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CS 540

HW8

04.15.2019

Q1)

Given the knowledge base p =⇒ (q =⇒ r)

use resolution to prove the query (p ∧ q) =⇒ (q =⇒ r).

Be sure to show what you convert to CNF (do not skip steps), and how you perform each resolution step.

Alpha:

P → (q → r)

¬ p ∨ (q → r) : implication

¬ p ∨ (¬ q ∨ r) : implication

¬ (p ∧ q) ∨ r : de Morgan's Law

¬p ∨ ¬ q ∨ r : distribute

Beta:

(p ∧ q) → (q → r)

¬(p ∧ q) ∨ (q → r) : implication

¬(p ∧ q) ∨ (¬ q ∨ r) : implication

(¬ p ∨ ¬ q) ∨ (¬ q ∨ r) : de Morgan's Law

¬p ∨ ¬ q ∨ r : distribute

Then, knowledge base ¬ p, ¬ q, r , ¬p ∨ ¬ q ∨ r

complements cancel each other, so contradicts.

therefore: alpha = beta

Q2.1)

• Every child loves Santa

∀ x Child (x) → loves (x, Santa)

Child(x) = x is child, Loves(x,y) = x loves

• Everyone who loves Santa loves any reindeer.

∀ x (loves (x, Santa) → ∀ y (reindeer(y) → loves (x, y)))

Reindeer(x) = x is reindeer

• Rudolph is a reindeer, and Rudolph has a red nose.

Reindeer (Rudolph) ∧ rednose(Rudolph)

Rednose(x) = x has red nose

• Anything which has a red nose is weird or is a clown

∀ x (rednose (x) → weird (x) ∨ clown (x))

Weird(x) = x is weird

• No reindeer is a clown.

¬∃x (reindeer(x) ∧ clown(x))

Clown(x) = x is clown

• Scrooge does not love anything which is weird.

∀ x (weird (x) → ¬ loves (Scrooge, x))

• Scrooge is not a child

¬ child (Scrooge)

2.2)

• Every Austinite who is not conservative loves some armadillo.

∀ x (austinite(x) ∧ ¬conservative(x) → ∃y (armadillo(y) ∧ loves(x, y)))

Austinite (x) = x is a Austinite, Conservative(x) = x is conservative, Armadillo(y) = y is an armadillo,

loves(x,y) = x loves

• Anyone who wears maroon-and-white shirts is an Aggie.

∀x (shirt(x) → Aggie(x))

shirt(x) = x wears a maroon-and-white shirt, Aggie(x) = x is Aggie

• Every Aggie loves every dog.

∀x (Aggie(x) -> ∀y (dog(y) → loves(x, y)))

• Nobody who loves every dog loves any armadillo.

¬∃x ((∀y (dog(y) → loves(x, y))) ∧ ∃z (armadillo(z) ∧ loves(x, z)))

• Clem is an Austinite, and Clem wears maroon-and-white shirts.

Austinite(Clem) ∧ shirt(Clem)

• There is a conservative Austinite.

∃x (Austinite(x) ∧ conservative(x))

Q3)

• Alice says that she is innocent. She says that Barney and Victor were friends, and Caddy and Victor were not friends.

• Barney says that he is innocent, plus that he and Victor were not friends.

• Caddy says that she is innocent, and that Barney and Victor were friends. As an astute detective you make the following assumptions about the world: CS 540 Spring 2019

• Friends don't murder each other.

• There is no more than one murderer.

• If a person isn't a murderer, they don't lie

(1) (1 point) Write a set of FOL sentences representing the information learned when interviewing the three suspects.

¬ murderer(alice) → (friends(barney, victor) ∧ ¬ friends(caddy, victor))

¬ murderer(barney) → ¬ friends(barney, victor)

¬ murderer(caddy) → friends(barney, victor)

Murderer(x) = x is murderer, friends(x, y) = x and y are friends

(2) (1 point) Write a set of FOL sentences representing the general knowledge assumptions you've made.

