

AI assignment 3

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1 Q 1.1

Let P and Q be the user-feature and feature-item matrix, respectively.

$$P \times R = \hat{r}_{ij}$$

The get the predicted rating value for $P_i^{th}user$ and Q_j^{th} item will be

$$\hat{r}_{ij} = \sum_{k=1}^k P_{ik} Q_{kj}$$

which can be written as the dot product of vectors as

$$\hat{r}_{ij} = P_i \cdot Q_j \quad (1)$$

$$e_{ij}^2 = (r_{ij} - \hat{r}_{ij})^2 \quad (2)$$

Putting equation 1 in equation 2

$$e_{ij}^2 = (r_{ij} - P_i \cdot Q_j)^2 \quad (3)$$

Now we will differentiate 3 with respect to each variable separately in order to know in which direction we have to modify P_i and P_j

$$\frac{\partial}{\partial P_i} e_{ij}^2 = -2(r_{ij} - P_i \cdot Q_j)(Q_j) = -2e_{ij}Q_j$$

$$\frac{\partial}{\partial Q_j} e_{ij}^2 = -2(r_{ij} - P_i \cdot Q_j)(P_i) = -2e_{ij}P_i$$

In order to minimize the error now we will change the value of P_i and P_j

$$P_i' = P_i + 2\alpha e_{ij} Q_j$$

$$Q_j' = Q_j + 2\alpha e_{ij} P_i$$

2 1.2

In order to cater user and item biases user bias vector (bU) and item bias vector (bI) are introduced to the predictions.

$$\hat{R}_{ij} = bU_i + bI_j + P_i \cdot Q_j \quad (4)$$

Now we will substitute 4 into the error equation, equation 2

$$e_{ij}^2 = (r_{ij} - (bU_i + bI_j + P_i \cdot Q_j))^2 \quad (5)$$

Now we will differentiate 5 with respect to each bU_i and bI_j

$$\frac{\partial}{\partial bU_i} e_{ij}^2 = 2(r_{ij} - (bU_i + bI_j + P_i \cdot Q_j)) = -2e_{ij}$$

$$\frac{\partial}{\partial bI_j} e_{ij}^2 = 2(r_{ij} - (bU_i + bI_j + P_i \cdot Q_j)) = -2e_{ij}$$

Now we move in the found direction and update bU_i and bI_j .

$$bU_i' = bU_i + \alpha \frac{\partial}{\partial bU_i} e_{ij}^2 = bU_i + 2\alpha e_{ij}$$

$$bI_j' = bI_j + \alpha \frac{\partial}{\partial bI_j} e_{ij}^2 = bI_j + 2\alpha e_{ij}$$