Assignment 10

MAA4211

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(Graded) 5.3.3. (a) Let $g(x) = h(x) - x$. Because $h(x)$ is differentiable on $[0, 3]$, it is also continuous on $[0, 3]$, making g also continuous on $[0, 3]$ by the Algebraic Continuity Theorem.

We can compute $g(0) = h(0) - 0 = 1$ and $g(3) = h(3) - 3 = -1$. In other terms, $g(3) < 0 < g(0)$, so by the Intermediate Value Theorem, there must exist $d \in (0, 3) \subset [0, 3]$ such that $g(d) = 0$, or equivalently, $h(d) = d$.

(b) We know that h is differentiable (and hence continuous) on $[0, 3]$. Then, by the Mean Value Theorem, there exists a point $c \in (0, 3)$ where

$$h'(c) = \frac{h(3) - h(0)}{3 - 0} = \frac{1}{3}.$$

(c) Let $g(x) = h(x) - \frac{1}{4}x$. Similar to in (a), we can deduce that g is continuous and differentiable on $[0, 3]$. Additionally, we have $g(0) = 3/4$, $g(1) = 7/4$, and $g(3) = 5/4$.

But since $g(0) < 5/4 < g(1)$, by the Intermediate Value Theorem, we must have some $c \in (0, 1)$ such that $g(c) = 5/4$. Then, since $g(c) = g(3)$, by Rolle's Theorem, we must have some $d \in (c, 3) \subset [0, 3]$ where $g'(d) = 0$. Because $g'(x) = h'(x) - \frac{1}{4}$, this is equivalent to $h'(d) = 1/4$, as desired.