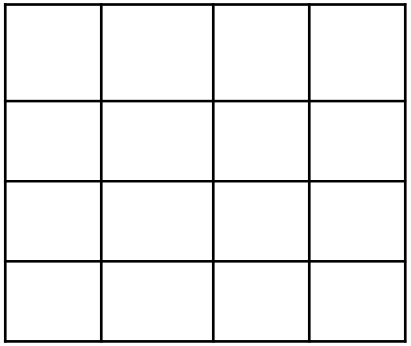
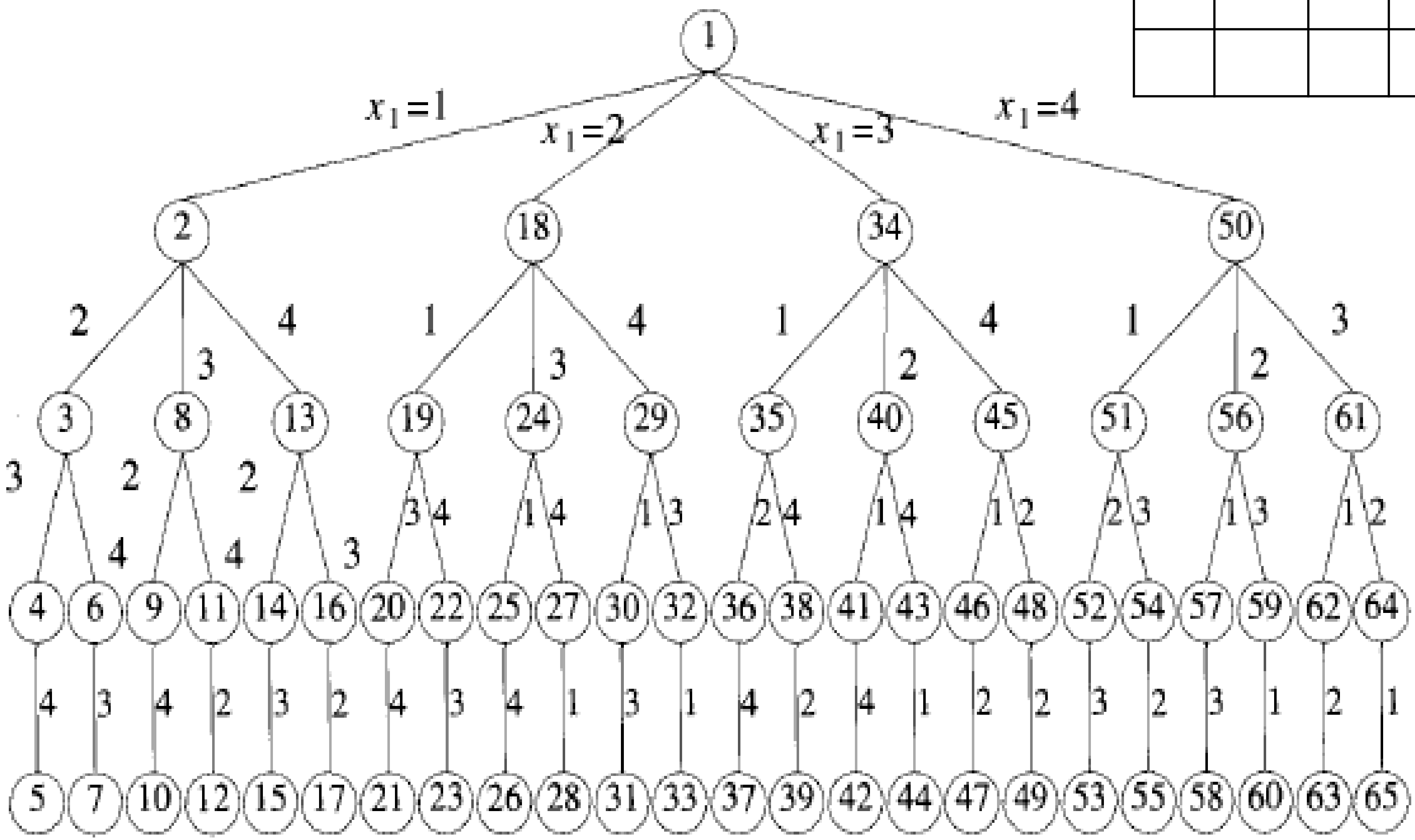
‘N’ Queens Problem

