Problem Set 2

Faisal Jayousi

Helpful proposition

Proposition 1. *Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ a convex separable function i.e.*

$$f(x) = \sum_{i=1}^{n} f_i(x_i) \tag{1}$$

where $f_i : \mathbb{R} \to \mathbb{R} \cup \{+\infty\}$ is proper and convex for any $i = 1, \dots, n$. Then, the proximal operator of f is given by

$$\operatorname{prox}_f(x) = \big(\operatorname{prox}_{f_1}(x_1), \cdots, \operatorname{prox}_{f_n}(x_n)\big). \tag{2}$$

Problem 1

Let $\tau > 0$, and $f \colon \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$. Recall that the proximal operator of τf, $\operatorname{prox}_{\tau f} \colon \mathbb{R}^n \to \mathbb{R}^n$, is given by

$$\operatorname{prox}_{\tau f}(x) = \underset{u \in \mathbb{R}^n}{\operatorname{argmin}} \frac{1}{2\tau} \|u - x\|_2^2 + f(u) \tag{3}$$

1. Does the RHS function admit a minimiser? Is it unique?
2. Let $f \colon \mathbb{R} \to \mathbb{R}$, $x \mapsto |x|$. Compute $\operatorname{prox}_{\tau f}(x)$.
3. Let $g \colon \mathbb{R}^n \to \mathbb{R}$, $x \mapsto \|x\|_1$. Using proposition 1, compute $\operatorname{prox}_{\tau g}(x)$.
4. Plot $\operatorname{prox}_{\tau f}(\cdot)$ and over $[-5, 5]$ with $\tau = 0.7$.
5. Observe the impact of τ on the graph of the function.

Problem 2

Let $\varphi \colon x \mapsto \|x\|_1 + \langle \alpha, x \rangle + \beta$ where $x, \alpha \in \mathbb{R}^n$ and $\beta > 0$. Show that

$$\operatorname{prox}_{\tau\varphi}(x) = \operatorname{prox}_{\tau\|\cdot\|_1}(x - \tau\alpha) \tag{4}$$