Exam Answers

Student Name

Answers

1. **Question 1(a):**
   Student's Answer:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{array}{cccc} & e_1 & e_2 & e_3 \\ A & 1 & 0 & 0 \\ B & 0 & 1 & 1 \\ C & 0 & 1 & 0 \\ D & 0 & 0 & 1 \end{array}$$

   e_1 connects node A & B
   e_2 connects node B & C
   e_3 connects node B & D
   Constructing a symmetric matrix since the graph is undirected.
   Adjacency Matrix Representation

$$\begin{array}{cccc} & A & B & C \\ D & & & \\ A & 0 & 1 & 0 \\ 0 & & & \\ B & 1 & 0 & 1 \\ 1 & & & \\ C & 0 & 1 & 0 \\ 0 & & & \\ D & 0 & 1 & 0 \\ 0 & & & \end{array} \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

2. **Question 1(b):**
   Student's Answer:
   B2) Erdős–Rényi (Random Network) Model
   In this model, every possible edge between node pairs is formed independently with equal probability p, regardless of other edges, unlike other models.

3. **Question 1(c):**
   Student's Answer:
   C) Nash Equilibrium
   This strategy profile has no player who can improve their outcome by unilaterally changing their strategy, assuming others stay the same.

4. **Question 1(d):**
   Student's Answer:
   B3) Assortative Mixing
   This refers to the preference of nodes to connect with others that are similar based on attributes like degree etc.

5. **Question 1(e):**
   Student's Answer:
   D) Because it quantifies how often a node lies on the shortest path between other nodes.
   Betweenness captures control over communication pathways, making it more useful than raw connection count.

6. **Question 1(f):**
   Student's Answer:
   C) The presence of many nodes with very high degrees (hubs) that maintain connectivity.
   Random failures are unlikely to hit hubs but targeted attacks on them disrupt global connectivity.

7. **Question 1(g):**
   Student's Answer:
   A) The average number of intra-community edges is significantly higher than expected in a random network with the same degree sequence.

8. **Question 1(h):**
   Student's Answer:

   $$X \to \{A, B, C, D\} Y \to \{C, D, E\} \text{Common} = \{C, D\} = 2 \text{Union} = \{A, B, C, D, E\} B) \frac{2}{5}$$

9. **Question 1(i):**
   Student's Answer:
   D) ICM activates nodes based on global network properties, while LTM uses only local information.

10. **Question 1(j):**
    Student's Answer:

B) Because aggregating features from dissimilar neighbors can blur the node's own representative features, making classification harder.
GCN assumes homophily, so in heterophilic graphs, feature aggregation mixes conflicting signals.

11. **Question 2:**
Student's Answer:
Given, only 5% of the population can be vaccinated preemptively.
Nodes = people, edges = contacts
Following an SIR which is Susceptible Infected, Recovered model, the following strategy can be applied together (two distinct SNA concepts)
(i) Degree Centrality: This has the numbers of direct connections a node has. Nodes with many contacts are at highest risk of getting infected early and can become super contagious.
Vaccinating them reduces the reproduction rate (R_0) and slows early transmission.
After selecting high-betweenness nodes, use remaining vaccine quota on nodes with the highest degrees.
Whenever an overlap occurs i.e., a node has both high betweenness and degree it prioritizes more.
Hence, the top 5% of nodes with the highest degree centrality should be identified.
(ii) Betweenness Centrality: This quantifies how often a node lies on the shortest path between other pairs of nodes in the network.
Nodes with high betweenness are often bridges between communities or regions in the network.
If the infection passes through such a node, it can spread to entire new subgroups.
By vaccinating these nodes, we can break critical transmission paths, especially inter-community spread.
Hence, computing betweenness centrality scores for all nodes, and vaccinating the top 2-3% of nodes with highest scores can give optimum results.
Combining both will give us targeted immunity in both early stage super contagious people and network bottlenecks.
Hence this approach maximizes impact by blocking critical transmission paths and high contact individuals by minimizing infection under limited resources.

