### **Characteristics of Functions**

1. Fully factor each of the following.

a) 
$$60x^5 - 12x^3$$

b) 
$$x^2 - 3x - 10$$

c) 
$$3x^2 - 75y^2$$

d) 
$$6x^2 - 7x - 5$$

2. Simplify each of the following, state restrictions:

a) 
$$\frac{x^2 + 4x - 12}{x^2 - 4} \times \frac{x^2 + 4x + 4}{x + 6}$$

b) 
$$\frac{4x-4}{x^2-1} + \frac{x+7}{x^2-2x-3}$$

3. Simplify the following. Your answer must be in simplest form.

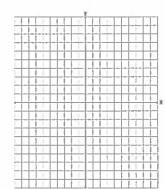
a) 
$$\sqrt{18} + 3\sqrt{8} - 2\sqrt{50}$$

b) 
$$3\sqrt{2}(\sqrt{2} + 9\sqrt{11})$$

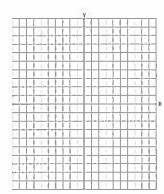
b) 
$$3\sqrt{2}(\sqrt{2} + 9\sqrt{11})$$
 c)  $(\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 4\sqrt{2})$ 

4. State the domain and range of the following functions. Provide a sketch of the given relation.

a) 
$$y = -2(x-3)^2 - 3$$



b) 
$$y = \sqrt{x+3} - 1$$



Domain \_\_\_\_\_

Range \_\_\_\_\_

Function? YES or NO (Circle one)

Domain \_\_\_\_\_

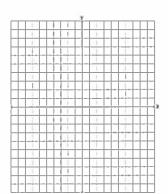
Range \_\_\_\_

Function? YES or NO (Circle one)

5. Determine the coordinates of the intersection point(s) for the following system of equations.

$$y = x^2 - 6x + 1$$

$$y = x - 5$$



6. The following transformations are applied to f(x)

- Stretch vertically by a factor of 8
- Stretch horizontally by a factor of 4
- Reflect in the y-axis.
- Translate 2 units left and 1 unit up

Write g(x) with its transformations.

7.	Describe, in the appropr	iate order, the transformations (example:	HS by 4) that must be applied to
th	e base function, $f(x) = \sqrt{x}$	to obtain the transformed function $g(x)$ .	

$$g(x) = \frac{1}{3}\sqrt{2x-2} + 11$$

- 8. A rocket is shot from a lighthouse. The height, h, in metres, after t seconds is given by  $h(t) = -3t^2 + 18t + 21$ .
  - a) What is the height of the launching pad?
  - b) What is the maximum height reached by the rocket?

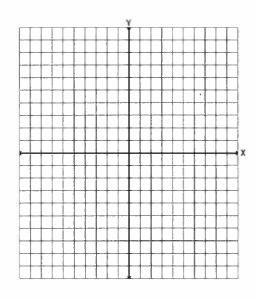
- c) At what time is the maximum height reached?
- d) When does the rocket hit the ground?

e) State the domain and range for this application

9. Find the inverse of the relation  $f(x) = 2x^2 + 4x - 2$ .

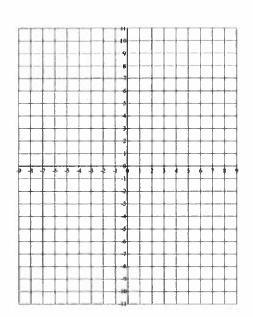
10.a) Given 
$$f(x) = (x+1)^2 - 3$$
,

- b) Graph f(x) and  $f^{-1}(x)$
- c) Determine the equation of  $f^{-1}(x)$



11. Given the table of values of f(x), sketch the graph of g(x) = -3f(2(x-1)) + 4

Points on f(x)	Mapping Rule, new points
(-5,0)	
(-3,2)	
(0,2)	
(1,-1)	
(2,1)	



## **Exponential Functions**

- 1. Evaluate each of the following. Show steps to show your use of the exponent laws
- a) (-5)<sup>2</sup>

