Homework 5

Put your name and student ID here

2020-10-07

Q1: Let $X_1, \dots, X_n$ be a sample from density

$$f(x; \sigma) = \frac{1}{2\sigma} e^{-|x|/\sigma},$$

where $\sigma > 0$. Find the maximum likelihood estimator for σ.

Q2: Let $X_1, \dots, X_n$ be a sample from $\mathbb{U}[\theta, \theta+1]$, where $\theta \in \mathbb{R}$. Prove that the maximum likelihood estimator (MLE) for θ is not unique, and find out all the MLEs.

Q3: Suppose that the random variable X is taken from $N(0,1)$ and $N(\mu, \sigma^2)$ with equal probability 1/2, where $\mu \in \mathbb{R}, \sigma^2 > 0$. The random variable X actually follows a mixed distribution with density

$$f(x; \mu, \sigma^2) = \frac{1}{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} + \frac{1}{2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Let $X_1, \dots, X_n$ be a sample from the mixed distribution. Prove that the maximum likelihood estimation for μ, σ^2 does not exist. Could you estimate μ, σ^2 via the method of moments?

Q3'(选做): Consider a more general mixed distribution with density

$$f(x; \lambda, \mu_1, \sigma_1, \mu_2, \sigma_2) = \frac{\lambda}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + \frac{1-\lambda}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}},$$

where $\lambda \in [0,1], \mu_1, \mu_2 \in \mathbb{R}, \sigma_1^2, \sigma_2^2 > 0$. How to estimate the five parameters via the method of moments?

Q4: Let $X_1, \dots, X_n$ be a sample from density

$$f(x; \theta) = \frac{\Gamma(\theta+1)}{\Gamma(\theta)\Gamma(1)} x^{\theta-1} 1\{0 < x < 1\},$$

where $\theta > 0$. Use the method of moments to estimate θ.

Q5: Let $X_1, \dots, X_n$ be a sample from density

$$f(x; c, \theta) = \frac{1}{2\theta} 1\{c - \theta \le x \le c + \theta\},$$

where $\theta > 0, c \in \mathbb{R}$. Use the method of moments to estimate c and θ.