Problem set 2 solution

Zhengting (Johnathan) He

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```
# Set up
setwd("D:/OneDrive - Johns Hopkins/Course/140.621.81 - Statistical Methods in Public Health I/Problem set/
jhsphbiostat621-assignment/Problem Set 2")
```

Problem 1. Vitamin A Supplementation to Prevent Children's Mortality in Nepal

Section 1: Probability, Binomial and Poisson Models

i. The Nepal data set is located in the `csv` data file named `nepal621.csv` . Refer to your Class Dataset Code Book for the file format. Load the tidyverse with: `library(tidyverse)` Open the data set and name it `nepal621` . Construct a 2x2 contingency table of treatment (placebo or Vitamin A) against status (alive or dead) at sixteen months of follow-up. Calculate the rate of child mortality in Nepal for children receiving placebo; Vitamin A. Summarize the difference in mortality in a sentence as if for a journal.

```
require(tidyverse)
nepal621 <- read_csv("./data/nepal621.csv")
```

```
CT <- table(nepal621$trt, nepal621$status)
addmargins(CT)
```

```
##
##          Alive  Died   Sum
##   Placebo 13099   290 13389
##   Vit A   13499   233 13732
##   Sum     26598   523 27121
```

```
prop.table(CT, margin=1)
```

```
##
##                Alive       Died
##   Placebo 0.97834043 0.02165957
##   Vit A   0.98303233 0.01696767
```

```
The rate of child mortality in Nepal for children receiving Vitamin A is 0.0170, while the rate for childr
en receiving placebo is 0.0217. The rate difference and ratio for children receiving Vitamin A compared to
receiving placebo is -0.0047 and 0.7834 respectively, indicating children receiving Vitamin A have a lower
mortality rate compared with children receiving placebo in Nepal.
```

ii. For a randomly chosen child from the study population, calculate the following probabilities from the 2x2 contingency table you constructed above.

- *Marginal Probabilities*

```
Pr(VitA) = 13732/27121 = 0.5063
Pr(Died) = 523/27121 = 0.0193
```

- *Joint Probabilities*

```
Pr(Died and VitA) = 233/27121 = 0.0086
Pr(Died and Placebo) = 290/27121 = 0.0107
```

- *Conditional Probabilities*

```
Pr(Died | VitA) = Pr(Died and VitA)/Pr(VitA) = 0.0086/0.5063 = 0.0170
Pr(Died | Placebo) = Pr(Died and Placebo)/Pr(Placebo) = 0.0107/(13389/27121) = 0.0217
```

By hand, use Bayes' Theorem and the 2x2 contingency table to calculate the probability that a child that died received Vitamin A. Use the observed rates for each term below to see how the calculations work.

```
Pr(VitA | Died) = Pr(VitA and Died) / Pr(Died)
                = Pr(Died | VitA)*Pr(VitA) / Pr(Died)
                = Pr(Died | VitA)*Pr(VitA) / (Pr(Died | VitA)*Pr(VitA) + Pr(Died | Placebo)*Pr(Placebo))
                = 0.0170*0.5063 / (0.0170*0.5063 + 0.0217*13389/27121)
                = 0.4455
```

iii. For each treatment group, construct the 2x2 contingency table of sex (male or female) versus vital status (alive or dead). From these tables, calculate the overall probability of dying for males and females separately by treatment group. Describe in a sentence as if for a journal the relationship between mortality and treatment. Does the effect of treatment appear to vary by sex? If so, we say: "sex is an effect modifier," or "sex modifies the effect of vitamin A on mortality" or there is an "interaction" of sex and treatment in causing mortality. Write another sentence or two describing differences in the treatment effect between boys and girls. Be quantitative and use the term "effect modification."

```
nepal.plac <- filter(nepal621, trt == "Placebo")
nepal.vit <- filter(nepal621, trt == "Vit A")

# Placebo group
CT <- table(nepal.plac$sex, nepal.plac$status)
addmargins(CT)
```

```
##
##          Alive  Died   Sum
##   Female  6376   166  6542
##   Male    6723   124  6847
##   Sum    13099   290 13389
```

```
prop.table(CT, margin=1)
```

```
##
##               Alive       Died
##   Female 0.97462550 0.02537450
##   Male   0.98188988 0.01811012
```

```
# Vit A group
CT <- table(nepal.vit$sex, nepal.vit$status)
addmargins(CT)
```

```
##
##          Alive  Died   Sum
##   Female  6544   121  6665
##   Male    6955   112  7067
##   Sum    13499   233 13732
```

```
prop.table(CT, margin=1)
```

```
##
##              Alive       Died
##   Female 0.98184546 0.01815454
##   Male   0.98415169 0.01584831
```

```
The overall probability of dying for males and females separately by treatment group are:
    Pr(Died | Male & Placebo) = 124/6847 = 0.0181
    Pr(Died | Female & Placebo) = 166/6542 = 0.0254
    Pr(Died | Male & Vit A) = 112/7067 = 0.0158
    Pr(Died | Female & Vit A) = 121/6665 = 0.0182

The mortality probability is 0.0023 and 0.0072 lower in vitamin A treatment group compared with placebo gr
oup and the ratio is 0.8751 and 0.7155 in treatment group compared with placebo group, in male and female
subgroups respectively, indicating treatment can reduce mortality probability.

The mortality probability is 0.0024 higher in female subgroup compared with male subgroup, and the ratio i
s 1.1455 in female subgroup compared with male subgroup, in vitamin treatment group. Compared to the overa
ll treatment group, the mortality probability is 0.0012 higher in female subgroup and 0.0011 lower in male
subgroup, and the ratio is 1.0699 and 0.9340 for female and male subgroups respectively compared to the ov
erall treatment group. These evidence indicates the effect of treatment appear to vary by sex; and, sex is
an effect modifier for the effect of vitmain A on mortality.
```

iv. Summarize in a table and/or figure the evidence (data) relevant to the null hypotheses that:

1. Vitamin A supplementation has no effect on mortality in Nepali pre-school children;

and

2. The treatment effect is the same for both boys and girls (i.e., “not modified by sex”).

Group	Evidence	Conclusion
Null hypothesis (1)	Vitamin A group compared with placebo group Death probability difference = -0.0047 Death probability ratio = 0.7834	Null hypothesis (1) may not hold, Vitamin A supplementation may have effect on mortality in Nepali pre-school children
Null hypothesis (2)	In vitamin A treatment group Male subgroup compared with female subgroup Death probability difference = -0.0023 Death probability ratio = 0.8730	Null hypothesis (2) may not hold, sex may be an effect modifier for the association of treatment effect and mortality.

v. Consider a family with 3 boys and 2 girls who received placebo. Suppose that each child’s survival is independent of all the other children in the family. Calculate the probability that 0, 1, 2 or 3 boys die during the study follow-up.

```
x ~ Bin(3, 0.0181)
P(x = 0) = 0.94668
P(x = 1) = 0.05235
P(x = 2) = 0.00097
P(x = 3) = 0.00001
```

vi. Use the Poisson approximation to the binomial distribution to recalculate probabilities of 0, 1, 2 or 3 boys dying in problem v.

μ = np = 3*0.0181 = 0.0543

```
x ~ Pois(0.0543)
P(x = 0) = 0.94715
P(x = 1) = 0.05143
P(x = 2) = 0.00140
P(x = 3) = 0.00003
```