- b)  $\frac{5^{-1}}{3^{-2}}$
- c)  $(6^{-2})^2$
- d) 16 4

2. Simplify each of the following. Show steps and your answer should have positive exponents only.

a) 
$$\frac{x^{\frac{2}{5}} \bullet x^{\frac{7}{10}}}{x^{\frac{1}{4}}}$$

b) 
$$(100x)^{\frac{1}{2}} \div (27x^{-2})^{\frac{2}{3}}$$

3. Verify that the tables represent exponential relationships. Find an equation for each set of data.

i)

×	У
0	-2
1	-10
2	-50
3	-250

ii)

х	У
-1	16
0	8
1	4
2	2

	4. The value of a car after it is purchased depreciates according to the formula $V(n) = 26500(0.77)^n$
	where $V(n)$ is the car's value after a number of years, n.
a)	What is the purchase price of the car?
b)	What is the annual rate of depreciation?
c)	What is the car's value at the end of 2 years?
	5. A city has 2 million people living in it in 2005. It experienced an average growth in the population of 8.5% per year.
	<ul> <li>a) Write an equation that models the population, P, in millions, of this country as a function of the number of years, n, since 2005.</li> </ul>
	b) What is the city population in 2027?
	c) Use your equation to determine when the population will double from 2005.
	<ul><li>6. A 7-g sample of radioactive plutonium has a half-life of 41 days.</li><li>a) State an equation to represent the amount of plutonium, A, in grams, that remains after t days.</li></ul>
	b) Determine that amount that remains after 90 days.

# **Trigonometric Functions**

- 1. Determine each exact value. Show all of your steps. (no calculators use special triangles)
- a) cos 120°

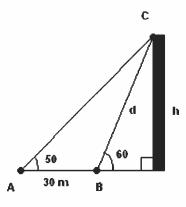
b) cot 225°

- 2. Solve for *two* values of  $\theta$  to the nearest degree where  $0^{\circ} \le \theta \le 360^{\circ}$ .
- a)  $\cos \theta = -0.1683$

c)  $\sec \theta = \sqrt{2}$ 

3. A is an angle in standard position. P(3, -4) is a point on the terminal arm of  $\theta$ . Determine the sin and cot ratios for the angle.

4. Find d and h, showing all your work.



5. In  $\triangle ABC$ ,  $\angle C = 50^{\circ}$ , c = 3.1 cm and b = 3.6 cm. Solve for the **two possible** values for  $\angle B$ . Include diagrams in your answer. (ambiguous case)

6. Prove the identity cosx(cscx + tanx) = cotx + sinx

- 7. The graph of the function  $y = \sin x$  is transformed as described below.
  - Stretch the graph horizontally by a factor of 5.
  - Stretch the graph vertically so the new amplitude of the graph is 3 times of the original amplitude.
  - Reflect the graph in the x axis.
  - Translate the graph 1 units up and shift the graph 45° to the right.

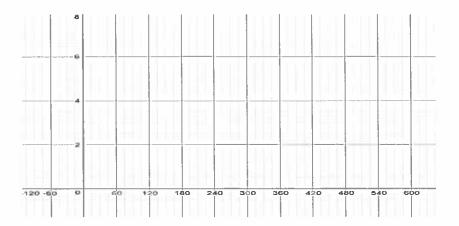
Determine an equation of the transformed function.

8. Complete the following chart for the given functions.

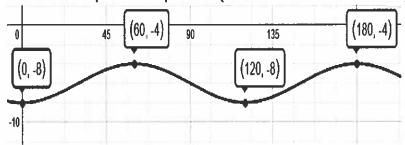
Equation	Amplitude	Period	Phase Shift	Vertical Translation
a) $y = -3\sin(\theta - 80^{\circ}) - 6$				
b) $y = 0.5\cos 2\theta - 60^{\circ} + 1$				

9. Sketch one cycle of  $y = 4 \sin 0.5(\theta + 90^{\circ}) + 3$ 

Key points	Mapping Rule:



10. Find two possible equations (one Sine and one Cosine) for the given function.



11. The blue seat on a ferris wheel begins at the top of the ride is at the height h, in m, at a time in t seconds that follows the equation.  $h(t) = -11 \sin(10t) + 12$ 

Sketch the graph

- a) Max. height
- b) Min. height
- c) Time for one rotation
- d) Radius of the wheel
- e) when the blue seat is at a maxiumum

- 1. Find the general term of the sequence 0, 12, 24....
- 2. Find the general term of 3, -3/2, 3/4, .......
- 3. For the arithmetic series -6 1 + 4 ...139 , determine  $S_n$ .

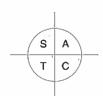
4. For the geometric series 8+32+128 ....., determine S<sub>5</sub>.

5. What is the final amount if you invest \$4444 at 5.3% p/a , compounded quarterly, for 11 years?

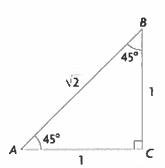
6. How much do you need to invest today so that it grows to \$155000 in 30 years? (5.2% interest per annum, compounded monthly)

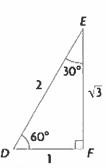
### **FORMULA PAGE**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$













<b>Identities Based on Definitions</b>	Identities Derived from Relation	nships
Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\csc \theta = \frac{1}{\sin \theta}$ , where $\sin \theta \neq 0$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$ , where $\cos \theta \neq 0$	$\sin^2\theta + \cos^2\theta = 1$
$\sec \theta = \frac{1}{\cos \theta}$ , where $\cos \theta \neq 0$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$ , where $\sin \theta \neq 0$	$1 + \tan^2 \theta = \sec^2 \theta$
$\cot \theta = \frac{1}{\tan \theta}$ , where $\tan \theta \neq 0$		$1 + \cot^2 \theta = \csc^2 \theta$

Finding sides: 
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Right triangles: SOHCAHTOA

$$\sin\theta = \frac{y}{r}$$

$$\sin \theta = \frac{y}{r}$$
  $\cos \theta = \frac{x}{r}$   $\tan \theta = \frac{y}{x}$ 

$$\tan\theta = \frac{y}{x}$$

$$t_n = a + d(n-1)$$

$$S_n = \frac{n}{2} [2a + d(n-1)]$$

$$S_n = \frac{n}{2} (a + t_n)$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{t_n \times r - a}{r - 1}$$

$$A = P(1+i)^n$$

$$P = A(1+i)^{-n}$$

### Characteristics of Functions

1. Fully factor each of the following.

a) 
$$60x^5 - 12x^3$$
  
=  $12 \times (5x^2 - 1)$ 

c) 
$$3x^2 - 75y^2$$
  
=  $3(x^2 - 25y^2)$   
=  $3(x+5y)(x-5y)$ 

- b)  $x^2 3x 10$ =(x-5)(x+2)
- d)  $6x^2 7x 5$ = (3x-5)(2x+1) -> 6x2 -6x
- 2. Simplify each of the following, state restrictions:

a) 
$$\frac{x^2 + 4x - 12}{x^2 - 4} \times \frac{x^2 + 4x + 4}{x + 6}$$

$$=\frac{(\chi+0)(\chi-2)}{(\chi+2)(\chi-2)} \times \frac{(\chi+2)(\chi+2)}{(\chi+4)}$$

b) 
$$\frac{4x-4}{x^2-1} + \frac{x+7}{x^2-2x-3}$$

$$= \frac{4(x_{7})}{(x+1)(x_{7})} + \frac{(x+7)}{(x-3)(x+1)}$$

$$= \frac{4}{x+1} + \frac{(x+7)}{(x-3)(x+1)}$$

$$= \frac{4(x-3) + x+7}{(x-3)(x+1)}$$

$$= \frac{4x-12+x+7}{(x-3)(x+1)} - \frac{5x-5}{(x-3)(x+1)}$$

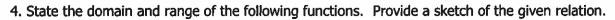
- 3. Simplify the following. Your answer must be in simplest form.
- a)  $\sqrt{18} + 3\sqrt{8} 2\sqrt{50}$ =352 + 652 - 1052

