Homework 10

Put your name and student ID here

2021-05-08

Q1: True or false, and state why:

1. The significance level of a statistical test is equal to the probability that the null hypothesis is true.
2. If the significance level of a test is decreased, the power of the test would be expected to increase.
3. The probability that the null hypothesis is falsely rejected is equal to the power of the test.
4. A type I error occurs when the test statistic falls in the rejection region of the test.

Q2: A coin is thrown independently 10 times to test the hypothesis that the probability of heads is 1/2 versus the alternative that the probability is not 1/2. The test rejects if either 0 or 10 heads are observed.

1. What is the significance level of the test?
2. If in fact the probability of heads is 0.1, what is the power of the test?

Q3: Suppose that X_1, X_2, X_3 are samples of Bernoulli $B(1, p)$ population. For testing the hypothesis $H_0 : p = 1/2$ *vs.* $H_1 : p = 3/4$, we use a rejection region:

$$W = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 \geq 2\}.$$

1. What are the probabilities of the two types of errors for W?
2. What is the power of the test?

Q4: Let $X_1, \ldots, X_n$ be an iid sample of $N(\mu, \sigma^2)$, where μ is known. Show that this model has a monotone likelihood ratio. Given a significance level α, derive a UMP test of the following hypotheses:

$$H_0 : \sigma^2 \geq \sigma_0^2 \ vs. \ H_1 : \sigma^2 < \sigma_0^2;$$

$$H_0 : \sigma^2 \leq \sigma_0^2 \ vs. \ H_1 : \sigma^2 > \sigma_0^2.$$

Q5: Let $X_1, \ldots, X_n$ be an iid sample of the double exponential distribution with PDF $f(x) = \frac{1}{2}\lambda \exp(-\lambda|x|)$, where $\lambda > 0$ is unknown. Show that this model has a monotone likelihood ratio. Given a significance level α, derive a UMP test of the following hypotheses:

$$H_0 : \lambda \geq \lambda_0 \ vs. \ H_1 : \lambda < \lambda_0;$$

$$H_0 : \lambda \leq \lambda_0 \ vs. \ H_1 : \lambda > \lambda_0.$$

Q6: Under the setting of Q5, derive a test (not necessarily UMP) of the two-sided hypothesis

$$H_0 : \lambda = \lambda_0 \ vs. \ H_1 : \lambda \neq \lambda_0$$

for a given level of significance α.

Q7: Under the setting of Q5, derive a UMP test of the hypothesis

$$H_0 : \lambda > \lambda_0 \ vs. \ H_1 : \lambda \leq \lambda_0$$

for a given level of significance α. (Hint: prove that the result in Q5 is also UMP for this case)