相对误差、相对误差限

Sunday, February 16, 2020 12:16 PM

100 1

1000 5

$$\frac{1}{100} \rightarrow \frac{10}{1000}$$

$$\frac{5}{1000}$$

准确性更高

$$e_r^* = \frac{x^* - x}{x} = e_r(x^*)$$

不知道 $\leftarrow x^*$

$$e_r^* = \frac{x^* - x}{x^*}$$

两种定义

$$|e_r^*| = \left|\frac{x^* - x}{x^*}\right| \leq \frac{\varepsilon^*}{|x^*|} = \varepsilon_r^*$$

$$\varepsilon_r(x^*)$$

$$\varepsilon^* = \frac{1}{2} \times 10^{p-n} \geq |x^* - x|$$

$$\varepsilon_r^* = \frac{\varepsilon^*}{|x^*|} \leq \frac{\frac{1}{2} \times 10^{p-n}}{|x^*|} \leq ?$$

$$|x^*| = 0.a_1 a_2 \cdots a_m \times 10^p$$

$$= (a_1 + 0.a_2 a_3 \cdots a_m) \times 10^{p-1}$$

$$= (\alpha_1 + \underline{0.\alpha_2\alpha_3\cdots\alpha_m}) \times 10^{\ }$$

$\alpha_1 \neq 0$ $\geq (\alpha_1 + 0) \times 10^{p-1} = \alpha_1 \times 10^{p-1}$ n大

$\alpha_2 \cdots \alpha_m \geq 0$

$$\underline{\varepsilon_r^*} \leq \frac{\frac{1}{2}\times 10^{p-n}}{\alpha_1 \times 10^{p-1}} = \boxed{\frac{1}{2\alpha_1} \times 10^{-(n-1)}}$$

值小

α

值大

$e^* = 2.718 \rightarrow e = 2.7182\cdots$ $n=4$

$$\underline{\varepsilon_r}^* \leq \frac{1}{2\times 2} \times 10^{-(4-1)} = \boxed{\frac{1}{4} \times 10^{-3}}$$

$= 0.2718 \times 10^1$

$\underline{\varepsilon_r^*} = \underline{\frac{1}{4} \times 10^{-3}}$

$$\varepsilon_r^* \leq \frac{1}{2(\alpha_1+1)} \times 10^{-(n-1)}$$

$$\frac{\varepsilon^*}{|x^*|} \leq \frac{1}{2(\alpha_1+1)} \times 10^{-(n-1)}$$

$$\varepsilon^* \leq \frac{|x^*|}{2(\alpha_1+1)} \times 10^{-(n-1)} \leq \cdots$$

$$|x^*| = 0.a_1a_2\cdots a_m \times 10^{p}$$

$$= (a_1 + \underbrace{0.a_2a_3\cdots a_m}_{1 > \quad \geq 0}) \times 10^{p-1}$$

$$\leq (a_1+1)\times 10^{p-1}$$

$$\varepsilon^* \leq \frac{\cancel{(a_1+1)}\times 10^{p-1}}{2\cancel{(a_1+1)}} \times 10^{-(n-1)}$$

$$= \frac{1}{2}\times 10^{p-1-n+1} = \frac{1}{2}\times 10^{p-n}$$

$\Rightarrow$ 有效数字个数为 n