12. **Question 3:**
Student's Answer:

Task: Improve "suggested collaborators" feature on a platform for academic researchers.
Link Prediction Algorithm: This algorithm tries to estimate the likelihood of a future link (i.e., collaboration) between two unconnected nodes based on several features such as network structure, node similarity & common neighbors etc.
By applying link prediction algorithms such as Jaccard Coefficient, Adamic-Adar index, preferential attachment etc can help identify pairs of researchers who
(i) share mutual co-authors
(ii) work in overlapping subfields
are more likely to collaborate in the future.
Node Embedding Technique: (Node2Vec)
This is an algorithm that learns vector representations of nodes using random walks through the network.
It captures both local & global structure, latent relationship between researchers.
This transforms researchers into dense vector based on their coauthorship and citation content.
This also allows one to measure some similarity or Euclidian distance between researchers embeddings.
It also combines structural and semantic features for better recommendations.
Role of Homophily: Homophily is the tendency of individuals to connect with others who are similar in some or other attribute.
Here, researchers in similar fields tend to collaborate more.
Embedding based similarity often reflects this as Node2Vec captures homophilic patterns.
Potential Way to promote cross-disciplinary collaboration
While homophily is useful, it can reinforce echo chambers. So adding diversity-boosting rules to this recommendation engine helps promote diversity.
Following scenarios can help promote true collaboration:
(i) Penalize similarity between researchers if they are from same department or cluster
(ii) Introduce controlled randomness using embedding clusters.
(iii) Use metadata to connect methodological experts with domain experts.
Hence, this recommendation logic can reward diversity and methodological complementarity.

13. **Question 4(a):**
Student's Answer:
Girvan-Newman Algorithm:
The core idea is to assume that edges connecting different community are used more frequently in shortest paths between nodes. So, these 'bridge' edges have high edges betweenness centrality which describes how many shortest paths pass through an edge.
By removing edges with the highest betweenness the network gradually falls apart into densely connected components, which are interpreted as communities.
All-in-all, it detects communities in a network by progressively removing edges that are likely to lie between communities.

14. **Question 4(b):**
Student's Answer:
Usage of Edge betweenness centrality iteratively.
Edge betweenness can be defined by the number of paths (shortest) between all node pairs that pass through a given edge.
1. Calculate all edge betweenness for all edges in the current network.
2. Remove the edge with the highest betweenness.
3. Recalculate edge betweenness after each removal since the graph's structure changes.
4. Repeat until the network breaks into disconnected components or a desired number of communities is reached.
This iterative recalculation is what makes the Girvan-Newman algorithm computationally intensive but also effective.

15. **Question 4(c):**
Student's Answer:
Computational limitation:
a) The high cost is the primary drawback.
b) Time complexity is evident. For a graph with n nodes & m edges, computing edge betweenness for all edges takes $O(nm)$ time. Since it's recomputed after each edge removal, the total complexity becomes $O(n^2m)$ in sparse graphs.
Hence, this becomes infeasible for large networks with thousands of nodes or edges.

16. **Question 4(d):**
Student's Answer:
The Louvain method is a fast, greedy optimization algorithm for modularity based community detection.

It works in two phases repeated iteratively.
1. Local Movement Phase: Nodes are moved to neighboring communities if doing so increases modularity.
2. Community Aggregation Phase:
a) Graph is collapsed so that each community becomes a supernode.
b) Edges between communities are preserved and weighted.
Since it uses greedy local optimization instead of global recalculations, it is highly scalable and balances speed and modularity quality.

17. **Question 5(a):**
Student's Answer:
Intuition behind the Page Rank Algorithm for determining node importance.
The Page Rank algorithm assigns an importance score to each node in a directed network based on the idea that any node is important if it is linked to by other important nodes.
Not all incoming links are equal, being linked by a highly ranked node contributes more to a score.
Ex: A paper cited by many influential papers is more important than the one cited by random or less-known papers.
Page rank uses link structure alone and not node metadata.
It models a random surfer who keeps clicking on links. This score represents the probability that the surfer lands on a node at any time.

