

# Homework 12

*Put your name and student ID here*

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**Q1:** True or false, and state why:

1. If the p-value is 0.03, the corresponding test will reject at the significance level 0.02.
2. If a test rejects at significance level 0.06, then the p-value is less than or equal to 0.06.
3. The p-value of a test is the probability that the null hypothesis is correct.

**Q2:** Mutual funds are investment vehicles consisting of a portfolio of various types of investments. If such an investment is to meet annual spending needs, the owner of shares in the fund is interested in the average of the annual returns of the fund. Investors are also concerned with the volatility of the annual returns, measured by the variance or standard deviation. One common method of evaluating a mutual fund is to compare it to a benchmark, the Lipper Average being one of these. This index number is the average of returns from a universe of mutual funds. The Global Rock Fund is a typical mutual fund, with heavy investments in international funds. It claimed to best the Lipper Average in terms of volatility over the period from 1989 through 2007. Its returns are given in the table below.

Year	Investment Return %	Year	Investment Return %
1989	15.32	1999	27.43
1990	1.62	2000	8.57
1991	28.43	2001	1.88
1992	11.91	2002	-7.96
1993	20.71	2003	35.98
1994	-2.15	2004	14.27
1995	23.29	2005	10.33
1996	15.96	2006	15.94
1997	11.12	2007	16.71
1998	0.37		

The standard deviation for the Lipper Average is 11.67%. Let  $\sigma^2$  denote the variance of the population represented by the return percentages shown in the table above. Consider the test

$$H_0 : \sigma^2 \geq (11.67)^2 \text{ vs. } H_1 : \sigma^2 < (11.67)^2.$$

- If the significance level  $\alpha = 0.05$ , what's your decision?
- Show up the p-value of your test.

**Q3:** The National Center for Health Statistics (1970) gives the following data on distribution of suicides in the United States by month in 1970. Is there any evidence that the suicide rate varies seasonally, or are the data consistent with the hypothesis that the rate is constant (the significance level  $\alpha = 0.05$ )? (Hint: Under the latter hypothesis, model the number of suicides in each month as a multinomial random variable with the appropriate probabilities and conduct a goodness-of-fit test.)

Month	Number of Suicides	Days/Month
Jan.	1867	31
Feb.	1789	28
Mar.	1944	31
Apr.	2094	30

Month	Number of Suicides	Days/Month
May	2097	31
June	1981	30
July	1887	31
Aug.	2024	31
Sept.	1928	30
Oct.	2032	31
Nov.	1978	30
Dec.	1859	31

**Q4:** Under (the assumption of) simple Mendelian inheritance, a cross between plants of two particular genotypes produces progeny 1/4 of which are “dwarf” and 3/4 of which are “giant”, respectively. In an experiment to determine if this assumption is reasonable, a cross results in progeny having 243 dwarf and 682 giant plants. If “giant” is taken as success, the null hypothesis is that  $p = 3/4$  and the alternative that  $p \neq 3/4$ .

- Let  $X_i, i = 1, \dots, n$  be the sample of the population  $B(1, p)$ . By central limit theorem (CLT), the distribution of  $\bar{X}$  can be approximated by a normal distribution  $N(p, p(1 - p)/n)$ . Please use this approximation to do the binominal test above.
- Actually, we can do the exact binominal test according to the formula given in P.114 of our Chinese textbook. Compare the results in the exact test and the approximate test for significance levels  $\alpha = 0.05, 0.01, 0.001$ .