Sung Joon Lim CS 540 03.11.2019

Q 1).

1.1. a) $\cancel{P(X|Y,Z)}$ $P(X,Y|Z)$ $= \dfrac{P(X,Y,Z)}{\cancel{P(Y,Z)}\ P(Z)} = \dfrac{P(Z|X,Y)\,P(X,Y)}{P(Z)}$

$$= \frac{P(Z|X,Y)\,P(Y)\,P(X|Y)}{P(Z)}$$

1.1. b) no such expression,

$P(X,Y,Z) = P(X,Y|Z)\,P(Z) = P(X|Z)\,P(Y|Z)\,P(Z)$ $\quad X \perp\!\!\!\perp Y | Z$

expression cannot be made with $p(Y), p(X|Y,Z), p(Y|X,Z), p(Z|X,Y)$

1.1. c) no such explession,

$X \quad Y|X \quad Z|Y \quad X|Y,Z \quad X \perp\!\!\!\perp Y \quad \rightarrow Z$

$\dfrac{p(Y,X)}{p(X)} = p(Y)$ | $\dfrac{p(X,Y,Z)}{p(Y,Z)} = p(X,Z)$

↑

$X \perp\!\!\!\perp Y$ $\quad \dfrac{p(Z,Y)}{p(Y)} \rightarrow$ ~~not~~ Y & Z is not independent.

expression cannot be made.

1.1. d) $P(Y|X,Z) = \dfrac{P(X,Y,Z)}{P(X,Z)}$

$p(X,Y,Z) = p(X)\,p(Y|X)\,p(Z|Y,X)$ $\quad \cancel{P(X,Z) = \sum p(\ldots)}$

$p(Y,X)$ $\quad \dfrac{p(X,Y,Z)}{\cancel{p(Y,X)}}$ $\quad p(X,Z) = \sum_{Y \in y} p(X,Y,Z)$

$$\therefore \frac{p(X)\,p(Y|X)\,p(Z|Y,X)}{\sum_{Y \in y} p(X)\,p(Y|X)\,p(Z|Y,X)}$$

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1.2.a) $P(X,Y) = P(X|Z)\cancel{P(Y)}$, $P(X,Y) = P(X|Y)\cancel{P(Y)}$

$P(X|Z) = P(X|Y)$ when $X \perp\!\!\!\perp Y$ & $X \perp\!\!\!\perp Z$ //

1.2.b) $P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)} = \frac{P(Y|X)P(X)}{P(Y)}$, no independence

1.2.c) $P(Z|X,Y) = \frac{P(X|Z)P(Y|Z)P(Z)}{P(X|Y)P(Y)}$

$\frac{P(X,Y,Z)}{P(X,Y)} = \frac{P(X|Y,Z) \cdot P(Y|Z) \cdot P(Z)}{P(X|Y)P(Y)}$

$P(X|Y,Z) \cdot P(Y|Z) \cdot P(Z) = P(X,Y,Z)$

$P(X|Y,Z) = P(X|Z)$

when $Y \perp\!\!\!\perp X | Z$ //

$Y \perp\!\!\!\perp Z | X$ //

$P(Z|X,Y) = \frac{P(X|Z)P(Y|Z)P(Z)}{P(X|Y)P(Y)}$

1.2.d). $P(X,Y) = \sum_{z \in Z} P(X|Y)P(Y|Z)P(Z)$

$\hookrightarrow P(X|Y)P(Y)$

$\hookrightarrow \sum_{Y \in Z} P(Y|Z)P(Z=z) \Rightarrow P(X|Y) \sum_{Y \in Z} P(Y|Z)P(Z=z)$

$\sum_{Y \in Z} P(X|Y)P(Y|Z)P(Z=z)$

no independence.

