Exam Answers

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Matched Answers

Question 1: Short answer/objective question.

(a) **Convert incidence matrix to adjacency matrix:**

```
Incidence Matrix to Adjacency Matrix

The incidence matrix provided is:
\[
\begin{bmatrix}
1 & 0 & 0 \\
1 & 1 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

The matrix structure has been given the name. Each edge in an
incidence matrix corresponds to a row and every node has its
designated column. Each entry of value '1' shows that the node
has an incidence with the edge. A simple undirected graph
contains exactly two adjacent nodes for every single edge withi

From the incidence matrix:
-> Edge 1 connects Node 1 and Node 2
-> Edge 2 connects Node 2 and Node 3
-> Edge 3 connects Node 2 and Node 4

The connection relationship between nodes in a graph structure
```

```
in adjacency matrix format. The adjacency matrix contains value
1 in position (i, j) and (j, i) when node i and node j having
an edge, otherwise both positions contain 0.

The adjacency matrix will represent node to node connection.
\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
\]
```

(b) **Network model for uniform probability:**

```
Ans -> Option-(B) -> Erd}os{Rényi (Random Network) Model.

In the Erd}os{Rényi model the edges connect randomly
because every edge formation remains isolated from other
edges.
```

(c) **Game theory situation:**

```
Ans -> Option-(C) -> Nash Equilibrium

Players reach a stable position known as Nash Equilibrium when
none of them achieve higher rewards through independent
strategy modifications.
```

(d) **Tendency for individuals to associate:**

```
Answer -> Option-(B) Assortative Mixing

Under the concept of Assortative Mixing, similar nodes establis
connects with each other.
```

(e) **Betweenness centrality relevance:**

```
Answer -> Option-(D) Because it quantifies how often a node lie
shortest paths between other nodes.
```

The measurement technique called Betweenness Centrality evaluates nodes based on their capacity to link other nodes because it determines information flow efficiencies.

(f) **Scale-free networks property:**

Answer -> Option-(C) The presence of many nodes with very high degrees (hubs) that maintain connectivity.

Scale-free Network (Hubs) operate through central nodes which serve as crucial connections. So their attack vulnerability becomes particularly pronounced.

(g) **Community detection for high modularity:**

Answer -> Option-(A) The number of intra-community edges is significantly higher than expected in a random network with the same degree sequence.

The goal of Community detection (Modularity) is to identify communities through dense connection within those groups compared to random chance expectation.

(h) **Jaccard Coefficient for link prediction:**

Answer -> Option-(B) 2/5

Jaccard Coefficient,

Neighbors of X: A, B, C, D
Neighbors of Y: C, D, B

Intersection: C, D (2 nodes)
Union: A, B, C, D, E (5 nodes)

Jaccard Coefficient = Intersection / Union = 2/5 = 0.4

The Jaccard's Coefficient calculates set (neighbors) similarity
dividing the overlapping intersection by their combined element

(i) **ICM vs LTM activation mechanism:**

```
Answer -> Option-(B) -> LTM uses edge probabilities independentl

LTM uses a weighted sum of active neighbors compared to a node t

ICM operates through independent edge probability, but LTM
adopts a threshold-based method which takes the
weighted sum of neighbors' impact.
```

(j) **GCN message-passing approach:**

```
Answer -> Option-(B) -> Because aggregating features from dissim
neighbors can blur the node's own representational features, mak
classification harder.

When heterophily occurs during GCN and heterophily
operations it becomes difficult to classify nodes because
they lose their individual characteristics through the
features aggregation process from dissimilar neighbors.
```

Question 2: Novel influenza strain strategy:

```
Influenza Spread and Vaccination

For reducing influenza transmission to 5% level a combined
approach of centrality measurement should be used to
determine who gets vaccinated first.

A high value of betweenness centrality reveals that the
node functions as a "bridge" through which the disease transmits
to numerous other connections. The immunization of these
specific people breaks transmission pathways between different
segments of the network.

High degree centrality helps identify "hubs" which possess
numerous connections since they have the potential to infect
multiple other people.

A computation between betweenness and degree centrality should be
performed for every individual:
```

-> A list of ranked people forms based on the centrality measurement result.
-> The group of people selected for vaccination needs a strong ranking in both centrality measures.
-> Additional individuals should be included for vaccination until 5% coverage is reached if the top selected candidates do not suffice.

Justification ->

A combined approach delivers better results since betweenness centrality identifies network bridges which link different network sections while degree centrality distinguishes hubs that spread widely.

Vaccinating key hubs and bridges provides maximum efficiency by controlling both small outbreaks and widespread distribution of the virus.

Question 3: Improving 'suggested collaborators' feature:

Suggested Collaborators Feature

Link Prediction -> System utilizes linking prediction algorithms that generate potential collaboration prediction through the analysis of current co-authorship and citation data.

When researchers A and B demonstrate numerous joint authors link with each other scientist a link prediction method could indicate potential collaboration between them.

