Assignment 7
MAA4211
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(Graded) 3.3.3. Let $K \subseteq \mathbb{R}$ be closed and bounded. Consider a sequence (a_n) in K. Because K is bounded by some M, all elements $a_k \leq M$, so (a_n) is also bounded. Thus, by Bolzano-Weierstrass, (a_n) has a subsequence that converges, say (a_{n_k}).

However, this convergent subsequence must also be a Cauchy sequence. Because K is closed, by Theorem 3.2.8., (a_{n_k}) has a limit that is also in K. Thus, by the definition of compactness, K is compact.