

# Homework 10

*Put your name and student ID here*

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**Q1:** Let  $X_1, \dots, X_n$  be an iid sample of  $N(\mu, \sigma^2)$ , where  $\mu$  is known. Show that this model has a monotone likelihood ratio. Given a significance level  $\alpha$ , derive a UMP test of the following hypotheses:

$$H_0 : \sigma^2 \geq \sigma_0^2 \text{ vs. } H_1 : \sigma^2 < \sigma_0^2;$$

$$H_0 : \sigma^2 \leq \sigma_0^2 \text{ vs. } H_1 : \sigma^2 > \sigma_0^2.$$

**Q2:** Let  $X_1, \dots, X_n$  be an iid sample of the double exponential distribution with PDF  $f(x) = \frac{1}{2}\lambda \exp(-\lambda|x|)$ , where  $\lambda > 0$  is unknown. Show that this model has a monotone likelihood ratio. Given a significance level  $\alpha$ , derive a UMP test of the following hypotheses:

$$H_0 : \lambda \geq \lambda_0 \text{ vs. } H_1 : \lambda < \lambda_0;$$

$$H_0 : \lambda \leq \lambda_0 \text{ vs. } H_1 : \lambda > \lambda_0.$$

**Q3:** Under the setting of Q2, derive a test (not necessarily UMP) of the two-sided hypothesis

$$H_0 : \lambda = \lambda_0 \text{ vs. } H_1 : \lambda \neq \lambda_0$$

for a given level of significance  $\alpha$ .

**Q4:** Under the setting of Q2, derive a UMP test of the hypothesis

$$H_0 : \lambda > \lambda_0 \text{ vs. } H_1 : \lambda \leq \lambda_0$$

for a given level of significance  $\alpha$ . (Hint: prove that the result in Q2 is also UMP for this case)