FYSS360 Numerical exercise

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A single particle electron model was used to estimate the loss cone of electrons in an ECR ion source. It is also a demonstration of the confinement of electrons (and ions) in a magnetic bottle formed by the generated solenoid and hexapole magnetic fields.

The on-axis solenoid field $B_z(r = 0, z)$ and the hexapole field $\mathbf{B}(x, y)$ were plotted (figures 1 and 2). Slices of the total magnetic field were plotted as well at $(x = 0, y, z)$ and $(x, y = 0, z)$ (figures 3 and 4).

Electrons with uniform random velocities (with constant energy 10 keV) and positions inside the cylindrical plasma chamber ($z_1 = -160\,\text{mm}$, $z_2 = 170\,\text{mm}$ and $r = 39\,\text{mm}$) were generated. Further selection was done with respect to the position: only electrons with position $\mathbf{r}$ such that $|\mathbf{B}(\mathbf{r})| < 0.5\,\text{T}$ were accepted for the simulation — these correspond to electrons within the ECR zone.

The paths of the electrons were tracked independently, one at a time. The Boris integrator was used to solve the equations of motion in the presence of a static magnetic field. In order to determine the timestep length sufficient to maintain stability, the trajectory of one random particle was tracked for $10^{-8}\,\text{s}$ and plotted with timestep lengths 10^{-10}, 10^{-11} and $10^{-12}\,\text{s}$ (figures 5, 6 and 7 respectively). From these figures it is immediately seen that at $\Delta t = 10^{-10}\,\text{s}$ the solution is unstable. At $\Delta t = 10^{-11}\,\text{s}$ it seems that for this trajectory stability is achieved. In the end, a timestep length $\Delta t = 5 \times 10^{12}\,\text{s}$ was chosen in order to have a safety margin.

In order to have sufficient statistics, the number of test particles was chosen to be $N = 10^5$. Particles were tracked until they hit any of the cylinder ends and walls or the confinement time of $10^{-6}\,\text{s}$ was elapsed. The locations where the electrons hit the geometry were recorded. Histograms for the end circles at $z_1 = -160\,\text{mm}$ and $z_2 = 170\,\text{mm}$ and the radial wall are shown in figures 8, 9 and 10 respectively.

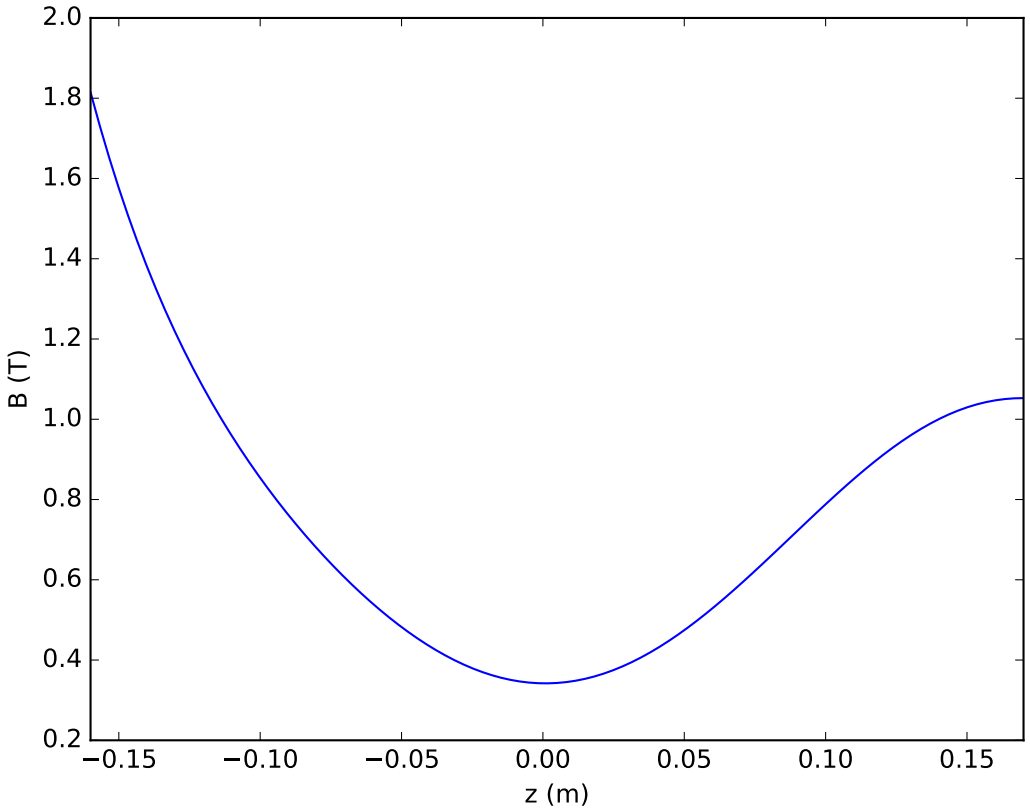

Figure 1: Solenoid on-axis magnetic field $B_z(r = 0, z)$.

