MAE5810 Independent Project

Zilong He

October 2023

1 Abstract

A Pan-Tilt-Zoom camera is a versatile device which has the capability of rotating in two axes and change focal length in order to capture target features and conduct surveillance tasks. The functionalities of the PTZ camera are useful for optimizing surveillance coverage, ensuring efficient system integration, and facilitating automated controls. The primary goal of this project is by simulating a PTZ camera in Matlab using different measurement models in order to study its Field-of-View visibility to gain a better understanding of the advantages and constraints of the fundamental pinhole camera sensor.

2 Theory and Methods

2.1 Constant Zoom Model

In order to efficiently simplify the model, the focal length can be set to a constant such that only two state variables, which are the pan and tile angles ψ and φ, are needed to describe the position of the camera. A detailed schematic of this model[1] is shown in Figure 1.

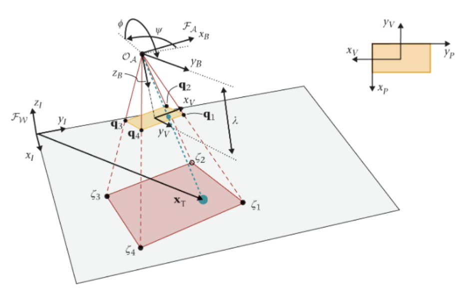


Figure 1: Pinhole camera model coordinate frames and FOV geometry[1].

In the schematic, there is an inertial frame $\mathcal{F}_\omega$ which corresponds to the physical workspace and a body frame $\mathcal{F}_\mathcal{A}$ which corresponds to the camera-based reference frame where the virtual image plane and optical axis reside. The yaw(ψ) and roll(φ) angles are defined by two successive right-hand rotations such that any vector X_T in the inertial frame can be resolved into the body frame using the following equation:

$$X_c = R_\varphi R_\psi X_T \tag{1}$$

where the 3D special orthogonal Euler rotational matrices have the following expression

$$R_\varphi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\varphi) & sin(\varphi) \\ 0 & -sin(\varphi) & cos(\varphi) \end{bmatrix}, R_\psi = \begin{bmatrix} cos(\psi) & sin(\psi) & 0 \\ -sin(\psi) & cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The four vertices of the virtual image plane can be easily found using the width(a) and height(b) of the image sensor. Then, the vector from the pinhole camera position(origin of the body frame) to the vertices can be determined. In order to derive the mathematical expression of the FOV tetrahedron, the vertex vectors need to be transformed into rays and the projection of the virtual image plane can be obtained using the four intersection points between the vertex rays and the inertial frame ground plane(xy plane).

2.2 Target Velocity

For a moving target, the projected velocity of the target onto the virtual image plane can be derived by taking derivative of the position vector in the same reference frame. The simplified expression is as follows,

$$\dot{P_T} = H \begin{bmatrix} R_\varphi^T R_\psi^T & o \\ 0 & -R_\varphi^T \end{bmatrix} \begin{bmatrix} \dot{X_T^T} & 0 & \dot{\varphi} & 0 & \dot{\psi} \end{bmatrix}^T \tag{3}$$

where the H denotes the image Jacobian matrix and has the following expression,

$$H := \begin{bmatrix} -\frac{\lambda}{q_z} & 0 & \frac{p_x}{q_z} & \frac{p_x p_y}{\lambda} & -\frac{\lambda^2 + {p_x}^2}{\lambda} & p_y \\ 0 & -\frac{\lambda}{q_z} & \frac{p_y}{q_z} & \frac{\lambda^2 + {p_x}^2}{\lambda} & -\frac{p_x p_y}{\lambda} & -p_x \end{bmatrix} \tag{4}$$

2.3 Deterministic and Stochastic Models

In ideal scenario, the camera is always able to capture target that is inside the FOV and will not have any response if the target is outside the FOV. The mathematical expression of the deterministic model is as follows:

$$\mathcal{P}(X_T) = \begin{cases} 1 & \text{if } X_T \in \mathcal{S} \\ 0 & \text{if } X_T \notin \mathcal{S} \end{cases} \tag{5}$$

However, to account for uncertainties and sensor noises in real world, stochastic models introduce the probabilistic perspective into the measurement of the

PTZ camera sensors and might be more accurate than the deterministic model depending on the environmental effects and hardware limitations.

