Assignment 9
MAA4211
Carson Mulvey

(Graded) 4.5.7. Consider the function $g(x) = f(x) - x$. By the Algebraic Continuity Theorem, g is continuous on interval $[0, 1]$. A fixed point of f will occur for some $x \in [0, 1]$ iff $g(x) = 0$.

Using the range of f, we know that $0 \leq g(0) \leq 1$ and $-1 \leq g(1) \leq 0$. If $g(0) = 0$ or $g(1) = 0$, then a fixed point occurs at $x = 0$ or $x = 1$, respectively. Now let $g(0) > 0$ and $g(1) < 0$. We have $g(0) > 0 > g(1)$, so by the Intermediate Value Theorem, there is a point $c \in (0, 1)$ where $g(c) = 0$, thus making $x = c$ a fixed point.