Friends don't murder each other.

∀x ∀y friends(x, y) → (¬ murder(x, y) ∧¬ murder(y, x))

There is no more than one murderer.

∀x ∀y ¬ Murderer(x) ∧ ¬ Murderer(y)

If a person isn't a murderer, they don't lie

∀x (person(x) ∧ ¬ murderer(x)) → ¬ lie(x)

(3) (2 point) Convert all of your sentences in (1) and (2) to CNF.

(murderer(Alice) ∨ friends(Barney, Victor)) ∧ (murderer(Alice) ∨ ¬ friends(Caddy, Victor))

murderer(Barney) ∨ ¬friends(Barney, Victor)

murderer(Caddy) ∨ friends(Barney, Victor)

¬friends(x, y) ∨ ¬murder(x, y) ∧ (¬friends(x, y) ∨ ¬murder(y, x))

¬murderer(x) ∨ ¬murderer(y)

¬ person(x) ∨ murderer(x) ∨ ¬lie(x)

(4) (1 point) State the goal to be solved as an FOL sentence.

∃x murder(x, Victor)

(5) (1 point) Caddy later tells you that she was a friend of Victor. Write an FOL sentence that represents this new piece of information.

¬ murderer(Caddy) → (friends(Barney, Victor) **∧ Friends(Caddy, Victor))**

(6) (2 point) Is the knowledge base containing all of the sentences satisfiable? If so, give an interpretation that makes it true. If not, prove unsatisfiability using resolution.

no, after cancelling out, murderer(Alice) , murderer(Barney), murderer(Caddy) is left out which show that knowledge base is unsatisfied.

Q4.1)