Here, we will introduce a simple stochastic model that only consists of four events: true positive(TP), missed detection(MD), true negative(TN), and false alarm(FA).

For example, suppose camera sensor noise and surrounding interference are present and have some influence on the measurement of the system such that when the target is inside the sensor FOV, there is a 80% chance that the sensor is able to capture the target and 20% chance that there is no detection signal, and when the target is outside the sensor FOV, there is a 95% chance that corresponds to the true negative and 5% chance there is a false alarm indicating the target capturing. Let F denote the event that the camera sensor declaring a detection signal, the mathematical expression is as follows:

$$\mathcal{P}(F) = \begin{cases} 0.8 & \text{if } X_T \in \mathcal{S} \\ 0.05 & \text{if } X_T \notin \mathcal{S} \end{cases} \tag{6}$$

$$\mathcal{P}(F^C) = \begin{cases} 0.2 & \text{if } X_T \in \mathcal{S} \\ 0.95 & \text{if } X_T \notin \mathcal{S} \end{cases} \tag{7}$$

A more clear expression of the probabilities is to use the conditional probability distribution notation,

$$\mathcal{P}(TP) = \mathcal{P}(F|X_T) = 0.8 \tag{8}$$

$$\mathcal{P}(MD) = \mathcal{P}(F|X_T^C) = 0.2 \tag{9}$$

$$\mathcal{P}(TN) = \mathcal{P}(F^C|X_T^C) = 0.95 \tag{10}$$

$$\mathcal{P}(FA) = \mathcal{P}(F^C|X^T) = 0.05 \tag{11}$$

2.4 State Space Representation

The kinematic and dynamic behaviors of the PTZ camera sensor can be effectively described using a state-space model, let $\mathbf{q} = \begin{bmatrix} \psi & \varphi & \dot{\psi} & \dot{\varphi} \end{bmatrix}^T$ denote the dynamic state vector and $\mathbf{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T$ denote the control input. The expression is as follows,

$$\mathbf{q}(k+1) = \mathbf{A}\mathbf{q}(k) + \mathbf{B}\mathbf{u}(k) \tag{12}$$

where,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_1 & 0 \\ 0 & b_2 \end{bmatrix} \tag{13}$$

and b_1, b_2 are constants that are related to specific hardware parameters. It is obvious that, when the sampling period Δt is constant, this becomes a linear time-invariant(LTI) system.

Because of the reference system convention, the pan and tilt angles are limited to certain ranges: $\psi \in [0, 2\pi)$, $\varphi \in [\frac{\pi}{2}, \pi]$. The angular velocities and normalized voltage control inputs are also bounded by physical constraints,

$$\begin{bmatrix} 0 \\ \frac{\pi}{2} \\ -\dot{\psi}_{max} \\ -\dot{\varphi}_{max} \end{bmatrix} \leq \mathbf{q}(k) \leq \begin{bmatrix} 2\pi \\ \pi \\ \dot{\psi}_{max} \\ \dot{\varphi}_{max} \end{bmatrix} \tag{14}$$

$$|\mathbf{u}_i| \leq 1 \qquad i = 1, 2, ..., N \tag{15}$$

2.5 Control Policy

Now we have the state space model at hand, given the camera sensor sampling frequency is high enough and for the sake of reducing integral wind-up and increasing noise robustness, one optimal control solution is PD controller which generally fast responses, let $e(t)$ denote the error signal and K_p, K_d denote the proportional and derivative gains,

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt} \tag{16}$$

Because of the electrical components constraints of the sensor and normalization of the input variable, the control input $u(t)$ saturates at value of 1. This operation also changes the ranges of the variables b_1 and b_2 in the $\mathbf{B}$ matrix from the state space model.

2.6 Dependency Analysis

Figure 2 demonstrates the potential relationships between variables in the measurement model, target features and operating conditions are directly related to the sensor measurements while themselves have potential correlation with sensor state and target state. The exact relationships differ with each distinct mathematical model.