Through Node2Vec the system creates vector representations of researchers based on network structural information, embedding that align with each other between researchers demonstrates similar research interest and collaboration potential.

```
Homophily ->

Due to the natural human technology of homophily the
system will provide recommendations related to researchers
within similar academic fields.

Due to encouraging diverse collaboration the system
requires a diversity metric within its recommendation
functionality. Although the link prediction score may
be slightly lower the system will give priority to
research links that combine fields from distinct areas.
```

Question 4: Girvan-Newman algorithm:

(a) **Core idea:**

```
Girvan-Newman Core Idea

The Girvan-Newman algorithm detects communities
through a recursive method which successively
eliminates edges connecting different communities.
```

(b) **Edge betweenness centrality:**

```
Edge Betweenness Centrality ->

Edge Betweenness Centrality analyzes the number of times
an edge lies between all pairs of nodes to discover
these edges.

Edges with high betweenness values link separate
communities according to this method.
```

(c) **Major computational limitation:**

```
Computational Limitation

The cost of computation increases substantially when
calculating edge betweenness centrality on extensive networks
especially when they involve frequent edge removal
operation.
```

(d) **Louvain method explanation:**

```
Louvain Method

The Louvain method functions as a scalable approach
which uses greedy optimization to shift nodes
between communities during its iterative process.
```

Question 5: PageRank algorithm:

(a) **Intuition behind PageRank:**

```
PageRank Algorithm

According to PageRank the importance of nodes
depends on both the quantity and quality
of incoming links passing through them.

Page importance derives from other pages that link
to it.
```

(b) **Role of 'damping factor':**

```
Damping Factor (d)

During random web navigation a surfer has a probability
explored through the Damping Factor (d) either
to pause their link clicks or to move to an
arbitrary page.
```

(c) **Problem with dangling nodes:**

```
Dangling Nodes

The lack of outgoing links from Dangling Nodes makes
PageRank flow through the network. We handle this
condition using an equal distribution among all network
nodes.

Each iteration distributes the rank of dangling
```

nodes across all network nodes in an equal way.

Question 6: Game theory payoff matrix:

(a) **Pure Strategy Nash Equilibria:**

A strategy pair where players can improve their payoff
unilaterally changing their strategy.

Check each pair:

(U, A): Payoff (3, 2)
-> Player 1: Switch to L -> 2 < 3, no improve
-> Player 2: Switch to B -> 1 < 2, no improve
-> Nash Equilibrium.

(U, B): Payoff (0, 1)
-> Player 1: Switch L -> 2 > 0, improve
-> Not Nash

(L, B): Payoff (2, 3)
-> Player 1: Switch to U -> 0 < 2, no improve
-> Player 2: Switch to A -> 0 < 3, no improvement
-> Nash Equilibrium.

There are two pure strategy Nash equilibria: (U, A) & (L, B).

(b) **Expected payoff calculation:**

Player 1 plays U with probability p, L with probability
1-p.

Player 2's strategy:

Expected payoff for Player 2 if choosing Strategy A
\[
E[A] = p \times 2 + (1-p) \times 0 = 2p
\]

Expected payoff for Player 2 if choosing Strategy B
\[

```
E[B] = p \times 1 + (1-p) \times 3
= p + 3(1-p)
= p + 3 - 3p = 3 - 2p
\]
```

(c) **Expected outcome for $p = 0.77$:**

```
For Player 2:

\[
E[A] = 2p = 2 \times 0.7 = 1.4
\]
\[
E[B] = 3 - 2p = 3 - 2 \times 0.7 = 1.6
\]

Since \(E[B] > E[A]\), Player 2 could choose Strategy B.

For Player 1: (0.7 U, 0.3 L)
Expected payoff = 0.7 \times 0 + 0.3 \times 2
= 0.6 when Player 2 chooses B

The expected outcome for Player 1 receives a payoff
of 0.6 if Player 2 receives a payoff of 1.6.
```

Question 7: Graph Neural Network feature update:

```
Graph Neural Network Feature Update

Given,

Directed edge A -> B
C -> B
D -> B

Weighted matrix W =
\[
\begin{bmatrix}
0.5 & 0 \\
0.1 & 0.2 \\
```

```
\end{bmatrix}
\]

Step 1: Aggregate neighbor features.

\[
h_A^{(0)} + h_C^{(0)} + h_D^{(0)} =
\begin{bmatrix}
1 \\
\end{bmatrix}
+
\begin{bmatrix}
3 \\
\end{bmatrix}
+
\begin{bmatrix}
3 \\
\end{bmatrix}
\]

\[
h_{N(B)}^{(0)} = \frac{1}{3}
\begin{bmatrix}
7 \\
6 \\
\end{bmatrix}
=
\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}
\]

Step 2: Transform

\[
W
\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}
```

```
=
\begin{bmatrix}
0.5 & 0 \\
0.1 & 0.2 \\
\end{bmatrix}
\begin{bmatrix}
1 \\
2 \\
\end{bmatrix}
\]

\[
=
\begin{bmatrix}
0.5 \\
0.1 \times 1 + 0.2 \times 2 \\
\end{bmatrix}
\]

\[
=
\begin{bmatrix}
0.5 \\
0.1 + 0.4 \\
\end{bmatrix}
\]

\[
=
\begin{bmatrix}
0.5 \\
0.5 \\
\end{bmatrix}
\]

Step 3: Activate

ReLU
\begin{bmatrix}
0.5, 0.5 \\
\end{bmatrix}
```

```
=
\begin{bmatrix}
0.5, 0.5 \\
\end{bmatrix}

Final Answer \(h_B^{(1)}\) =
\begin{bmatrix}
0.5, 0.5 \\
\end{bmatrix}
```