A

B

C

$A \wedge B \Rightarrow D$

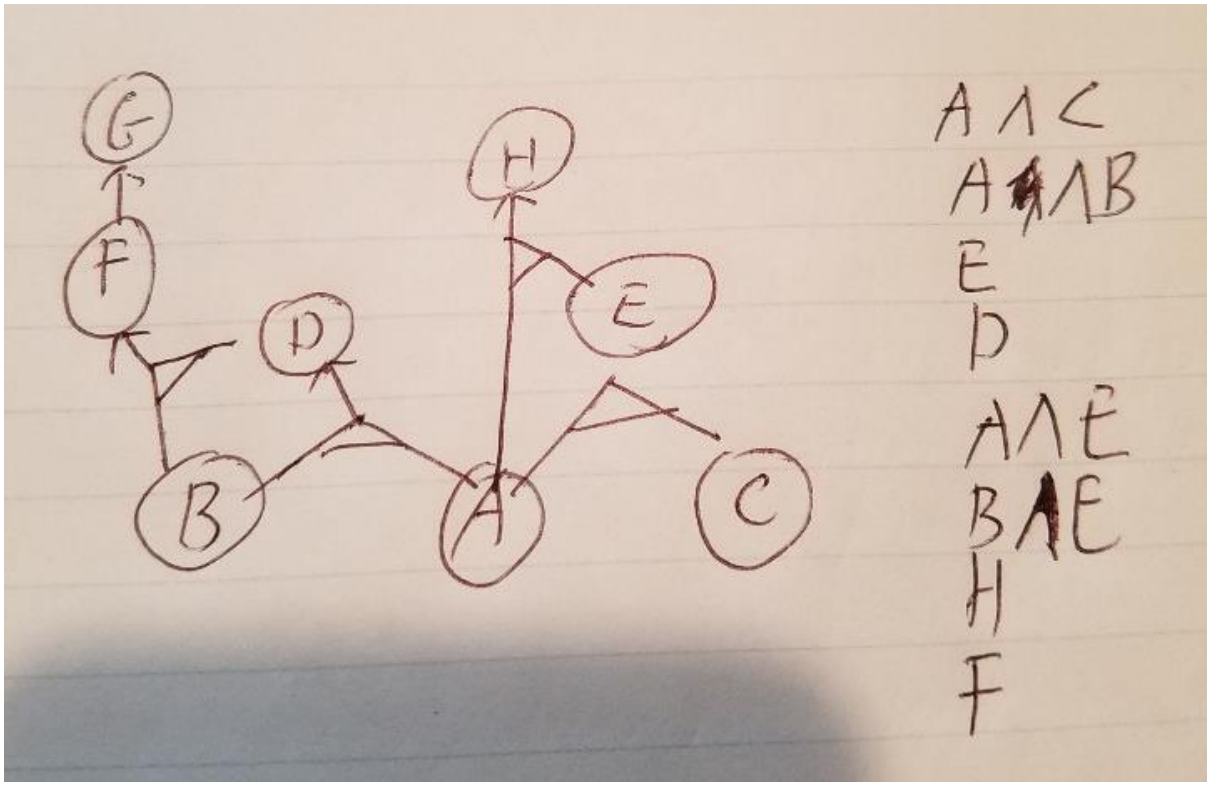
$B \wedge D \Rightarrow F$

$F \Rightarrow G$

$A \wedge E \Rightarrow H$

$A \wedge C \Rightarrow E$

Is H true? Draw a tree to illustrate the search for a proof.



H is true.

Q4.2)