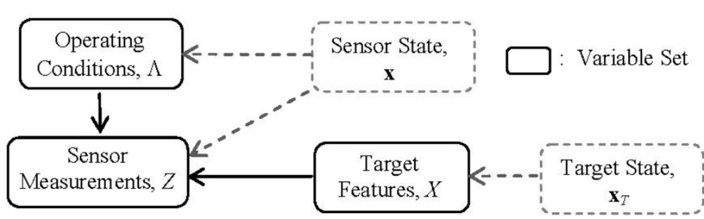


Figure 2: Causal relationships among variables[1].

For the interest of stochastic model simulation, consider a probability heat map over the virtual image plane of the sensor such that the central region has 100% chance of detection and the peripheral region has 50% chance of detection(only inside-FOV region is discussed), and the probability distribution in between is a Gaussian bell curve. The probability of true detection can be described using the formula below:

$$p(x,y) = 0.5 + 0.5 \times e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{17}$$

Now suppose there exists random environmental noise that is also a Gaussian distribution of 0 mean and known standard deviation to both x and y axes. This certainly changes the original probability heat map since the function now becomes:

$$p_{noisy}(x,y) = 0.5 + 0.5 \times e^{-\frac{(x+\epsilon_x)^2+(y+\epsilon_y)^2}{2\sigma^2}} \tag{18}$$

Now if we want to conduct sensitivity analysis on this equation, this can somehow get difficult because ϵ_x and ϵ_y are Gaussian random variables and taking partial derivatives are not trivial. A more realistic approach is to compute sensitivity numerically by evaluating change in $p_{noisy}(x,y)$ when a small disturbance is given to σ_{noise}.

$$\frac{\partial p_{noisy}}{\partial \sigma_{noise}} \approx \frac{p_{noisy}(x,y;\sigma_{noise}+\delta) - p_{noisy}(x,y;\sigma_{noise})}{\delta} \tag{19}$$

Figure 3 below shows how small changes of the standard deviation of noise distribution can affect the FOV detection probability distributions.

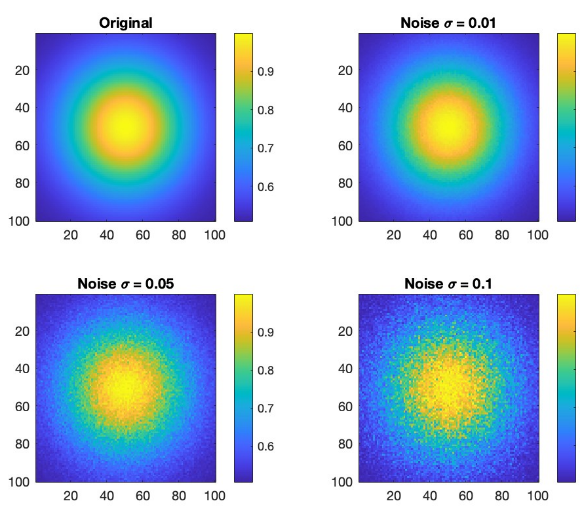


Figure 3: Sensitivity to different noise levels.

3 Procedure

In this project, we will specifically study how the employment of deterministic measurement model and stochastic measurement model affect the sensor measurements and tracking behaviors by comparing Matlab simulation results for two different models.

First we need to construct various functions according to the derived formulas in the Methods section. One for the camera measurements based on current camera state and target state, one for desired angle computation based on camera states and measurements, and one for calculating next move of the camera based on camera state and control input. Then, we can use these functions to iteratively simulate target movements and camera measurements. The sample block diagram describing the simulation is shown in Figure 4,

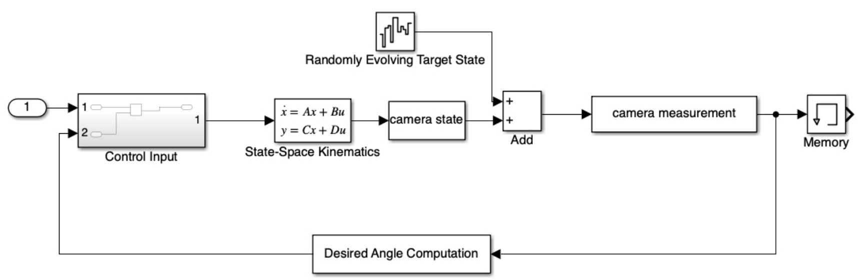


Figure 4: Matlab simulation block diagram.

