COT3100 Exam 2 review

Section 26872 (Carson, Zac)

Feb 27, 2024

Topics

Exam details

- Time: 8:20 to 10:20 PM
- Topics:
  - 2.3 to 2.5 (no matrices!)
  - 3.1 to 3.3
  - Somewhat cumulative (proof techniques, etc.)
- Things to bring:
  - Writing utensils
  - Handwritten reference sheet (8.5x11)
  - 4 function calculator
  - ID (UF ID, state ID, or ID on phone)

Common mistakes!

1. What is the parity of 0?
2. List all primes less than 10.
3. What is 0! ?
4. What is $\lfloor -3.2 \rfloor$?

Topics

Functions

Definition

A *function* f is written as

$$f\colon A \to B,$$

where A is the *domain* of f and B is the *codomain* of f. Every element of the domain is mapped to exactly one element of the codomain.

Definition

The *range* of a function f is the set of all values of the codomain that are actually mapped to by some value of the domain.

Be careful in using codomain and range! The codomain is the *possible* values that can be mapped to, but they aren't necessarily all mapped to.

One-to-one and onto

Definition

A function f is *one-to-one* or *injective* if every element of the comain has *at most* one domain element mapped to it.

Definition

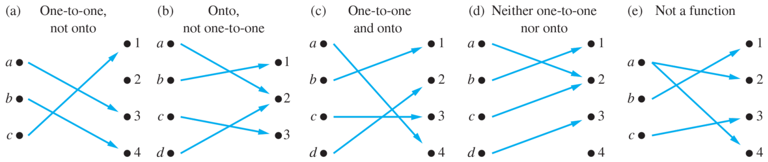
A function f is *onto* or *surjective* if every element of the codomain has *at least* one domain element mapped to it.

See the relationship?

Bijections

Definition

A function f is a *one-to-one correspondence* or *bijective* if it is both one-to-one and onto. Equivalently, each codomain element has *exactly one* domain element mapped to it.



Mathmatical definitions

Definition

A function f is *one-to-one* if for any two elements x_1 and x_1 in the domain of f,

$$f(x_1) = f(x_2) \implies x_1 = x_2.$$

Definition

A function f is *onto* if for any element y in the codomain of f, there exists an x in the domain where

$$f(x) = y.$$

Note

An onto function has the same range and codomain (why?).

Practice

Problem

Given $f : \mathbb{R} \to \mathbb{R}^+$, where $f(x) = x^4$, prove or disprove if f is one-to-one. Prove or disprove if f is onto. What if the domain was $\mathbb{R}^+$?

Topics

Important series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} 1 = ?$$

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$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} 1 = n$$

Practice

Problem

Evaluate

$$\sum_{j=0}^{4} \sum_{k=98}^{100} \frac{1}{25}(k-97)(j+1)^3.$$

Topics

Countability

Definition

A *countable* set S has a bijection

$$f : \mathbb{Z}^+ \to S.$$

A set where this bijection does not exist is *uncountable*.

Note

The integers $\mathbb{Z}$, the set of even integers, and (surprisingly) the set of rationals $\mathbb{Q}$ are all examples of countable sets.

Note

The set of reals, $\mathbb{R}$ is uncountable. Any interval of $\mathbb{R}$ is also uncountable. The set of irrationals is also uncountable (why?).

Practice

Problem

Prove that the subset of a countable set is countable.

Topics

Searching and sorting complexities

Search algorithms:

- Linear search - $O(n)$
- Binary search - $O(\log_2 n)$

Sorting algorithms:

- Bubble sort - $O(n^2)$
- Selection sort - $O(n^2)$
- Insertion sort - $O(n^2)$

Refer to the textbook and discussion slides for details of each algorithm.

Pseudocode

When writing pseudocode, getting the message across is most important

- Will likely look similar to Python
- Can replace technical syntax (`array[i][j]=n`) with description ("set array element at (i, j) to n")

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Warning!

You will likely need to write algorithms beyond searching/sorting!
Be ready to work with arrays (lists), strings, and numbers.

Practice

Problem

For an integer array `nums` of size at least 2, let a *peak* be an index i of `nums` where the element at i is strictly greater than the elements at indices $i-1$ (if it exists) and $i+1$ (if it exists). Describe an algorithm `peak_count` that finds the number of peaks of a given `nums`.