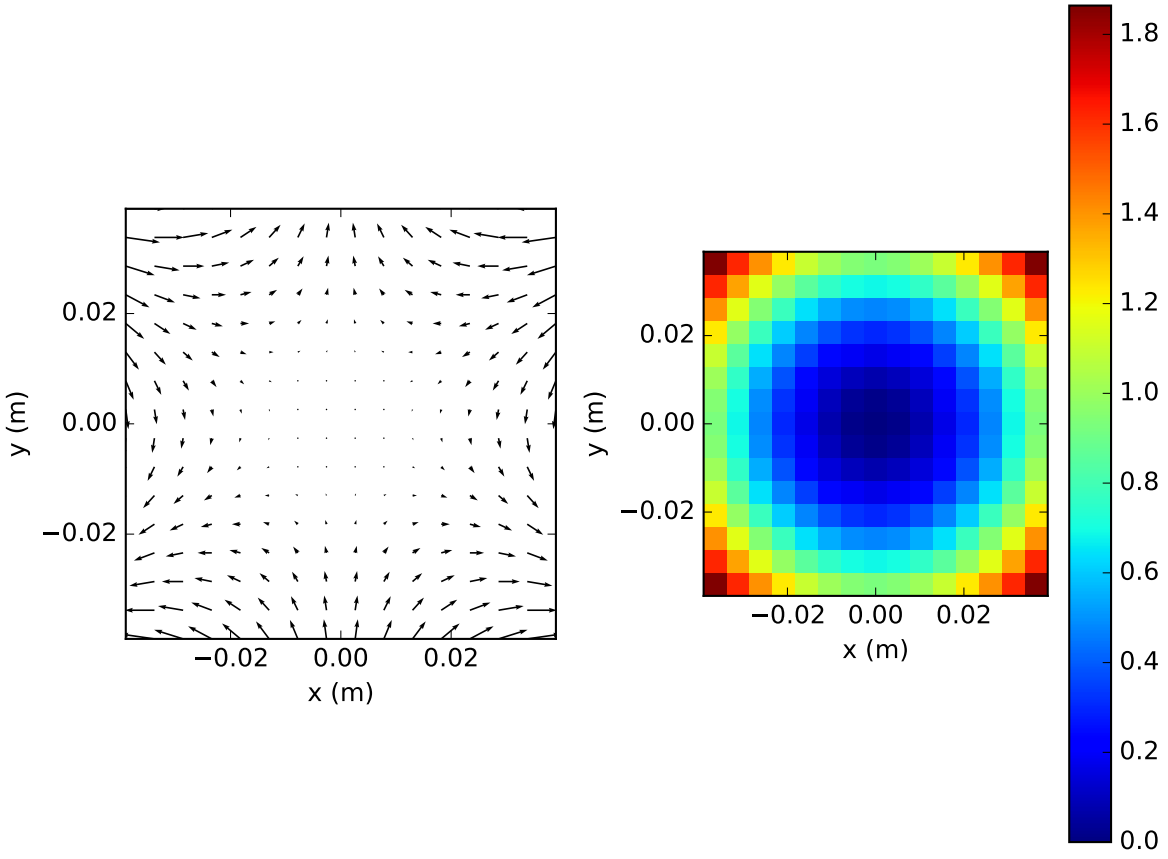

Figure 2: The direction and magitude of the hexapole B field.