'N' Queens problem

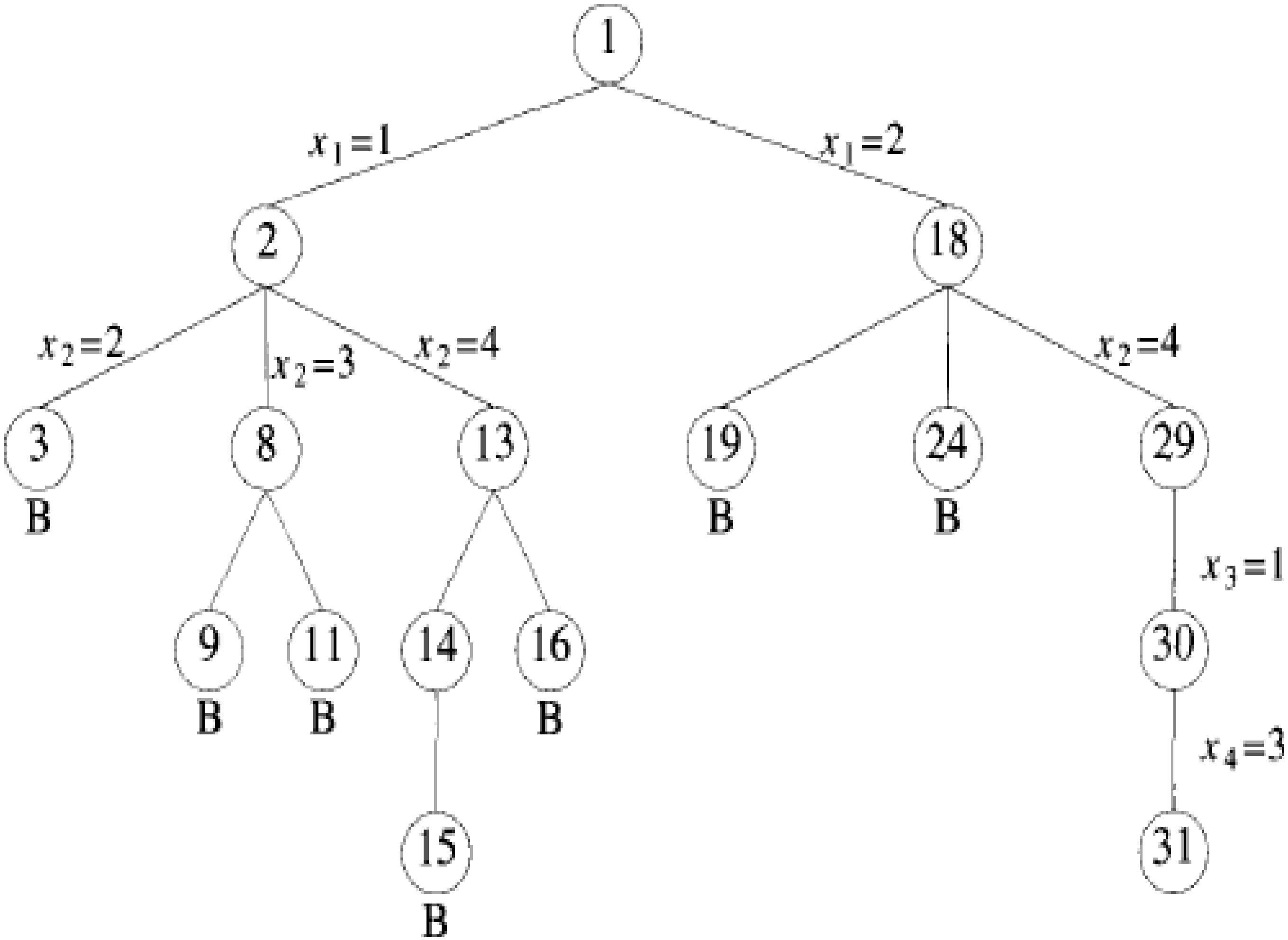
- Now 'n' queens are to be placed on an n x n chessboard so that no two attack; that is, no two queens are on the same row, column, or diagonal. Generalizing the discussion the solution space consists of all n! permutations of the n-tuple(1,2,,n)
- Explicit constraints for this problem that restrict each x_i to take on values only from a given set ie $1 <= x_i <= n$
- The implicit constraints for this problem are that no two x_i's can be the same(i.e. all Queens must be on different columns)and no two queens can be on the same diagonal
- The first of these two constraints implies that all solutions are permutations of the 4-tuple(1,2,3,4).This realization reduces the size of the solution space from 4^4 tuples to 4! tuple

4 Queens problem





4 Queens problem



BACKTRACKING (4 Queens problem)

Example :

1			

1			
.	.	2	

1			
			2

1			
		2	
.	3 .	. .	

	1		

	1		
			2
3 .	,		
		4	

BACKTRACKING (Contd..)

- We start with root node as the only live node. The path is ( ); we generate a child node 2.
- The path is (1).This corresponds to placing queen 1 on column 1 .
- Node 2 becomes the E node. Node 3 is generated and immediately killed. (because $x1=1,x2=2$).
- As node 3 is killed, nodes 4,5,6,7 need not be generated.

The n-queens problem and solution

- In implementing the n – queens problem we imagine the chessboard as a two-dimensional array A (1 : n, 1 : n).
- The condition to test whether two queens, at positions (i, j) and (k, l) are on the same row or column is simply to check i = k or j = l
- The conditions to test whether two queens are on the same diagonal or not are to be found

The n-queens problem and solution contd..

Observe that

i) For the elements in the the upper left to lower Right diagonal, the row - column values are same or row- column = 0, e.g. 1-1=2-2=3-3=4-4=0

(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)
(3,1)	(3,2)	(3,3)	(3,4)
(4,1)	(4,2)	(4,3)	(4,4)

ii) For the elements in the upper right to the lower left diagonal, row + column value is the same e.g. 1+4=2+3=3+2=4+1=5

The n-queens problem and solution contd..

Thus two queens are placed at positions (i, j) and (k, l), then they are on the same diagonal only if

- $i - j = k - l$ or
- $i + j = k + l$

Two queens lie on the same diagonal if and only if

- $|j - l| = |i - k|$

The n-queens problem -Algorithm

```
Algorithm Place(k, i)
// Returns true if a queen can be placed in kth row and
// ith column. Otherwise it returns false. x[ ] is a
// global array whose first (k − 1) values have been set.
// Abs(r) returns the absolute value of r.
{
    for j := 1 to k − 1 do
        if ((x[j] = i) // Two in the same column
            or (Abs(x[j] − i) = Abs(j − k)))
                // or in the same diagonal
            then return false;
    return true;
}
```

The n-queens problem -Algorithm contd..

```
Algorithm NQueens(k, n)
// Using backtracking, this procedure prints all
// possible placements of n queens on an n × n
// chessboard so that they are nonattacking.
{
    for i := 1 to n do
    {
        if Place(k, i) then
        {
            x[k] := i;
            if (k = n) then write (x[1 : n]);
            else NQueens(k + 1, n);
        }
    }
}
```

Time Complexity O(n!)