高 1D 第 11 週復習問題

$|\overrightarrow{a}| = 2,\ |\overrightarrow{b}| = 3,\ \overrightarrow{a} \cdot \overrightarrow{b} = 4$ とする.

(1) $(\overrightarrow{a} - \overrightarrow{b}) \cdot (\overrightarrow{a} + 3\overrightarrow{b})$ を求めよ.

(2) $|\overrightarrow{a} - \overrightarrow{b}|^2$ を求めよ.

(3) $|\overrightarrow{a} - \overrightarrow{b}|$ を求めよ.

(405) の残りの解答

(3) (あ)

$$|\overrightarrow{a} + 2\overrightarrow{b}|^2 = (\overrightarrow{a} + 2\overrightarrow{b}) \cdot (\overrightarrow{a} + 2\overrightarrow{b})$$
$$= |\overrightarrow{a}|^2 + 4\overrightarrow{a} \cdot \overrightarrow{b} + |\overrightarrow{b}|^2$$
$$= 4 + 4 \times 1 + 4 \times 9 = \boxed{44}$$

(い) (あ) より $|\overrightarrow{a} + 2\overrightarrow{b}| = \boxed{2\sqrt{11}}$

(う)

$$|3\overrightarrow{a} + \overrightarrow{b}|^2 = (3\overrightarrow{a} + \overrightarrow{b}) \cdot (3\overrightarrow{a} + \overrightarrow{b})$$
$$= 9|\overrightarrow{a}|^2 + 6\overrightarrow{a} \cdot \overrightarrow{b} + |\overrightarrow{b}|^2$$
$$= 9 \times 4 + 6 \times 1 + 9 = 51$$

なので $|3\overrightarrow{a} + \overrightarrow{b}| = \boxed{\sqrt{51}}$

(4) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC},\ \overrightarrow{BD} = -\overrightarrow{AB} + \overrightarrow{BC}$ なので

$$AC^2 + BD^2 = |\overrightarrow{AB} + \overrightarrow{BC}|^2 + |-\overrightarrow{AB} + \overrightarrow{BC}|^2$$
$$= (|\overrightarrow{AB}|^2 + 2\overrightarrow{AB} \cdot \overrightarrow{BC} + |\overrightarrow{BC}|^2) + (|\overrightarrow{AB}|^2 - 2\overrightarrow{AB} \cdot \overrightarrow{BC} + |\overrightarrow{BC}|^2)$$
$$= 2(|\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2) = 2 \times (4 + 9) = \boxed{26}$$