Product Level Weight Inference

The utility form
$u(p) = w_0 * v_0(p) + w_1 * v_1(p) + \cdots + w_n * v_n(p)$
where $w_i, v_i(p)$ is the *weight* and *value function* on i-th attribute for product p.

How to calculate the possible (uncertain) weights?

When weights are uncertain, a set of possible product orderings is induced. When some preferences are given as input, the feasible weights can be computed as shown in the Figure 1.

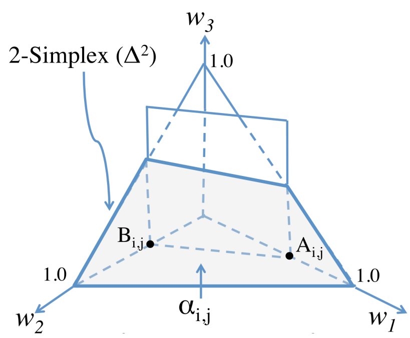

Figure 1: Geometrical representation of feasible weights

For example, if given the product p^* contains the highest utility among the product $p_i \in \mathcal{P}$, we have the following constraints, defined as follows:

$$\begin{gathered}
w_0 * v_0(p^*) + w_1 * v_1(p^*) + \cdots + w_n * v_n(p^*) > w_0 * v_0(p^1) + w_1 * v_1(p^1) + \cdots + w_n * v_n(p^1) \\
w_0 * v_0(p^*) + w_1 * v_1(p^*) + \cdots + w_n * v_n(p^*) > w_0 * v_0(p^2) + w_1 * v_1(p^2) + \cdots + w_n * v_n(p^2) \\
\cdots > \cdots \\
w_0 * v_0(p^*) + w_1 * v_1(p^*) + \cdots + w_n * v_n(p^*) > w_0 * v_0(p^k) + w_1 * v_1(p^k) + \cdots + w_n * v_n(p^k) \\
w_0 + w_1 + \cdots + w_n = 1
\end{gathered}$$

With the constraints as defined as above, we can get some feasible weights which can be geometrically represented as polyhedron.

How to infer the weights ordering given the feasible weights?

we can randomly draw sample weights from the feasible region, each sample weights will induce a weight ordering. These weight orderings from different weight samples can be aggregated into one final weight orderings. This final ordering represent the most possible weight ordering given the feasible weight region.

After we get the weight ordering, the attributes which have the top 2 weights can be identified as the most-important attributes, the attributes which have low 2 weights can be identified as the least-important attributes, and the other 3 attributes can be identified as important attributes.

Alternation 1: Conditional Probability

The probability of h given the evidence e

$$p(h|e) = \frac{N(h \wedge e)}{N(e)} (2)$$

where $N(e)$ denotes the number of observations.

Alternative 2: Formal definition of *Weight of Evidence (WOE)*

The *weight* in favour of a hypothesis h, provided by evidence e:

$$woe(h : e) = \log \frac{O(h|e)}{O(h)} (1)$$

where

$$O(h) = \frac{p(h)}{p(\bar{h})} = \frac{p(h)}{1 - p(h)} (1)$$

is the *prior* odds of the hypothesis, h being true, and

$$O(h|e) = \frac{p(h|e)}{p(\bar{h}|e)} = \frac{p(h|e)}{1 - p(h|e)} (2)$$

is the *posterior* odds of the hypothesis h being true condition on evidence e having been observed. WOE could be used to estimate the probability that hypothesis h is true, based on the the presence of evidence e.

WOE *measures likelihood of what has been observed:*

$$woe(h : e) = \log \frac{p(e|h)}{p(e|\bar{h})} (2)$$

From this expression, we see that the WOE can be viewed as how much more likely we would be to see the evidence given that the hypothesis were true, relative to the likelihood of observing the same evidence were it to be false.

WOE is additive

$$woe(h : e_1 \wedge e_2) = woe(h : e_1) + woe(h : e_2|e_1) (2)$$

This property states that the weight in favor of a hypothesis provided by two sources of evidence taken together is equal to the weight provided by the first piece of evidence, plus the weight provided by the second piece of evidence.