Homework 4

MAD4204

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1. For P a finite poset, let $J(P)$ be the set of ideals in P and $A(P)$ be the set of antichains.

   (a) Find $\#J(P)$ and $\#A(P)$ for a chain. For an antichain.

   (b) Find $\#J(P)$ and $\#A(P)$ for B_3.

   (c) Must $\#J(P) = \#A(P)$? Why or why not? Explain.

   Solution.

   (a) For chain P, all pairs of elements are comparable, so only singleton antichains exist (and the empty antichain). Thus $\#A(P) = \#P + 1$.

   For an antichain P, all pairs of elements are incomparable, so any subset of P is another antichain. Thus $\#A(P) = 2^{\#P}$.

   (b) test

2. (a) For P a poset with n elements, prove P contains a chain with at least $\sqrt{n}$ elements or an antichain with at least $\sqrt{n}$ elements.

   (b) Prove Hall's theorem using Dilworth's theorem.

3. For P a finite poset, show the number of elements in a maximum chain equals the number of antichains in the smallest antichain cover.

4. Let $M(n,k)$ be the multiset consisting of k copies of each element in $[n]$. Let $P(n,k)$ be the poset on submultisets of $M(n,k)$ ordered by containment, e.g.

$$\{\{1,1,4\}\} \subseteq \{\{1,1,1,3,3,4,5,5\}\} \quad \text{but} \quad \{\{1,1,4\}\} \not\subseteq \{\{1,3,3,4,4\}\}.$$

   Find a general formula for $\mu_{P(n,k)}(x,y)$, and explain how it relates to Example 16.20.