P ⇒ Q

E ⇒ B

R ⇒ Q

M ∧ N ⇒ Q

A ∧ B ⇒ P

A ⇒ M

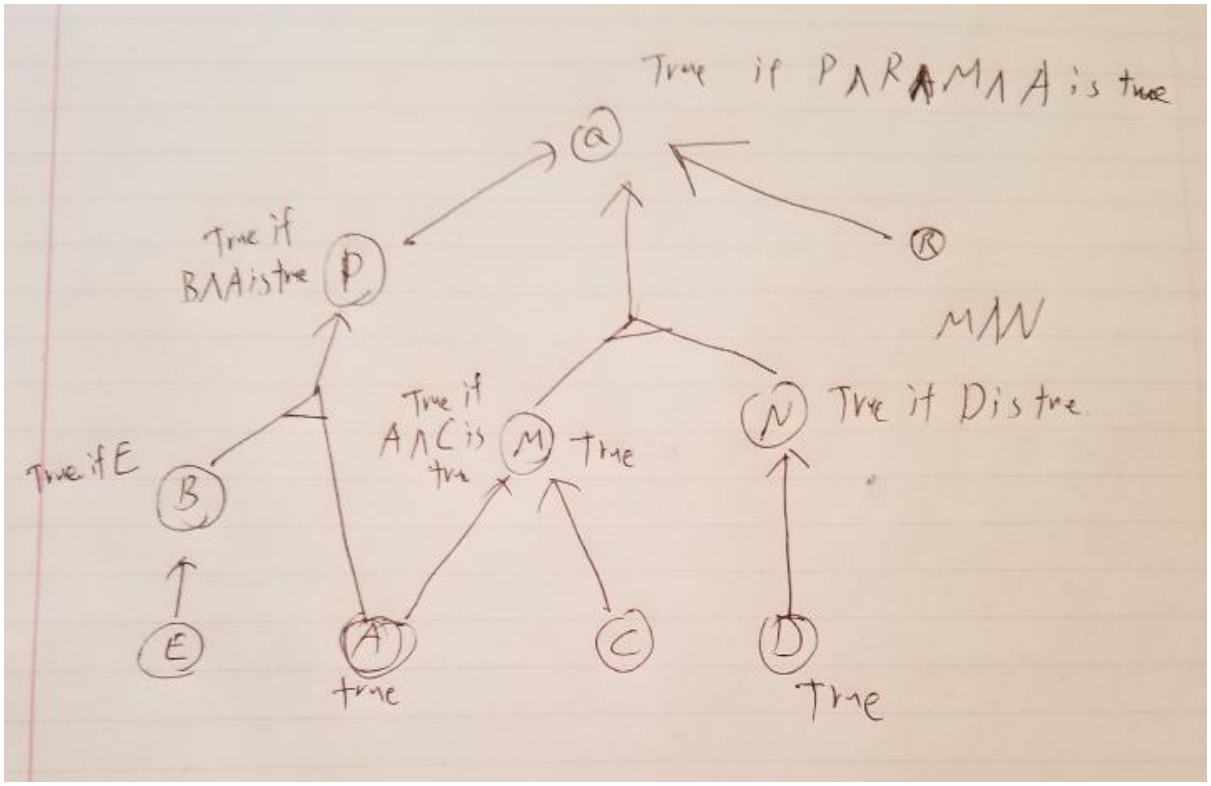
C ⇒ M

D ⇒ N

D

A

Draw a tree to illustrate the search for a proof. Mark the nodes that are satisfied in this KB. What is the proof of Q? (Please show the steps)



Q is true

Q5.1)

* I calculated distances in miles because that's what it said on the question, but was actually unnecessary ☹

	BOS	NY	DC	MIA	SLC	SEA	SF	LA	DEN	ATL
BOS	0									
NY	156.43	0								
DC	392.42	249.79	0							
MIA	1257.66	1153.98	923.97	0						
SLC	2090.5	1953.67	1843.6	2086.73	0					
SEA	2480.92	2369.36	2321.98	2727.7	696.08	0				
SF	2688.44	2551.2	2435.3	2588.24	597.94	677.1	0			
LA	2584.77	2438.5	2292.51	2331.69	577.29	956.65	347.09	0		
DEN	1764.34	1620.85	1491.94	1722.32	371.59	1019.19	945.17	824.37	0	
ATL	934.55	787.89	542.14	598.5	1587.45	2182.75	2140.17	1935.18	1215.87	0

156.43 BOS & NY

	BOS,NY	DC	MIA	SLC	SEA	SF	LA	DEN	ATL
BOS,NY	0								
DC	249.79	0							
MIA	1153.98	923.97	0						
SLC	1953.67	1843.6	2086.73	0					
SEA	2369.36	2321.98	2727.7	696.08	0				
SF	2551.2	2435.3	2588.24	597.94	677.1	0			
LA	2438.5	2292.51	2331.69	577.29	956.65	347.09	0		

DEN	1620.85	1491.94	1722.32	371.59	1019.19	945.17	824.37	0	
ATL	787.89	542.14	598.5	1587.45	2182.75	2140.17	1935.18	1215.87	0

249.79 (BOS, NY) & DC

	B,N,D	MIA	SLC	SEA	SF	LA	DEN	ATL
B,N,D	0							
MIA	923.97	0						
SLC	1843.6	2086.73	0					
SEA	2321.98	2727.7	696.08	0				
SF	2435.3	2588.24	597.94	677.1	0			
LA	2292.51	2331.69	577.29	956.65	347.09	0		
DEN	1491.94	1722.32	371.59	1019.19	945.17	824.37	0	
ATL	542.14	598.5	1587.45	2182.75	2140.17	1935.18	1215.87	0

371.59 DEN & SLC

	B,N,D	MIA	SLC,DEN	SEA	SF	LA	ATL
B,N,D	0						
MIA	923.97	0					
SLC,DEN	1491.94	1722.32	0				
SEA	2321.98	2727.7	696.08	0			
SF	2435.3	2588.24	597.94	677.1	0		
LA	2292.51	2331.69	577.29	956.65	347.09	0	
ATL	542.14	598.5	1215.87	2182.75	2140.17	1935.18	0

347.09 SF & LA

	B,N,D	MIA	SLC,DEN	SEA	SF,LA	ATL
B,N,D	0					
MIA	923.97	0				
SLC,DEN	1491.94	1722.32	0			
SEA	2321.98	2727.7	696.08	0		
SF,LA	2292.51	2331.69	577.29	677.1	0	
ATL	542.14	598.5	1215.87	2182.75	1935.18	0

