Homework 4

Put your name and student ID here

2020-09-29

Q1: Let $X_1, \ldots, X_n$ be an iid sample of $N(1, 2^2)$. Which of the following items are true? ( )

A. $\frac{\bar{X}-1}{2/\sqrt{n}} \sim t(n)$

B. $\frac{1}{4}\sum_{i=1}^{n}(X_i - 1)^2 \sim F(n, 1)$

C. $\frac{\bar{X}-1}{\sqrt{2}/\sqrt{n}} \sim N(0, 1)$

D. $\frac{1}{4}\sum_{i=1}^{n}(X_i - 1)^2 \sim \chi^2(n)$

Q2: Let $X_1, \ldots, X_n$ be a simple random sample of normal population $N(\mu, \sigma^2)$.

1. Find the mean and variance of S_n^2.
2. Show that $S_n^2 \sim Gamma((n-1)/2, n/(2\sigma^2))$.

Q3: Let $X_1, \ldots, X_n$ be a simple random sample of a distribution with mean μ and variance σ^2. Find the mean of S_n^2 and S_n^{*2}, respectively.

Q4: An iid sample $X_1, \ldots, X_6$ is taken from the density $f_X(x) = 3x^2,\ 0 < x < 1$. Find $P(X_{(6)} > 0.75)$.

Q5: Let $X_1, \ldots, X_m$ be a simple random sample of $N(\mu_1, \sigma_1^2)$, and $Y_1, \ldots, Y_n$ $(n > 3)$ be a simple random sample of $N(\mu_2, \sigma_2^2)$, and the two samples are independent. Denote S_X^{*2} and S_Y^{*2} by the modified sample variances of X_is and Y_is, respectively.

1. Show the PDF of the ratio of the two modified sample variances S_X^{*2}/S_Y^{*2}.
2. Find the mean of S_X^{*2}/S_Y^{*2}, and compare it with the ratio of two population variances σ_1^2/σ_2^2.

(Hint: the mean of $F(m, n)$ distribution is $n/(n-2)$ when $n > 2$)