About the deterministic and stochastic settings, first thing to be determined is that if the target is within the FOV. And an probability value need to be randomly generated at each step in order to simulate the stochastic detection setting. If the target is inside the FOV and the probability value is within the designated range, the camera will be able to detect the target and thus initiate next-step kinematics and measurements, if not, there will be no valid measurement of the target and the new control input for this specific step will be zero. If the target is outside the FOV and the probability value is within the designated range, this will also be a no-detection scenario; on the other hand, if the target is outside the FOV and the probability value falls outside the designated range, this would be a false alarm scenario where the sensor falsely declares a detection signal due to environmental noise. In this case we need to randomly generate a fake target state that is within the FOV. The flow diagram is shown in figure 5. The range and proportionality of the probability value is set according to the previously defined probability distribution in section 2.4.

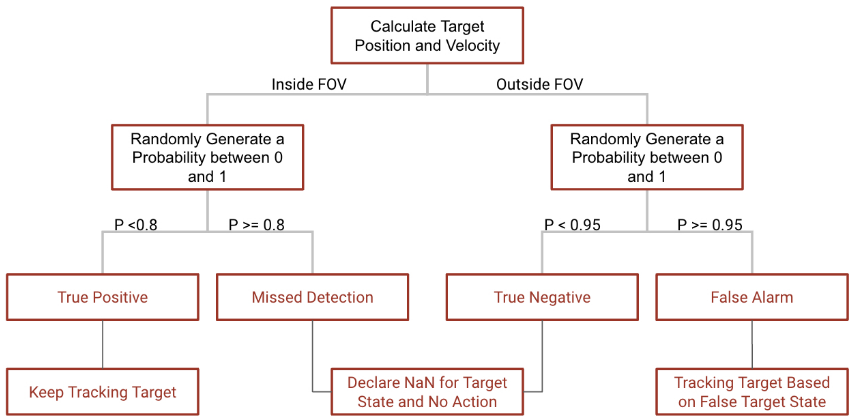


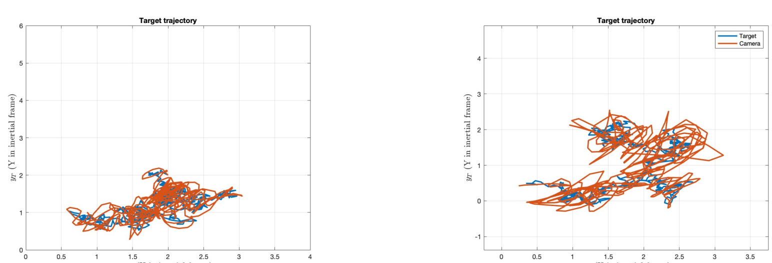
Figure 5: Decision-making flow diagram.

4 Results

In this section, we will examine how the employment of deterministic and uniformly-distributed stochastic models affect the PTZ camera simulation in target tracking. The simulation assumes the target follows the random walk pattern in order to fully demonstrate randomness of target movement and showcase how well the sensor performs in this scenario.

4.1 Trajectory Simulation

One important thing to look at is the trajectory of the camera compared to the trajectory of the target. Considering the uncertainties of the random walk pattern, control input saturation and sampling frequency, it can be foreseen that the tracking will not be perfect as there will be errors and even more so for the stochastic model.



(a) Deterministic Setting

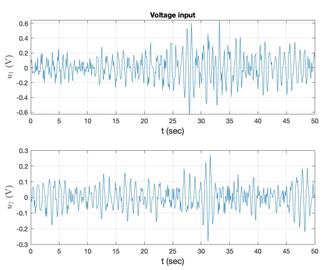
(b) Stochastic Setting

Figure 6: Target and camera trajectories under two settings.

