Question 2

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a) $H = \text{Energy} = \frac{1}{2}mv^2$ $\qquad p(\vec{v}_i) = \dfrac{e^{-\overset{H=\frac{1}{2}mv^2}{E_i}/kT}}{\underbrace{N}_{\text{normalization constant like } Q}}$ which means $\sum_i e^{-E_i/kT}/N = 1$

∴ Because velocities are continuous, take integral

$$\int_{-\infty}^{\infty} e^{-m\overbrace{|\vec{v}|^2}^{|\vec{v}|^2 = v_x^2+v_y^2+v_z^2}/2kT}\, d\vec{v} = N = \int_{-\infty}^{\infty} e^{-mv_x^2/2kT} dv_x \int_{-\infty}^{\infty} e^{-mv_y^2/2kT} dv_y \int_{-\infty}^{\infty} e^{-mv_z^2/2kT} dv_z$$

Use: $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a}$ $\qquad a = m/2kT$

$$\therefore \left(\sqrt{\frac{\pi \cdot 2kT}{m}}\right)^3 = \left(\frac{2\pi kT}{m}\right)^{3/2} N.$$

$$\therefore \boxed{p(\vec{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m|\vec{v}|^2/2kT}}$$

if $p(v_x) = p(v_y) = p(v_z)$

$$\int_{-\infty}^{\infty} e^{-mv_x^2/2kT} dv_x = \int_{-\infty}^{\infty} e^{-mv_y^2/2kT} dv_y = \ldots f$$ which will make $v_x = v_y = v_z$

Thus $\langle \vec{v} \rangle = \langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle$

$$\langle v_x \rangle = \int_{-\infty}^{\infty} p(v_x) \cdot v_x \, dv_x = \int_{-\infty}^{\infty} \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv_x^2/2kT} \cdot v_x \cdot dv_x \Rightarrow a = \frac{m}{2kT}$$

$$= \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{-\infty}^{\infty} e^{-v_x^2 \cdot a} \cdot v_x \, dv_x$$

$u = v_x^2$
$du = 2v_x dv_x$
$du/2 = v_x dv_x$

$$= \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{-\infty}^{\infty} \frac{e^{-u \cdot a}}{2} \cdot du = \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{2}\left[\cancel{e^{-\infty \cdot a}} - \cancel{e^{-\infty a}}\right] = ?$$ goes to infinity?

b) $$\langle v_x^2 \rangle = \int_{-\infty}^{\infty} p(v_x) \cdot v_x^2 = \int_{-\infty}^{\infty} \left(\frac{m}{2\pi kT}\right)^{3/2} \cdot e^{-mv_x^2/2kT} v_x^2 \, dv_x. \qquad a = m/2kT$$

$$= \left(\frac{m}{2\pi kT}\right)^{3/2} \cdot \int_{-\infty}^{\infty} e^{-av_x^2} \cdot v_x^2 \, dv_x$$ as $\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$

$$= \left(\frac{\cancel{m}}{\cancel{2\pi kT}}\right)^{3/2} \cdot \frac{\cancel{2}kT}{\cancel{2} \cdot m} \sqrt{\frac{\pi \cdot 2kT}{\cancel{m}}} = \boxed{\frac{1}{2\pi}}$$