1.3.a) $p(X|Y,Z)$

- $\boxed{\dfrac{\sum_{w\in W} P(X,Y,Z,w)}{\sum_{w\in W} P(Y,Z,w)}}$ $\qquad P(X,Y,Z) = \dfrac{P(X,Y,Z)}{P(Y,Z)}$

using Bayes

- $\boxed{\sum_{w\in W} P(X|Y,Z,w)} = \dfrac{P(X,Y,Z)}{P(Y,Z)} = p(X|Y,Z)$

1.3.b). $p(X|Z)$

- $\boxed{\dfrac{P(X,Y|Z)}{P(Y|Z)}} \rightarrow \dfrac{P(X|Z)\cancel{P(Y|Z)}}{\cancel{P(Y|Z)}}$

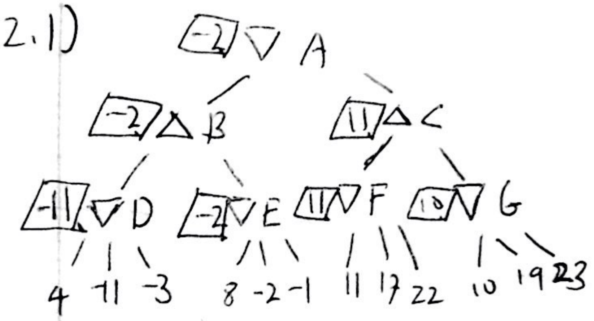
- $\boxed{\dfrac{\sum_{Y\in Y} P(X,Y|Z)P(Z)}{P(Z)}}$ $\qquad \sum_{Y\in Y} p(X,Y|Z) = p(X|Z)\cancel{\sum_Y P(Y|Z)}$

$= p(X|Z)$

Q2)

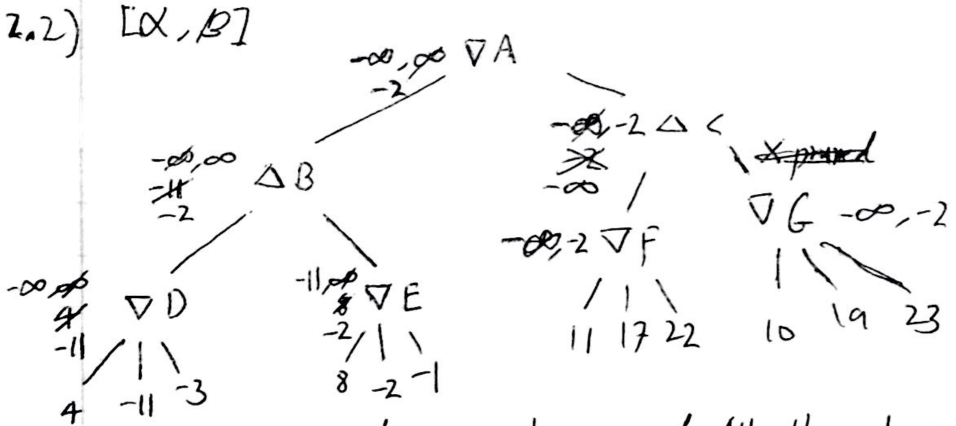
△ max ▽ min

2.1)



A: -2
B: -2
C: 11
D: -11
E: -2
F: 11
G: 10

2.2) [α, β]



A: $-\infty, -2$
B: $-2, \infty$
C: $-\infty, -2$
D: $-\infty, -11$
E: $-11, -2$
F: $-\infty, -2$
G: $-\infty, -2$

no nodes can be pruned. All the values ~~of~~ of α ~~and~~ were smaller than β.

2.3) Since there was no pruning in this tree, alpha-beta pruning does not seem to have advantage over naive minimax. However, for larger trees, using alpha-beta pruning is better since it can ~~save~~ be more efficient by not searching unnecessary nodes.

