2024_0114_0148_12

1 Instructions

The properties of a conditional (pair of hypothesis-conclusion) determine whether it can be linearized by a specific transformation technique or not. For each transformation techniques listed in section 2, the requirements on the properties of the input conditional will be clearly discussed. Therefore, it is necessary to firstly explain these properties.

1.1 Properties

TODO

1.2 Legend of Values for Properties

- In red color: indicating that the current transformation technique is not applicable.
- In blue color: indicating that the current transformation technique is applicable, but some adaptation is needed.
- In green color: indicating that the current transformation technique is applicable and can be directly applied.
- In teal color: indicating that the current transformation technique is applicable but too complex, and it seems that there are some other simpler transformation that is enough.

2 Useful Transformations

2.1 Difference-Based Binary-Hypothesis Selector

- Hypothesis
  - **hypothesis:variable_side:range_space**? binary | linearly_separable | neither |
- Conclusion
  - **conclusion:direction**? only_R_side | eq | neq | leq | geq | le | ge |
  - **conclusion:L_side:variable**?
  - **conclusion:L_side:range**?
  - **conclusion:R_side:R_0_type**? ignored | constant | variable |
  - **conclusion:R_side:R_1_type**? ignored | constant | variable |
  - **conclusion:R_side:$(R_0 - R_1)$_type**? any_ignored | constant | variable |

$$
\begin{aligned}
&(L) \quad (\text{compare}) \quad \begin{cases} R_0, & (\mathbf{a}^T\mathbf{x} + b) = 0 \\ R_1, & (\mathbf{a}^T\mathbf{x} + b) = 1 \end{cases} \\
&\Updownarrow \\
&(L) \quad (\text{compare}) \quad R_0 + (\mathbf{a}^T\mathbf{x} + b) \cdot (R_1 - R_0)
\end{aligned} \tag{1}
$$

2.2 Big-M-Based Binary-Hypothesis Selector

- Hypothesis
  - **hypothesis:variable_side:range_space**? binary | linearly_separable | neither |
- Conclusion
  - **conclusion:direction**? only_R_side | eq | neq | leq | geq | le | ge |
    - geq: using $\geq$ and $-|M|$ instead.
    - eq : decomposing into one $\leq$ and one $\geq$.
  - **conclusion:L_side:variable**?
  - **conclusion:L_side:range**?
  - **conclusion:R_side:R_0_type**? ignored | constant | variable |
  - **conclusion:R_side:R_1_type**? ignored | constant | variable |
  - **conclusion:R_side:$(R_0 - R_1)$_type**? any_ignored | constant | variable |

$$
\begin{gathered}
L \leq \begin{cases} R_0, & (\mathbf{a}^T\mathbf{x}+b)=0 \\ R_1, & (\mathbf{a}^T\mathbf{x}+b)=1 \end{cases} \\
\Updownarrow \\
L \leq \begin{cases} R_0, & (\mathbf{a}^T\mathbf{x}+b)=0 \\ \text{ignored}, & (\mathbf{a}^T\mathbf{x}+b)=1 \end{cases} \quad \wedge \quad L \leq \begin{cases} \text{ignored}, & (\mathbf{a}^T\mathbf{x}+b)=0 \\ R_1, & (\mathbf{a}^T\mathbf{x}+b)=1 \end{cases} \\
\Updownarrow \\
L \leq R_0 + ((\mathbf{a}^T\mathbf{x}+b)-0)\cdot|M| \quad \wedge \quad L \leq R_1 + (1-(\mathbf{a}^T\mathbf{x}+b))\cdot|M|
\end{gathered} \tag{2}
$$

2.3

(3)

(4)

(5)

(6)