= - 12

- - = 6+27522
- a)  $\sqrt{18} + 3\sqrt{8} 2\sqrt{50}$ b)  $3\sqrt{2}(\sqrt{2} + 9\sqrt{11})$ c)  $(\sqrt{3} 4\sqrt{2})(\sqrt{3} + 4\sqrt{2})$ =  $\sqrt{9} + 4\sqrt{6} 4\sqrt{6} 16\sqrt{4}$

$$=3-16(2)$$

$$= 3 - 32$$
  
=  $-29$ 

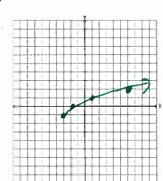


a) 
$$y = -2(x-3)^2 - 3$$
  $\sqrt{(3-3)}$ 

-2,-4,-10 ...

₩ Stepx-2

b) 
$$y = \sqrt{x+3} - 1$$



JX	(x-3,y-1)
(0,0)	(-3, -1)
(111)	(-2.0)
(4,2)	(+1,1)
(9,3)	(6,2)

Domain 5x6R3

Range YES or NO (Circle one)

Domain  $\{x \in R, x \ge -3\}$ Range  $\{y \in R, y \ge -1\}$ Function? (ES) or NO (Circle one)

5. Determine the coordinates of the intersection point(s) for the following system of equations.

$$y = x^2 - 6x + 1$$
  $(x-3)^2 - 8$ 

$$y = x - 5$$



POIs are (6,1) and (1,-4)

x2-6x+1=x-5  $x^{2}-7x+6=0$ (x-6)(x-1)=0 x=6 x=6 x=1 y=6-5 y=1-5 y=-4

- 6. The following transformations are applied to f(x)
  - Stretch vertically by a factor of 8
  - Stretch horizontally by a factor of 4
  - Reflect in the y-axis.
  - Translate 2 units left and 1 unit up

Write g(x) with its transformations.

$$g(x) = 8 f [-4(x+2)] + 1$$



7. Describe, *in the appropriate order*, the transformations (example: HS by 4) that must be applied to the base function,  $f(x) = \sqrt{x}$ , to obtain the transformed function g(x).

$$g(x) = \frac{1}{3}\sqrt{2x-2} + 11$$

Factor & PLX-1)

VC x 3 HC x Z H.T. I up

- 8. A rocket is shot from a lighthouse. The height, h, in metres, after t seconds is given by  $h(t) = -3t^2 + 18t + 21$ .
  - a) What is the height of the launching pad?

b) What is the maximum height reached by the rocket?

$$h(t) = -3(t^2-6t)+21$$

$$= -3(t^2-6t+9-9)+21$$

$$= -3(t-3)^2+27+21$$

$$= -3(t-3)^2+48$$
so max height is 48 m

 $\begin{array}{ccc}
-3t(t-6) \\
-3t(t-6) \\
4 & 4 & 4 \\
+v=3 \\
h(3) = -3(3)^2 + 18(3+2) \\
h(3) = 48
\end{array}$ 

c) At what time is the maximum height reached?

d) When does the rocket hit the ground?

$$0 = -3(t^2 - 6t - 7)$$

$$0 = -3(t - 7)(t + 1)$$

$$6 = 7$$

$$6 = 7$$

or quad formula t= -b + Jb2-4ac 2a

So it hits The grand @ 7 seconds.

e) State the domain and range for this application

9. Find the inverse of the relation  $f(x) = 2x^2 + 4x - 2$ 

$$y = 2(x^{2} + 2x + 1 - 1) - 2$$

$$y = 2(x + 1)^{2} - 1$$

$$x = 2(y+1)^{2} - 4$$
 $\frac{x+4}{2} = (y+1)^{2}$ 
 $\frac{1}{\sqrt{x+4}} = y+1$ 

$$\frac{0}{x^{2}-2(y+1)^{2}-4}$$

$$\frac{x+4}{2}=(y+1)^{2}$$

$$\frac{1}{x^{2}}=y+1$$

- 10.a) Given  $f(x) = (x+1)^2 3$ , Vertex (-1, -3)
  - b) Graph f(x) and f<sup>-1</sup>(x)
  - c) Determine the equation of  $f^{-1}(x)$

