1 PRELIMINARIES 1

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## 1 Preliminaries

## 2 Useful Transformations

## 2.1 Difference-Based Binary-Hypothesis Selector

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• Hypothesis
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- hypothesis:variable_side:range_space? binary | linearly_separable | neither |
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• Conclusion

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- \  \, {\bf conclusion: direction?} \qquad {\bf only\_R\_side} \mid {\bf eq} \mid {\bf neq} \mid {\bf leq} \mid {\bf geq} \mid {\bf le} \mid {\bf ge} \mid \\ - \  \, {\bf conclusion: L\_side: variable?} \\ - \  \, {\bf conclusion: L\_side: range?} \\ - \  \, {\bf conclusion: R\_side: } R_0 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: R\_side: } R_1 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: R\_side: } R_1 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: R\_side: } R_1 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: R\_side: } R_1 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: } R_1 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored} \mid {\bf constant} \mid {\bf variable} \mid \\ - \  \, {\bf conclusion: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf type?} \qquad \qquad {\bf ignored: } R_2 \_ {\bf t
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(L) (compare) 
$$\begin{cases} R_0, & (\mathbf{a}^T \mathbf{x} + b) = 0 \\ R_1, & (\mathbf{a}^T \mathbf{x} + b) = 1 \end{cases}$$

$$\updownarrow$$
(L) (compare) 
$$R_0 + (\mathbf{a}^T \mathbf{x} + b) \cdot (R_1 - R_0)$$

any\_ignored | constant | variable |

any\_ignored | constant | variable |

## 2.2 Big-M-Based Binary-Hypothesis Selector

- conclusion:R\_side: $(R_0 - R_1)$ \_type?

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- Hypothesis
  - hypothesis:variable\_side:range\_space? binary | linearly\_separable | neither |
- Conclusion

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- conclusion:direction? only_R_side | eq | neq | leq | geq | le | ge | 
* geq: using ≥ and -|M| instead.

* eq: decomposing into one ≤ and one ≥.

- conclusion:L_side:variable?

- conclusion:L_side:range?

- conclusion:R_side:R_0_type? ignored | constant | variable | 
- conclusion:R_side:R_1_type?
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$$L \leq \begin{cases} R_0, & (\mathbf{a}^T \mathbf{x} + b) = 0 \\ R_1, & (\mathbf{a}^T \mathbf{x} + b) = 1 \end{cases}$$

$$\updownarrow$$

$$L \leq \begin{cases} R_0, & (\mathbf{a}^T \mathbf{x} + b) = 0 \\ \text{ignored}, & (\mathbf{a}^T \mathbf{x} + b) = 1 \end{cases} \qquad \land \qquad L \leq \begin{cases} \text{ignored}, & (\mathbf{a}^T \mathbf{x} + b) = 0 \\ R_1, & (\mathbf{a}^T \mathbf{x} + b) = 1 \end{cases}$$

$$\updownarrow$$

$$L \leq R_0 + ((\mathbf{a}^T \mathbf{x} + b) - 0) \cdot |M| \qquad \land \qquad L \leq R_1 + (1 - (\mathbf{a}^T \mathbf{x} + b)) \cdot |M|$$

2.3