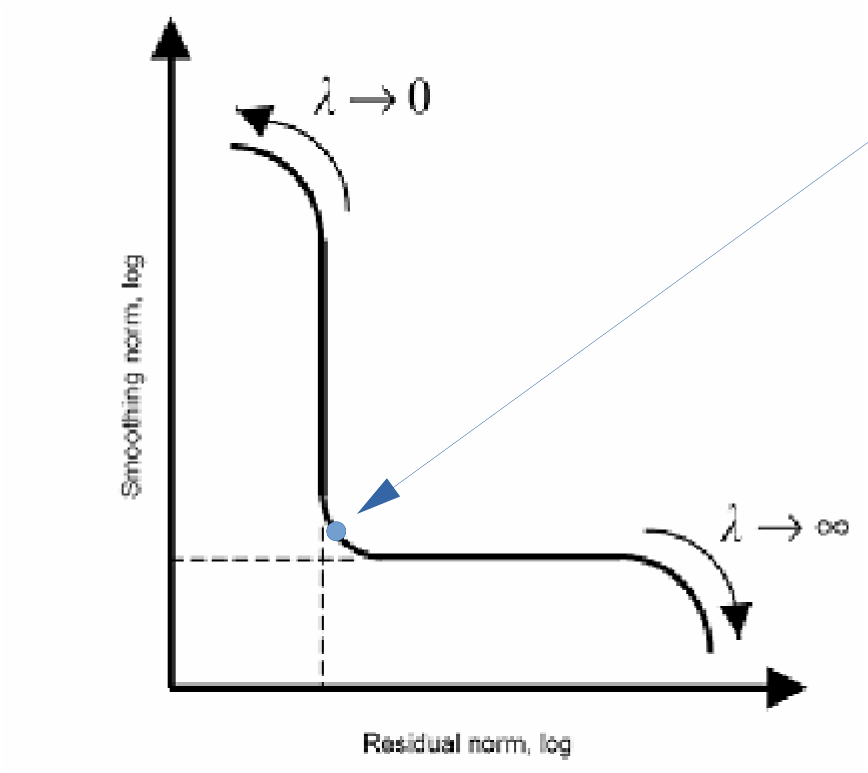
Solution:

$$\begin{aligned}\mathbf{J}_\lambda &= \left(\mathbf{G}^T\,\mathbf{G} + \lambda\,\mathbf{I}\right)^{-1}\mathbf{G}^T M \\ &= \mathbf{G}^T\left(\mathbf{G}\,\mathbf{G}^T + \lambda\,\mathbf{I}\right)^{-1} M\end{aligned}$$

More efficient.

Imaging method (L-curve)

How to find a proper value for λ.



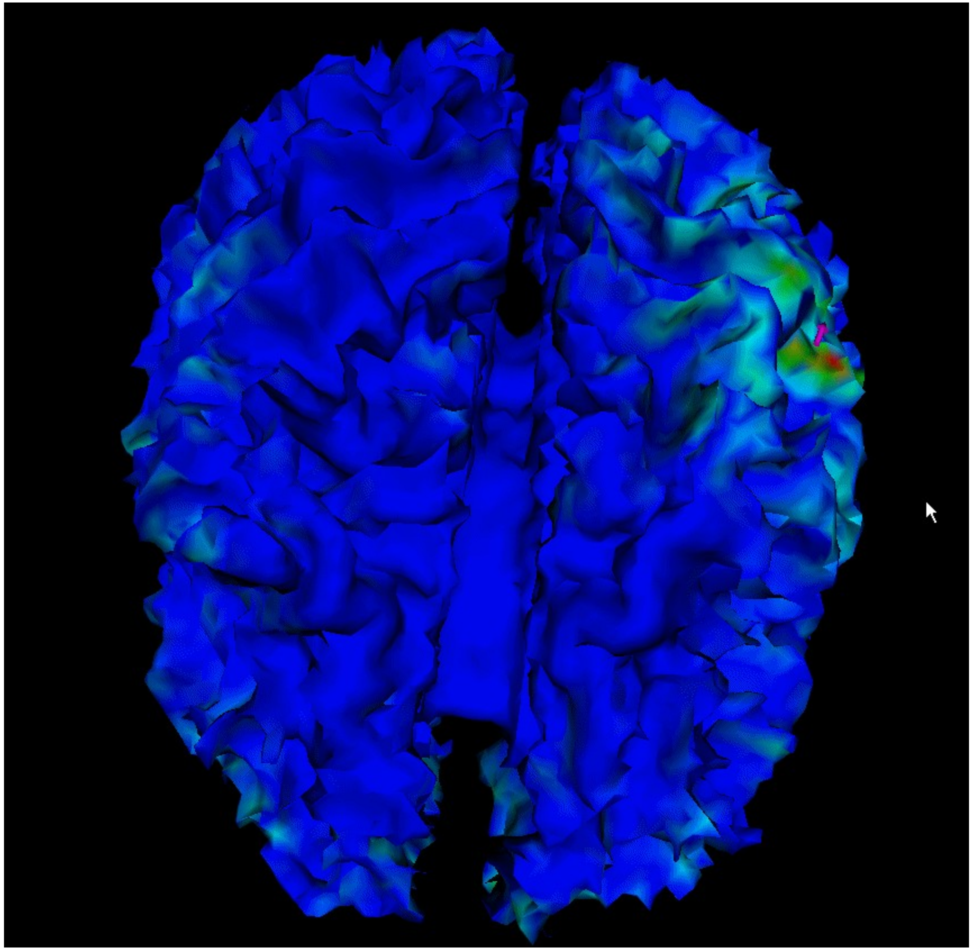
The best compromise between “smoothness” and “data attachment”.