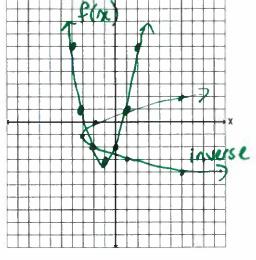
ermine the equation of 
$$f^{-1}(x)$$

$$y = (x+1)^{2} - 3$$

$$x = (y+1)^{2} - 3$$

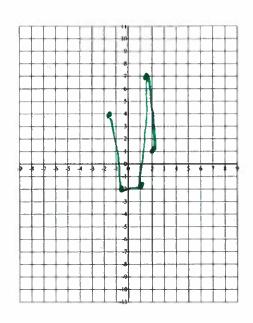
$$x + 3 = (y+1)^{2}$$

$$\pm (x+3) = (y+1)^{2}$$



11. Given the table of values of f(x), sketch the graph of g(x) = -3f(2(x-1)) + 4

Points on f(x)  (x,y)-	Mapping Rule and new points $(\frac{x}{2} + 1, -3y + 1)$
(-5,0)	9 (-1.5, 4)
(-3,2)	7 (-0.5 , -2)
(0,2) -	1 (1, -2)
(1,-1) -	7 (1.5, 7)
(2,1)	2 (1)
94-	



## **Exponential Functions**

1. Evaluate each of the following. Show steps to show your use of the exponent laws

a) 
$$(-5)^2$$
 b)  $\frac{3}{3^{-2}}$ 

$$= 25$$

$$= \frac{3^2}{5}$$

$$= \frac{9}{4}$$

c) 
$$(6^{-2})^2$$
 d)  $16^{\frac{3}{4}}$   
=  $6^{-4}$  =  $\sqrt{10}$   
=  $\frac{1}{1296}$  =  $8$ 

2. Simplify each of the following. Show steps and your answer should have positive exponents only.

a) 
$$\frac{x^{\frac{2}{5}} \cdot x^{\frac{7}{10}}}{x^{\frac{1}{4}}} = \frac{x^{\frac{7}{10}}}{x^{\frac{1}{10}}} = \frac{x^{\frac{7}{10}}}{x^{\frac{1}{10}}} = \frac{x^{\frac{17}{10}}}{x^{\frac{17}{20}}} = \frac{x^{\frac{17}{20}}}{x^{\frac{5}{120}}} = x^{\frac{17}{20}}$$

b) 
$$(100x)^{\frac{1}{2}} \div (27x^{-2})^{\frac{2}{3}}$$

$$= \sqrt{100} \times \sqrt{12}$$

ii)

3. Verify that the tables represent exponential relationships. Find an equation for each set of data.

, <b>x</b>	У	Common lat
0	-2	7:5
1	-10	K: <
2	-50	
3	-250	コノッ う

<b>x</b> .	v	1.
-1	16	5: 112
0 .	8	112
1	4	-
2	2	1): 112

Common Cato

- 4. The value of a car after it is purchased depreciates according to the formula  $V(n) = 26500(0.77)^n$ where V(n) is the car's value after a number of years, n.
- a) What is the purchase price of the car?

$$V(0) = 26500 (0.77)$$

$$= $26500$$
b) What is the annual rate of depreciation?

c) What is the car's value at the end of 2 years?

$$V(2) = 26500(0.77)^{2}$$
  
= \$15 711,85

- 5. A city has 2 million people living in it in 2005. It experienced an average growth in the population of 8.5% per year.
- a) Write an equation that models the population, P, in millions, of this country as a function of the number of years, n, since 2005.

$$P(n) = 2(1.085)^n$$

b) What is the city population in 2027?

$$P(22) = 2(1.085)^{22}$$

$$12 \text{ million people in } 2027$$

c) Use your equation to determine when the population will double from 2005.

$$4 = 2(1.085)^{2}$$
  
 $2 = 1.085^{9}$   
 $\log 2/\log 1.085$   $n = 8.5$  years.

