EECE562 (Jan. 2017 – Apr. 2017)
HW Assignment 3: mainly based on the Part-2 notes.
Due day: Mar. 13 (in class)
Notes: The problems are originally from Prof. Vikram Krishnamurthy.

1.
Consider the state space model:

$$x[k+1] = \begin{bmatrix} -1 & -2 \\ 1 & 0 \end{bmatrix} x[k] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[k]$$

$$y[k] = \begin{bmatrix} c_1 & c_2 \end{bmatrix} x[k]$$

(a) Express the following state-space system in ARMA form

(b) Suppose c_1 and c_2 are known. Given the output $y[k]$ is it possible to exactly reconstruct the initial condition $x[0]$?

(c) Suppose the parameters c_1 and c_2 were unknown. Obtain a least squares estimator for them.

2.
Simulate using Matlab, 1000 points of a second order stochastic AR model:

$$y[k] = a_1 y[k-1] + a_2 y[k-2] + w_k$$

where w_k is white Gaussian noise. Choose $a_1 = 1$, $a_2 = 0.1$ and variance of $w_k = 1$.

For what values of a_1, a_2 is the above process asymptotically stationary.

3.
For the stationary case of the above AR model, generate 1000 points. Implement using Matlab a least squares estimator for a_1, a_2.

Examine the accuracy (mean square error in estimate) of the least squares estimate with increasing data length – i.e., 1000 points, 2000 points, 3000 points.

Now pick a_1, a_2 so that the above AR model is not asymptotically stationary. Run your least squares estimator and examine its accuracy versus data length.

4.
From the paper "Linear Prediction: A Tutorial Review" by J . Makhoul, Proc. IEEE, Vol.63, 1975, (or any other paper that deals with Yule Walker based linear predictors), devise a Yule-Walker equation estimator for the above AR model and simulate its performance.

5.

Suppose you are given a system

$$y[k] = \psi[k]\theta + v[k]$$

Here $\psi[k]$ denotes a known regression scalar – simply generate $\psi[k]$ as a zero mean white Gaussian noise process. $v[k]$ is unit variance zero mean white Gaussian noise and θ denotes an unknown scalar parameter that varies slowly with time according to the following system:

$$\theta[k+1] = \theta[k] + \epsilon w[k]$$

where $w[k]$ denotes zero mean unit variance white Gaussian noise and $\epsilon > 0$ is a small constant.

(a) Show by simulation using Matlab that the smaller ϵ is, the slower the parameter $\theta[k]$ evolves with time.

(b) One way of recursively estimating this slowly time varying parameter θ is to use the recursive least squares algorithm with a forgetting factor. Implement this algorithm in Matlab and simulate its performance for small ϵ.