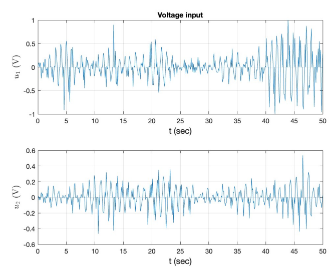
The purpose of the comparison is to observe to what extent the stochastic setting influences the tracking error. It is quite clear that despite the difficulty of tracking a target that is random walking, the deterministic model did a good job minimizing the tracking state error. The trajectories in Figure 6a are mostly overlapped while the situation is a little different for the stochastic model. The camera is still performing well most of the time, but the motion of the camera sometimes becomes wild due to wrong data feed. In the meantime, the state error vector is obviously not as well controlled.

4.2 Camera State Simulation

Besides the trajectory, it is also helpful to keep track of the states of the camera for this simulation. The camera sensor, being a mechatronic device, has some physical constraints that are directly related to its performance. Rotational angles, angular velocities and control voltages are all essential to the stability of the sensor.



(a) Deterministic Setting



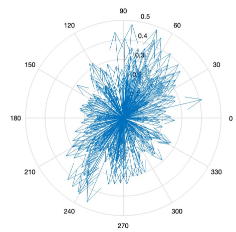
(b) Stochastic Setting

Figure 7: Control input voltages under two settings.

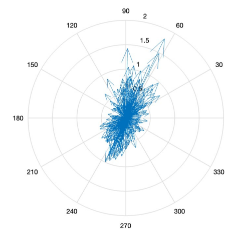
Figure 7 demonstrates the normalized input voltages across the entire simulation. The oscillation pattern is considered to be reasonable for the random walk pattern. As much as the two subfigures look quite alike, we notice that the voltage fluctuation is actually more intense under the stochastic setting given the vertical axis of the Figure 6b covers a larger range, during a few steps the control input is actually pushed close to saturation from zero because the controlled is trying to account for the change in target state over two steps where missed detection or false alarm takes place in between. Since controller input directly impacts camera yaw and roll angles, the pattern will be similar for the rotational motion of the camera. It will undergo more abrupt rotations with significantly larger angular velocities, this will be a challenge on controller design and hardware implementation in a real-world engineering problem.

4.3 Stochastic Model Data Analysis

The probability distribution is defined in section 2.4 and applied to the simulation according to Figure 5. Displaying state error evolution for two distinct settings can be potentially insightful in terms of quantitatively correlating measurement accuracy and sensor capturing expectation.



(a) Deterministic Setting



(b) Stochastic Setting

Figure 8: State error vector compass diagram under two settings.

From Figure 8, we can tell that the state error vectors under the deterministic setting is, generally speaking, uniformly distributed and have magnitude no larger than 0.5 and mean magnitude of 0.2165; while the errors under the stochastic setting has significantly larger magnitude(some approaching 2.0) and the distribution is rather not uniform over all directions due to limited sampling quantity. The mean state error magnitude is 0.3552, increased by 64.1%.

This simulation is based on the assumption that there is a 20% chance of missed detection and 5% chance of false alarm, as we can imagine, if the environmental interference is getting worse thus further influencing the probability distribution, causing more mismatch events, this compass diagram will have state error vectors of larger magnitude that can possibly lead to instability and failure of the physical system.

5 Conclusion

To sum up, this project mainly focuses on PTZ camera sensor performance degradation when stochastic measurement model is introduced, the result section demonstrates how inaccuracy in camera measurement can lead to large errors, abrupt control commands and unexpected sensor movement profiles that are potentially harmful to the stability of the system. Measurements' dependencies on environmental conditions and target state are discussed also under probabilistic setting. Further improvement can be done on the model by simulating more various probability distributions and investigate statistical inference from collected data to make the argument more persuasive.

References

[1] S. Ferrari and T. A. Wettergren, *Information-Driven Planning and Control.* Cambridge, MA: The MIT Press, 2021.