

Note: Some problems have been weakened.

## A

There are at most 3 contest. We consider the hardest case that  $k = 3$ . For each account, we compute the set of contests it participated in.

Let  $cnt_S$  ( $S \subseteq \{1, 2, 3\}$ ) be the number of accounts whose participated contest is set  $S$ .

It is easy to see for an account that participated in the contests  $\{1, 2, 3\}$ , it can't participate other contests.

For an account that participated in the contest  $\{1, 2\}$ , it can match with an account that only participated in the contest  $\{3\}$ . It's the same for  $\{1, 3\}$  and  $\{2, 3\}$ .

For the remaining account, we need  $\max(cnt_{\{1\}}, cnt_{\{2\}}, cnt_{\{3\}})$  peoples.

## B

First, let's see how to compute the size of  $S_u$ . When two sets  $S_u$  and  $S_v$  merge, the size becomes  $|S_u| + |S_v| - |S_u \cap S_v|$ . We can see  $S_u \cap S_v$  is exactly the set when these two sets merged last time. So we can compute it in  $O(n + m)$ .

For the original problem, we can solve the problem backward. The meaning of the set  $S_u$  becomes which vertices that  $u$  can reach.

## C

For a query  $[l, r]$  and a vertex  $[L, R]$  on the segment tree.

- If  $[L, R]$  intersects with  $[l, r]$  and  $[L, R]$  is not included in  $[l, r]$ , it will contribute to the number of visited vertices by 1.
- If  $[L, R]$  is included in  $[l, r]$  and  $L \neq R$ , it will contribute to the number of visited vertices by  $-1$ .
- If  $[L, R]$  is included in  $[l, r]$  and  $L = R$ , it will contribute to the number of visited vertices by 1.

Thus the contribution of a vertex only depends on the vertex itself now, instead of the structure of the tree (whether its father is included in  $[l, r]$ ).

We can use the partial sum technique to compute the contribute of each pair  $[L, R]$ , denoted as  $w_{L,R}$ .

Let  $dp_{L,R}$  be the minimum total contribution of a subtree with the root  $[L, R]$ ,  
 $dp_{L,R} = \min(dp_{L,k} + dp_{k,R} + 1, R + w_{L,R})$ .

## D

Recall the circle union algorithm, we need to find the border of the circle and find the answer.

In this problem, we can easily solve it in a similar way. For each piece of arcs on the circle, we just simply compute the probability that it's on the border and contribute it to the answer.

## E

We can enumerate first 3 elements in  $A$  and calculate the fourth element in  $O(1)$ .

## F

Let  $dp_{i,j}$  be the number of permutations with  $i$  elements and  $j$  inverse pairs,  
 $dp_{i,j} = \sum_{k=0}^{i-1} dp_{i-1,j-k}$ .

The answer  $f_i = \sum_{j=0}^{i(i-1)/2} dp_{i,j} j^k$ . We can compute first  $O(k)$  terms in  $O(k^3)$ .

We can prove sequence  $\{f_n/(n-2k)!\}$  is a polynomial, thus we can compute first terms and find  $n$ -th term by interpolation.

However, CaiDui said  $\{f_n/n!\}$  is a linear recurrence sequence, we can solve it by Berlekamp-Massey algorithm.

## G

We can regard the modulo inverse as a random sequence. If we compute the first  $\Theta(\sqrt{p})$  term and the minimum value can become  $O(\sqrt{p})$ . Thus for the remaining term, we can enumerate the value and compute the position of the value to see if it is the minimum. The expected time complexity is  $O(\sqrt{p})$ .

## H

First, let's see how to compute the distance  $d$  from  $O$  to plane  $ABC$ . We can compute the volume  $V$  of  $OABC$  by the determinant. Then we have  $\frac{Sd}{3} = V$ , where  $S$  is the area of triangle  $ABC$ , which can be computed stably.

Then we can try some constructions and test locally. One intuitive way is that find an equilateral triangle and make some numerical perturbations.

```
cout << "999999 1000000 0" << endl;  
cout << "-999997 0 999999" << endl;  
cout << "0 -999996 -999997" << endl;
```

## I

$n$  is small, we can compute the entire matrix in  $O(n^2)$ . Then find the maximum by 2d segment tree.

## J

We need to find the rank of binary vectors. It can be solved in  $O(2^{6k}/w)$  by Gaussian elimination and bitset.