Homework 9

Put your name and student ID here

2020-11-04

Q1: Let $X_1, \ldots, X_n$ be an iid sample of Possion distribution with parameter $\lambda > 0$. Find an approximate $100(1-\alpha)\%$ confidence interval for λ.

Q2: Suppose that an event A was observed 36 times out of 120 independent experiments. Use CLT to find an approximate 95% confidence interval for $P(A)$.

Q3: Let $X_1, \ldots, X_n$ be an iid sample from a distribution with CDF $F(x)$.

(a) Show that the empirical CDF $\hat{F}_n(x)$ is an unbiased estimate of $F(x)$ for any fixed $x \in \mathbb{R}$.

(b) Find the variance of $\hat{F}_n(x)$.

(c) Now suppose that $F(x) = 1 - \exp(-\lambda x)$ for $x > 0$ and 0 otherwise. Inspecting whether the variance of $\hat{F}_n(x)$ attains the lower bound of Cramer-Rao inequality for estimating $F(x)$ with fixed $x > 0$. (In fact, there exists a better unbiased estimator for $F(x)$ than the empirical CDF for this case.)

Q4: True or false, and state why:

1. The significance level of a statistical test is equal to the probability that the null hypothesis is true.
2. If the significance level of a test is decreased, the power of the test would be expected to increase.
3. The probability that the null hypothesis is falsely rejected is equal to the power of the test.
4. A type I error occurs when the test statistic falls in the rejection region of the test.

Q5: A coin is thrown independently 10 times to test the hypothesis that the probability of heads is 1/2 versus the alternative that the probability is not 1/2. The test rejects if either 0 or 10 heads are observed.

1. What is the significance level of the test?
2. If in fact the probability of heads is 0.1, what is the power of the test?

Q6: Suppose that X_1, X_2, X_3 are samples of Bernoulli $B(1, p)$ population. For testing the hypothesis $H_0 : p = 1/2$ *vs.* $H_1 : p = 3/4$, we use a rejection region:

$$W = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 \geq 2\}.$$

1. What are the probabilities of the two types of errors for W?
2. What is the power of the test?