高 1D 冬期第 3 週 宿題解答

(205)

(2)

$$\begin{aligned} I_2 &= \int_{\alpha}^{\beta} -(x-\alpha)(x-\beta)dx = \int_{0}^{\beta-\alpha} -x\{x-(\beta-\alpha)\}dx \\ &= \left[-\frac{1}{3}x^3 + \frac{1}{2}(\beta-\alpha)x^2\right]_0^{\beta-\alpha} = -\frac{1}{3}(\beta-\alpha)^3 + \frac{1}{2}(\beta-\alpha)^3 \\ &= \boxed{\frac{1}{6}(\beta-\alpha)^3} \end{aligned}$$

(3)

$$I_4 = 2\int_{-1}^{3}(x+1)(x-3)dx = -2\times\frac{1}{6}\times(3+1)^3 = \boxed{-\frac{64}{3}}$$

(4)

$$I_5 = 6\int_{-\frac{1}{3}}^{\frac{1}{2}}\left(x+\frac{1}{3}\right)\left(x-\frac{1}{2}\right)dx = -6\times\frac{1}{6}\times\left(\frac{1}{2}-\frac{1}{3}\right) = \boxed{-\frac{125}{216}}$$

(206)

(1)

$$\begin{aligned} I_1 &= \int_{2-\sqrt{3}}^{2+\sqrt{3}}\left\{x-\left(2-\sqrt{3}\right)\right\}\left\{x-\left(2+\sqrt{3}\right)\right\}dx \\ &= -\frac{1}{6}\times\left\{(2+\sqrt{3})-(2-\sqrt{3})\right\}^3 = \boxed{-4\sqrt{3}} \end{aligned}$$

(2)

$$\begin{aligned} I_2 &= \int_1^2 (x-1)^2(x-2)dx = \int_0^1 x^2(x-1)dx \\ &= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_0^1 = \frac{1}{4}-\frac{1}{3} = \boxed{-\frac{1}{12}} \end{aligned}$$

♠ 一般に

$$\int_{\alpha}^{\beta}(x-\alpha)^2(x-\beta)dx$$

の値はどうなるか？