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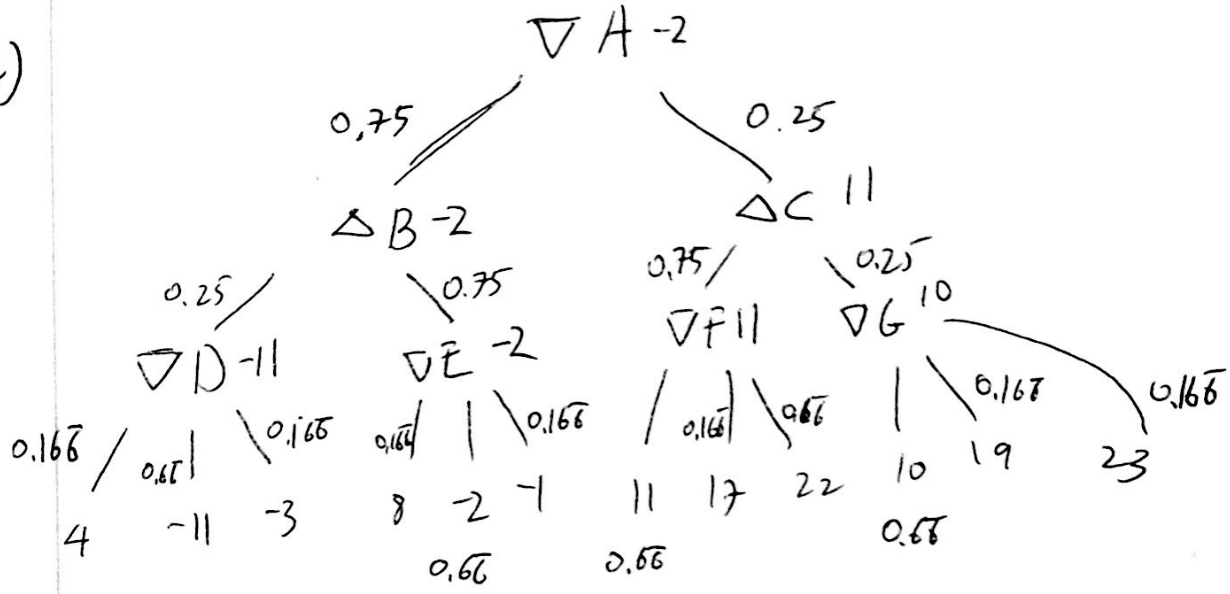
2.4)



2.5) A: $(0.75)(-2)+(11)(0.25) = 1.25$

B: $(0.25)(-11)+(0.75)(-2) = -4.25$

C: $(0.75)(11)+(0.25)(10) = 0.75$

D: $0.1667(4+-3)+0.6667(-11) = -7.167$

E: $0.1667(8+-1)+0.6667(-2) = -0.1665$

F: $0.1667(17+22)+0.6667(11) = 13.835$

G: $0.1667(19+23)+0.6667(10) = 13.6684$

Q3)

3.1) h_1 is not admissible. $h_1(E)=6$ & cheapest path from E to G is E-G which costs 5. So, $h_1(E)$ > cheapest path.
h_2 is admissible. All values of h_2 is smaller than or equal to cheapest path

3.2) A* using h_1,

A → $f(b) = 2+14 = 16$ $f(d) = 5+5 = 10$ → $f(b) = 5+9+14 = 28$ $f(c) = 5+1+10 = 16$ $f(f) = 5+2+2 = 9$ $f(e) = 5+9+6 = 20$ → $f(e) = 5+2+4+6 = 17$ $f(g) = 5+2+8+0 = 15$ END

So, A - D - F - G

A* using h_2,

A → $f(b) = 2+12 = 14$ $f(d) = 5+10 = 15$ → $f(d) = 2+9+10 = 21$ $f(c) = 2+1+11 = 14$ → $f(d) = 2+1+1+10 = 14$ →

→ $f(f) = 4+2+5.5 = 11.5$ $f(e) = 4+9+5 = 18$ → $f(e) = 6+4+5 = 15$ $f(g) = 6+8+0 = 14$ END

So, A - B - C - D - F - G

h_1 and h_2 return different value.

3.3) $h_3(D) \leq$ cheapest path, cheapest path from D to G is D-F-G which cost 10. So, interval for admissible is $0 \leq h_3(D) \leq 10$

3.4) A → $f(b) = 2+4 = 6$ $f(d) = 5 + h_3(d)$

for A* search to expand to b, f(b) must be smaller than f(d). ~~so, f(b) = 6~~

$f(b) = 6 < 5 + h_3(d)$
$= \quad 1 < h_3(d)$

B → $f(D) = 2+9+h_3(d)$ $f(C) = 2+1+10 = 13$

$f(c) = 13 < 2+9+h_3(d)$
$= \quad 2 < h_3(d)$

f(C) must be smaller than f(d)

So, $2 < h_3(d) \leq 10$ is interval of $h_3(D)$ to be admissible and A* search to expand in A-B-C order.