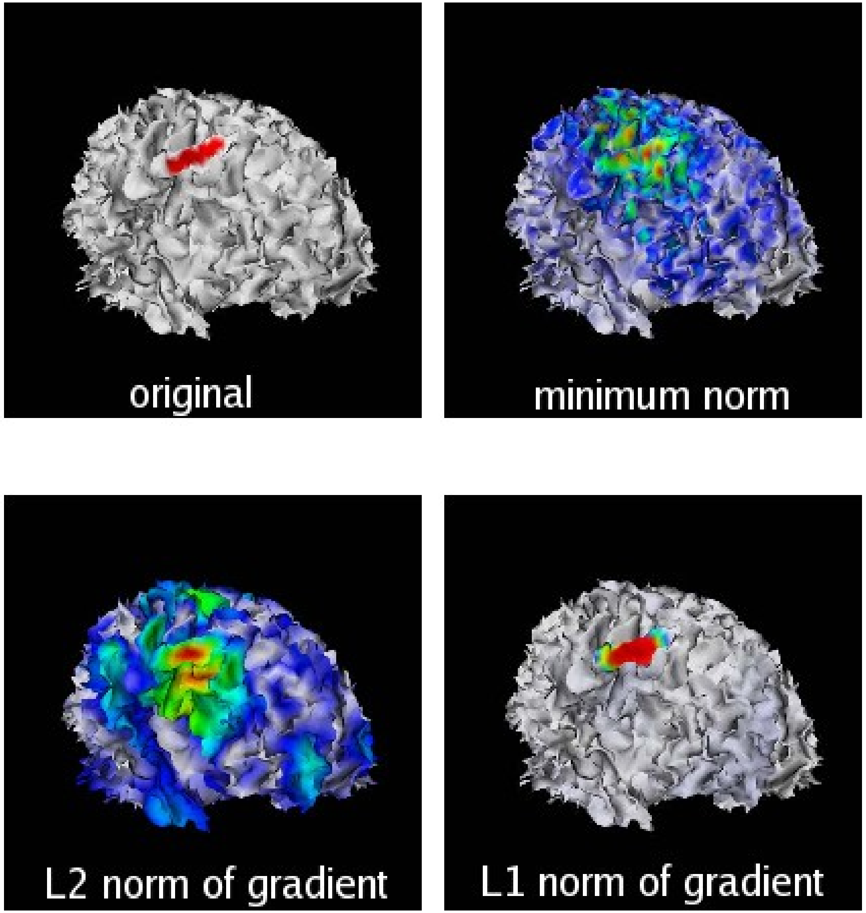
Imaging method (Leave one out)

How to find a proper value for λ.

- With multiple trials for the same task.
- Keep one sample as a test-set.
  Use the others for finding the solution $\mathbf{J}_{\lambda}$.
- Select the value of λ that minimizes the reconstruction error (data attachment) over all choices of the "leaved out" sample.

Imaging method (examples of solution)





Simulated data (10% of noise).