Practice

Problem

For an integer array `nums` of size at least 2, let a *peak* be an index i of `nums` where the element at i is strictly greater than the elements at indices $i - 1$ (if it exists) and $i + 1$ (if it exists). Describe an algorithm `peak_count` that finds the number of peaks of a given `nums`.

Example

If `nums=[2,1,5,3,1]`, then `nums` has a peak at $i = 0$ (2) since $2 > 1$, and a peak at $i = 2$ (5) since $5 > 1$ and $5 > 3$.

Topics

Growth rates

Note

The following order represents the growth rates of functions from slowest to fastest:

$$1 \ll \log n \ll n \ll n \log n \ll n^2 \ll (\text{polynomials}) \ll 2^n \ll n! \ll n^n$$

Big O

Definition

A function $f(x)$ is $O(g(x))$ if there are C and k such that

$$|f(x)| \leq C|g(x)|$$

for all $x > k$.

We think of big O as an *upper bound* for the growth of f. In other words, g either grows faster or at the same rate as f.

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Example

If $f(x) = x^2 + 3$, then $f(x)$ is $O(x^2)$. However, $f(x)$ is also $O(x^3)$, $O(2^x)$, $O(x!)$, and many more.

Big Ω

Definition

A function $f(x)$ is $\Omega(g(x))$ if there are C and k such that

$$|f(x)| \geq C|g(x)|$$

for all $x > k$.

We think of big Ω as a *lower bound* for the growth of f. In other words, g either grows slower or at the same rate as f.

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Example

If $f(x) = x^2 + 3$, then $f(x)$ is $\Omega(x^2)$. However, $f(x)$ is also $\Omega(x)$, $\Omega(1)$, etc.

Big Θ

Definition

A function $f(x)$ is $\Theta(g(x))$ if it is both $O(g(x))$ and $\Omega(g(x))$

We think of big Θ as giving a "class" of functions growing at a similar rate.

Note

The "optimal" big O refers to the *slowest* growing big O possible.
Similarly, the "optimal" big Ω refers to the *fastest* growing big Ω possible.
These both end up the same as big Θ.

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The "optimal" big O refers to the *slowest* growing big O possible.
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These both end up the same as big Θ.

Example

If $f(x) = x^2 + 3$, then $f(x)$ is $\Theta(x^2)$. Its "optimal" big O and big Ω are $O(x^2)$ and $\Omega(x^2)$, respectively.

Practice

Problem

Order the following functions of n by their growth rates from *fastest* to *slowest*.

$$\begin{array}{llllll}
(1) & 2024 & (1.0001)^n & log(n^n) & 4n^2-1 & n^{2024} \\
(2) & 2024 & (1.0001)^n & log(n^n) & 4n^2-1 & n^{2024} \\
(3) & 2024 & (1.0001)^n & log(n^n) & 4n^2-1 & n^{2024} \\
(4) & 2024 & (1.0001)^n & log(n^n) & 4n^2-1 & n^{2024} \\
(5) & 2024 & (1.0001)^n & log(n^n) & 4n^2-1 & n^{2024}
\end{array}$$

Problem

For $f(n) = 4n^3 + n - 5$, prove that $f(n)$ is $\Theta(n^3)$.

Practice

Problem

Order the following functions of n by their growth rates from *fastest* to *slowest*.

$$
\begin{array}{llllll}
(1) & 2024 & \color{red}{(1.0001)^n} & log(n^n) & 4n^2-1 & n^{2024} \\
(2) & 2024 & (1.0001)^n & log(n^n) & 4n^2-1 & \color{red}{n^{2024}} \\
(3) & 2024 & (1.0001)^n & log(n^n) & \color{red}{4n^2-1} & n^{2024} \\
(4) & 2024 & (1.0001)^n & \color{red}{log(n^n)} & 4n^2-1 & n^{2024} \\
(5) & \color{red}{2024} & (1.0001)^n & log(n^n) & 4n^2-1 & n^{2024}
\end{array}
$$

Problem

For $f(n) = 4n^3 + n - 5$, prove that $f(n)$ is $\Theta(n^3)$.

Topics

Practice

Problem

Find the time complexity of the `peak_count` algorithm.