Homework 3

Put your name and student ID here

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Q1: Let $X_1, \ldots, X_{15}$ be a simple random sample of $N(0, 2^2)$. What is the distribution of

$$Y = \frac{X_1^2 + \cdots + X_{10}^2}{2(X_{11}^2 + \cdots + X_{15}^2)}?$$

Q2: Let $\boldsymbol{Z} = \boldsymbol{c} + \boldsymbol{A}\boldsymbol{Y}$, where $\boldsymbol{Y}$ is a random vector, $\boldsymbol{A}$ is a fixed matrix, and $\boldsymbol{c}$ is a fixed vector. Prove that

1. the expected value of $\boldsymbol{Z}$: $\mathbb{E}[\boldsymbol{Z}] = \boldsymbol{c} + \boldsymbol{A}\mathbb{E}[\boldsymbol{Y}]$,
2. the covariance matrix of $\boldsymbol{Z}$: $\mathrm{Var}[\boldsymbol{Z}] = \boldsymbol{A}\mathrm{Var}[\boldsymbol{Y}]\boldsymbol{A}^\top$.

Q3: Let $X_1, \ldots, X_n$ be iid sample from $N(\mu, \sigma^2)$, where μ, σ are unknown parameters. Which of the following are statistics? ( ) 多选题

A. $X_1 + X_n - 2\mu$

B. $(X_1 - \mu)/\sigma$

C. $(\bar{X} - 10)/5$

D. $\frac{1}{n}\sum_{i=1}^n (X_i - S_n)^2$

Q4: Suppose that $n = 15, \bar{x}_n = 168, s_n = 11.43, x_{n+1} = 170$. Find the values for $\bar{x}_{n+1}, s_{n+1}^2$ and s_{n+1}^{*2}.

Q5: Show that if X and Y are independent exponential random variables with $\lambda = 1$, then X/Y follows an F distribution. Also, identify the degrees of freedom.

Q6: Suppose that $X = (X_1, \ldots, X_n)^\top \sim N(\mathbf{0}, I_n)$. Show that for any orthogonal matrix U, then $UX \sim N(\mathbf{0}, I_n)$.