Homework 8

Put your name and student ID here

2021-04-22

Q1: Let $X_1, \ldots, X_n$ be i.i.d. sample of $X \sim N(\mu_1, \sigma^2)$, and $Y_1, \ldots, Y_m$ be i.i.d. sample of $Y \sim N(\mu_2, \sigma^2)$, where X_is and Y_js are independent, $\mu_1, \mu_2 \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Denote $S_X^2 = \frac{1}{n}\sum_{i=1}^n (X_i - \bar{X})^2$ and $S_Y^2 = \frac{1}{m}\sum_{i=1}^m (Y_i - \bar{Y})^2$. Let $S_w = \sqrt{(nS_X^2 + mS_Y^2)/(n+m-2)}$

(a) Prove that S_w^2 is an unbiased estimator for σ^2

(b) Prove that

$$T_{a,b} := \frac{a(\bar{X} - \mu_1) - b(\bar{Y} - \mu_2)}{S_w\sqrt{\frac{a^2}{n} + \frac{b^2}{m}}} \sim t(n+m-2),$$

where a, b are non-zero constants.

(c) Based on the result in (b), find a $100(1-\alpha)\%$ confidence interval for the parameter $\vartheta = a\mu_1 - b\mu_2$.

Q2: Problem 23 in Page 61 of our Chinese textbook.

Q3: Problem 27 in Page 62 of our Chinese textbook.

Q4: Problem 28 in Page 62 of our Chinese textbook.

Q5: Suppose that $X_1, \ldots, X_n$ is an iid sample from the exponential density, $f(x) = \lambda e^{-\lambda x}, x > 0$, and $\lambda > 0$.

(a) Show that $\hat{\lambda}_n = X_1$ is not consistent for λ.

(b) Show that $\hat{\lambda}_n = X_1 + \cdots + X_n$ is not consistent for λ.

Q6: Let $X_1, \ldots, X_n$ be i.i.d. sample from a double exponential distribution whose density is given by

$$f(x) = \frac{1}{2\lambda}\exp(-|x|/\lambda),$$

where $\lambda > 0$.

(a) Find the maximum likelihood estimator (MLE) for λ.

(b) Show that the MLE in Part (a) is consistent for λ.