Imaging methods

Advantages

- Very simple method.
- Problem with unique solution.
- No need to choose a number of dipoles.
- Efficient (closed-form) solutions.

Drawbacks

- Depends on the regularization parameter.
- Complex solution which has to be interpreted by a human.
- Exploration of the solution.

Scanning method

Intermediate between “Moving dipole” and “Imaging methods”:

- As in moving dipole, assume a limited number of dipoles (choice of this number).
- As in imaging methods, possible dipole positions a fixed a priori.

<u>Selection of columns (positions) in the leadfield matrix.</u>

<u>Spatio-temporal methods.</u>

- Two families: MUSIC and beamformers/LCMV (Linear Constrained Minimum Variance).

Scanning method: MUSIC

MUltiple SIgnal Classification

- **Gain matrix assumption:** The $\mathbf{G}$ matrix for p dipoles is full rank (i.e. of rank r).

Source k

- **Asynchronous assumption:** The correlation matrix $R_{\mathbf{Q}} = E(\mathbf{Q}_k \mathbf{Q}_k^T)$ matrix for p dipoles is full rank (i.e. of rank r).

- **Noise whiteness assumption:** The noise is considered additive and temporally and spatially zero-mean white noise with variance σ^2. When a good noise model can be established, a prewhitening phase ensures that this is the case. Additionally the signal and noise are assumed to be uncorrelated.

Scanning method: MUSIC

Compute the matrix $\mathbf{F} = E(\mathbf{M}_k \, \mathbf{M}_k^T)$ or $\mathbf{F} \approx E(\mathbf{M}_k)\, E(\mathbf{M}_k)^T$.

Data for trial k.

Find the eigenvectors $\mathbf{U}$ of $\mathbf{F}$.

Split $\mathbf{U}$ between signal and noise spaces $\mathbf{U} = [\mathbf{U}_r, \mathbf{U}_{m-r}]$.

$$C(x_i) = \frac{\|\mathbf{U}_{m-r}^T \mathbf{G}_i\|}{\|\mathbf{G}_i\|} = \frac{\|P_{\mathbf{U}_r}^{\perp} \mathbf{G}_i\|}{\|\mathbf{G}_i\|}$$

Find the position(s) x_i (corresponding to $\mathbf{G}_i$) that minimize(s) the projection $C(x_i)$ of the measurements on the noise space (i.e. maximize the contribution in the signal space).

Scanning method: MUSIC

Find the position(s) x_i (corresponding to $\mathbf{G}_i$) that minimize(s) the projection $C(x_i)$ of the measurements on the noise space (i.e. maximize the contribution in the signal space).

- Extract the first p maxima (Standard MUSIC).
  $\rightarrow$ Problem: close sources often explain the same signal.

- Greedy approach (RAP-MUSIC):
  1. Extract the biggest maximum.
  2. Remove the contribution of that source to the signal.
  3. Re-apply MUSIC on this new signal (p times) to succesive sources.

- Many other variants (TRAP-MUSIC)...

Scanning method: Beamformers

- the noise $\mathbf{N}$ is zero-mean, with covariance C_N.;
- the sources are decorrelated: if $i \neq k$, $E\left([J(x_i) - \overline{J(x_i)}]\,[J(x_k) - \overline{J(x_k)}]^T\right)$ is the 3×3 null matrix;
- the noise and the source amplitudes are decorrelated

Scanning method: Beamformers

Similar ideas as with MUSIC but for the criterion:

$$\widehat{VarJ(x_0)} = \frac{Tr\left((G(x_0)^T C_{\mathbf{M}}^{-1} G(x_0))^{-1}\right)}{Tr\left((G(x_0)^T C_{\mathbf{N}}^{-1} G(x_0))^{-1}\right)} \; .$$

$J(x_0)$ is the reconstructed source by applying a filter $W(x_0)$ to the data.

The concept behind beamforming is, for a given spatial position x_0, to apply a spatial filtering to the measurements, which filters out sources which do not come from a small volume around x_0. Let $W(x_0)$ be a $m \times 3$ matrix representing the spatial filter: the source amplitude in the vicinity of x_0 will be estimated by

$$S(x_0) = W(x_0)^T \mathbf{M} \; .$$

$W(x_0)$ is computed to minimize the strength of the reconstructed source under the constraint that $W(x_0)\mathbf{G}(x_0)^T = \mathbf{I}$.