18. **Question 5(b):**
Student's Answer:
Role of damping factor:
The damping factor d simulates the probability that a user continues following links instead of jumping to a random page.
Page Rank is given by:

$$v = \frac{1-d}{N} + d \cdot \sum_{u \in In(v)} \frac{PageRank(u)}{OutDegree(u)}$$

d = damping factor
N = Total number of nodes
$In(v)$ = Nodes linking to v
This prevents random surfers getting stuck in loops or dead ends.
This ensures convergence of Page Rank values.
Introduces a small chance $(1-d)$ of jumping to a random node at every step.

19. **Question 5(c):**
Student's Answer:
Problem with dangling nodes:
1. It has no outgoing links
2. When a random surfer reaches such a node, they have nowhere to go.
In the page rank formula, this causes a leak.
This can be handled by 2 standard solutions:
1) Rank Redistribution Method:
a) Treat dangling nodes as if they link to all other nodes uniformly.
b) This means the rank is equally distributed if node u is dangling, then

$$PageRank(u) = \frac{PageRank(u)}{N}$$

is added to every node.
2) Add damping injection globally:
a) During each iteration, handle total page rank lost from dangling nodes.
b) Then redistribute it equally to all nodes.
The above methods ensure convergence.

20. **Question 6(a):**
Student's Answer:
Given,

	Strategy A	Strategy B
Strategy U	$(3,2)$	$(0,1)$
Strategy L	$(2,0)$	$(2,3)$

Pure Strategy Nash Equilibria:
A Nash equilibria occurs when neither player can improve their payoff by unilaterally changing their strategy.
Players 1:
1. If player 2 chooses A:
Player 1 gets U $\rightarrow$ 3, L $\rightarrow$ 2 $\rightarrow$ U
If player 2 chooses B:
Player 1 gets U $\rightarrow$ 0, L $\rightarrow$ 2 $\rightarrow$ L
Players 2:
1. If player 1 chooses U:
Player 2 gets U $\rightarrow$ 2, B $\rightarrow$ 1 $\rightarrow$ A
2. If player 1 chooses L:
Player 2 gets A $\rightarrow$ 0, B $\rightarrow$ 3 $\rightarrow$ B

By mutually neutral best response:
(U,A) and (L,B)
Thus, Nash Equilibria: (U,A) and (L,B)

21. **Question 6(b):**
Student's Answer:
Player 1 $\to$ Strategy U
Strategy L
probability p
probability $1-p$
Player 2 $\to$?
If player 2 chooses A:
P1 chooses U $\to$ P2 gets 2
P1 chooses L $\to$ P2 gets 0

$$E[\text{Payoff to P2 — A}] = p \cdot 2 + (1-p) \cdot 0 = 2p$$

If player 2 chooses B:
P1 chooses U $\to$ P2 gets 1
P1 chooses L $\to$ P2 gets 3

$$E[\text{Payoff to P2 — B}] = p \cdot 1 + (1-p) \cdot 3 = p + 3 - 3p = 3 - 2p$$

$$p = 0.7$$

For strategy A:

$$E[\text{Payoff to P2 — A}] = 2 \cdot 0.7 = 1.4$$

For strategy B:

$$E[\text{Payoff to P2 — B}] = 3 - 2 \cdot 0.7 = 3 - 1.4 = 1.6$$

Thus, if player 1 plays U with probability 0.7, player 2 should choose strategy B.

22. **Question 7:**
Student's Answer:
A $\to$ B, C $\to$ B, D $\to$ B

$$h_B^{(1)} = ?$$

`graph.png`

Given,

$$W = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix}$$

$$h_B^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad h_C^{(0)} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \quad h_D^{(0)} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Aggregate neighbor features

$$\begin{aligned} h_N^{(0)}(B) &= \frac{1}{3}(h_A^{(0)} + h_C^{(0)} + h_D^{(0)}) \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{aligned}$$

Apply transformations

$$W \cdot h_N^{(0)}(B) = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= 0.5 \times 1 + 0 \times 2 = 0.5$$

$$= 0.1 \times 1 + 0.2 \times 2 = 0.5$$

$$W \cdot h_N^{(0)}(B) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

ReLU Activation:

$$ReLU\left(\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}\right) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

since all elements are already ≥ 0
Thus, $h_B^{(1)} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$