- A 7-g sample of radioactive plutonium has a half-life of 41 days.
  - a) State an equation to represent the amount of plutonium, A, in grams, that remains after t days.

b) Determine that amount that remains after 90 days.

$$A = 7(1/2)^{90/41}$$

1.5g after 90 days

6. It will be.

double in

## **Trigonometric Functions**

1. Determine each exact value. Show all of your steps. (no calculators – use special triangles)

S A T C

a)  $\cos 120^{\circ}$   $\int_{-1}^{2} \cos 120^{\circ} = -\frac{1}{2}$ 

 $\cos \Theta = \frac{\text{adj}}{\text{hyp}}$   $\cos 120^{\circ} = -\frac{1}{2}$ 

tan 45' = 1/1 tan 225° = 1/1 tan 225° = 1/1 cot 225 = 1 cot 9 = adj/opp

Cot 8 = -1/-1 = 1

2. Solve for *two* values of  $\theta$  to the nearest degree where  $0^{\circ} \le \theta \le 360^{\circ}$ .

a)  $\cos \theta = -0.1683$ 

d= cus-1 (0.1683)

cost is neg.

 $Q2 \Theta = 180 - 80$  = 100  $Q3 \Theta = 180 + 80 = 260$ 

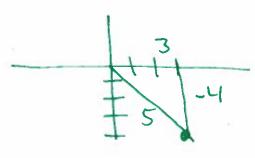
c)  $\sec \theta = \sqrt{2}$   $\cos \theta = \frac{1}{\sqrt{2}}$ 

special triangles & = 45°

600 10 pos in Q 144

Q4: 0=360-45

3. A is an angle in standard position. P(3, -4) is a point on the terminal arm of  $\theta$ . Determine the sin and cot ratios for the angle.

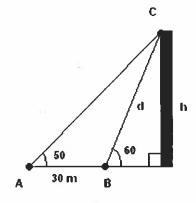


32 +42=12 25=12  $\sin \Theta = \frac{-4}{5}$ 

 $\tan \theta = -\frac{4}{3} \rightarrow \cot \theta = -\frac{3}{4}$ 

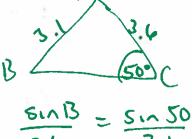
4. Find d and h, showing all your work.

$$d = \frac{30 \sin 50}{\sin 10}$$

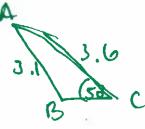


 $h = 114.6 \sim$ 

5. In  $\triangle ABC$ ,  $\angle C = 50^{\circ}$ , c = 3.1 cm and b = 3.6 cm. Solve for the **two possible** values for  $\angle B$ . Include diagrams in your answer. (ambiguous case)



$$3.6$$
  $3.1$   $R = 63^{\circ}$ 



6. Prove the identity cosx(cscx + tanx) = cotx + sinx

COSTX (CSCX+ tanx)

<u>es</u> cotatsing

LS=RS Boidenty 15 true.

- 7. The graph of the function  $y = \sin x$  is transformed as described below.
  - Stretch the graph horizontally by a factor of 5.
  - Stretch the graph vertically so the new amplitude of the graph is 3 times of the original amplitude.
  - Reflect the graph in the x axis.
  - Translate the graph 1 units up and shift the graph 45° to the right.

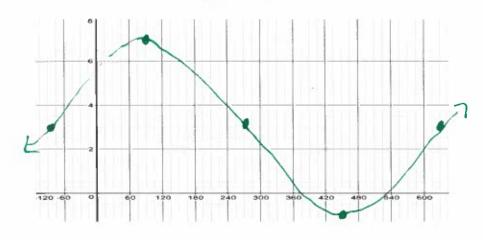
Determine an equation of the transformed function.

