Assignment 6

MAA4211

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(Graded) 2.7.3. (a) Suppose $\sum_{k=1}^{\infty} b_k$ converges. Let $\epsilon > 0$. Then there exists and $N \in \mathbb{N}$ such that for $n > m \geq N$, we have

$$|b_{m+1} + b_{m+2} + \cdots + b_n| < \epsilon.$$

However, because $0 \leq a_k \leq b_k$ for all $k \in \mathbb{N}$, we also see that

$$\begin{aligned} & a_{m+1} + a_{m+2} + \cdots + a_n \leq b_{m+1} + b_{m+2} + \cdots + b_n \\ \implies & |a_{m+1} + a_{m+2} + \cdots + a_n| \leq |b_{m+1} + b_{m+2} + \cdots + b_n| \\ & |a_{m+1} + a_{m+2} + \cdots + a_n| < \epsilon. \end{aligned}$$

Thus, taking the same N as picked for (b_n), we can apply the Cauchy Criterion for Series again, showing that $\sum_{k=1}^{\infty} a_k$ converges.

(b) Suppose that