542.14 (BOS, NY, DC) & ATL

	B,N,D,A	MIA	SLC,DEN	SEA	SF,LA
B,N,D,A	0				
MIA	598.5	0			
SLC,DEN	1215.87	1722.32	0		
SEA	2182.75	2727.7	696.08	0	
SF,LA	1935.18	2331.69	577.29	677.1	0

577.29 (SF, LA) & (SLC, DEN)

	B,N,D,A	MIA	S,D,S,L	SEA
B,N,D,A	0			
MIA	598.5	0		
S,D,S,L	1215.87	1722.32	0	
SEA	2182.75	2727.7	677.1	0

598.5 (BOS, NY, DC, ATL) & MIA

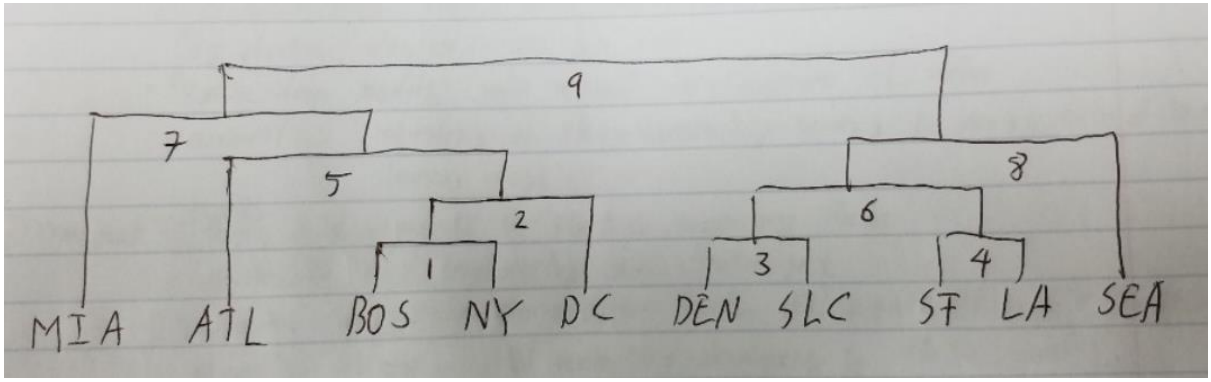
	B,N,D,A,M	S,D,S,L	SEA
B,N,D,A,M	0		
S,D,S,L	1215.87	0	
SEA	2182.75	677.1	0

677.1 (SF, LA, SLC, DEN) & SEA

	B,N,D,A,M	S,D,S,L,S
B,N,D,A,M	0	
S,D,S,L,S	1215.87	0

1215.87 Final Cluster

a)



b)

Last three clusters using single-linkage are (MIA, ATL, BOS, NY, DC), (DEN, SLC, SF, LA) and (SEA).

5.2)

c1	50	90
c2	30	100

Distance between c1 and c2 = 22.36

a) list of cities in the initial clusters

city	distance c1	distance c2	cluster
BOS	20.37081	31.44789	1
NY	18.02471	28.51123	1
DC	17.09415	24.66191	1
MIA	26.109	20.24055	2

SLC	23.75395	16.07016	2
SEA	32.38904	28.40863	2
SF	34.6208	23.71919	2
LA	32.3736	18.6561	2
DEN	18.19588	10.91284	2
ATL	17.26789	16.1301	2

b)

city	new center	new center
BOS	46.2	80.55
NY	45.85	82
DC	44.45	83.5
MIA	27.9	90.1
SLC	35.4	105.95
SEA	38.8	111.15
SF	33.9	111.2
LA	32.05	109.1
DEN	34.85	102.5
ATL	31.85	92.15

c)

city	distance c1	distance c2
BOS	10.18541	25.31289
NY	9.012353	23.9838
DC	8.547076	21.93291
MIA	22.10023	10.12028
SLC	21.62319	8.035079
SEA	23.93246	14.20431
SF	26.62048	11.8596
LA	26.21092	9.328049
DEN	19.64109	5.456418
ATL	18.2769	8.065048

BOS, NY, DC is in cluster 1 and rest in in cluster 2.