8. Complete the following chart for the given functions.

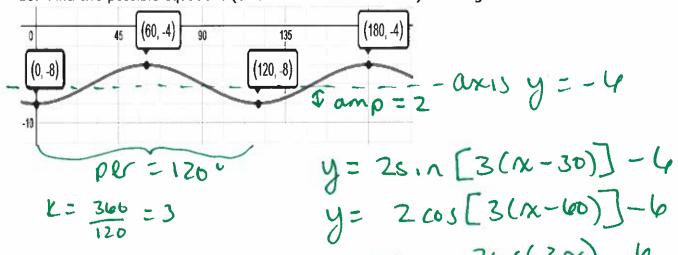
Equation	Amplitude	Period	Phase Shift	Vertical Translation
a) $y = -3\sin(\theta - 80^{\circ}) - 6$	3	360°	80°right	Cedour
b) $y = 0.5\cos 2\theta - 60^{\circ} + 1$	0.2	180	35° right	Lup.

9. Sketch one cycle of  $y = 4\sin 0.5(\theta + 90^\circ) + 3$ 

J. Sketch One cycle	OI .
Key points	(2x-90, 4y+3)
(0,0) -	) (-40,3)
(90,1) -	) (90,7)
(180,0) -	) (270,3)
(270,-1)	> (WOD)
(360,0)	(630, 3)

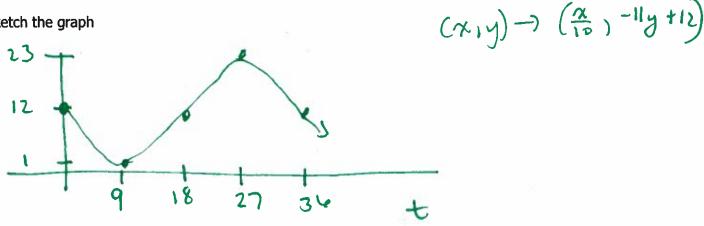


amp= 4 per = 720° axis y=3 10. Find two possible equations (one Sine and one Cosine) for the given function.



or y = - 2605(3x)-6 11. The blue seat on a ferris wheel begins at the top of the ride is at the height h, in m, at a time in t seconds that follows the equation.  $h(t) = -11 \sin 10t + 12$ 

Sketch the graph



a) Max. height

b) Min. height

c) Time for one rotation

d) Radius of the wheel

e) when the blue seat is at a maxiumum

### **Discrete Functions**

1. Find the general term of the sequence 0, 12, 24....

$$t_n = a + d(n-1)$$
  
= 0 + 12(n-1)

2. Find the general term of 3, -3/2, 3/4, .......

$$t_n = ar^{n-1}$$
 $t_n = 3(-1/2)^{n-1}$ 

アニーララ3 ニーランタ ニー3/6=-112

3. For the arithmetic series -6 - 1 + 4  $\dots$ 139 , determine  $S_n$ .

$$t_n = a + (n-1)d$$
 $139 = -6 + (n-1)(5)$ 
 $139 = -6 + 5n-5$ 
 $150 = 5n$ 
 $30 = n$ 

 $S_{30} = \frac{30}{2} \left[ 2(-6) + (30-)(5) \right]$   $S_{30} = 15 \left( -12 + 145 \right)$   $= 15 \left( 133 \right)$   $S_{30} = 1995$ 

4. For the geometric series 8+32+128 ...., determine  $S_5$ .

$$S_5 = 8(1023)$$

$$S_5 = 8(1023)$$

 $S_n = \frac{\alpha(r^{n-1})}{r^{n-1}}$  $S_{\bar{r}} = 2728$ 

5. What is the final amount if you invest \$4444 at 5.3% p/a , compounded quarterly, for 11 years?

05 Final anators 15 \$7930,58

6. How much do you need to invest today so that it grows to \$155000 in 30 years? (5.2% interest per annum, compounded monthly)

$$P = 155 000 (1 + 0.052)$$

So Insest \$ 32 681.05 today.