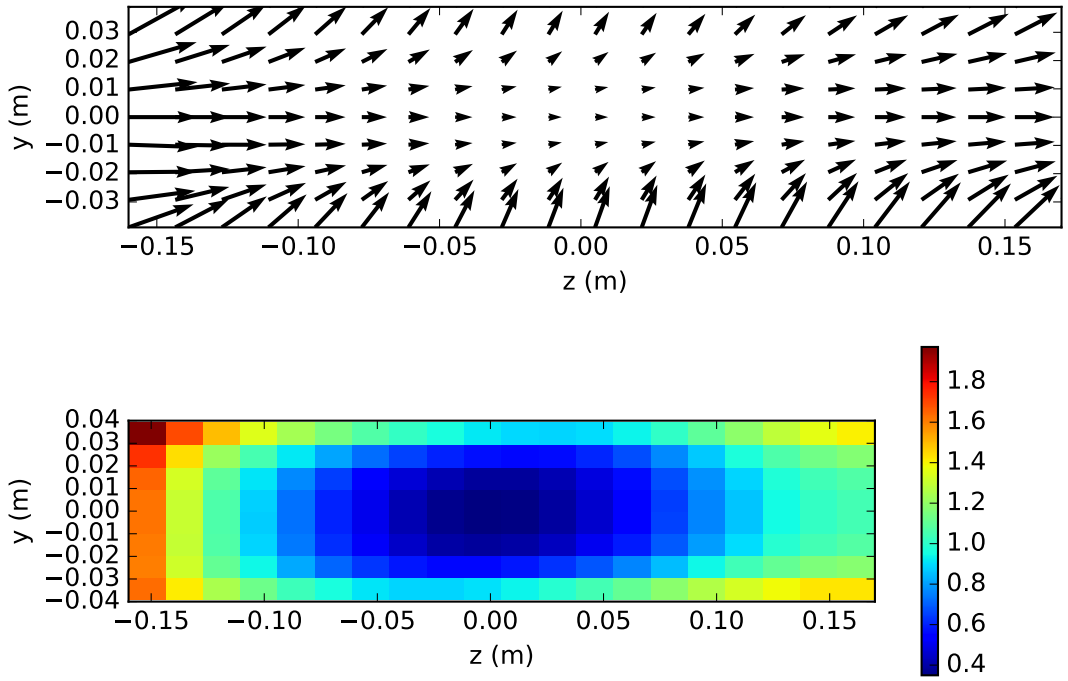

Figure 3: A slice of the total magnetic field $\mathbf{B}(x = 0, y, z)$.

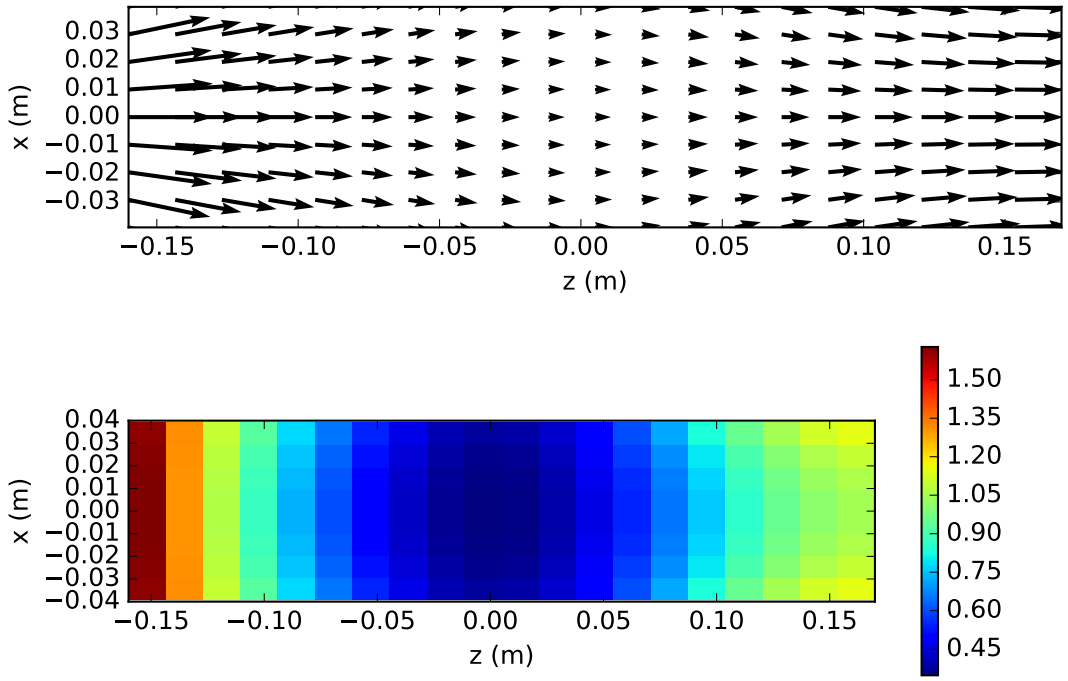

Figure 4: A slice of the total magnetic field $\mathbf{B}(x, y = 0, z)$.

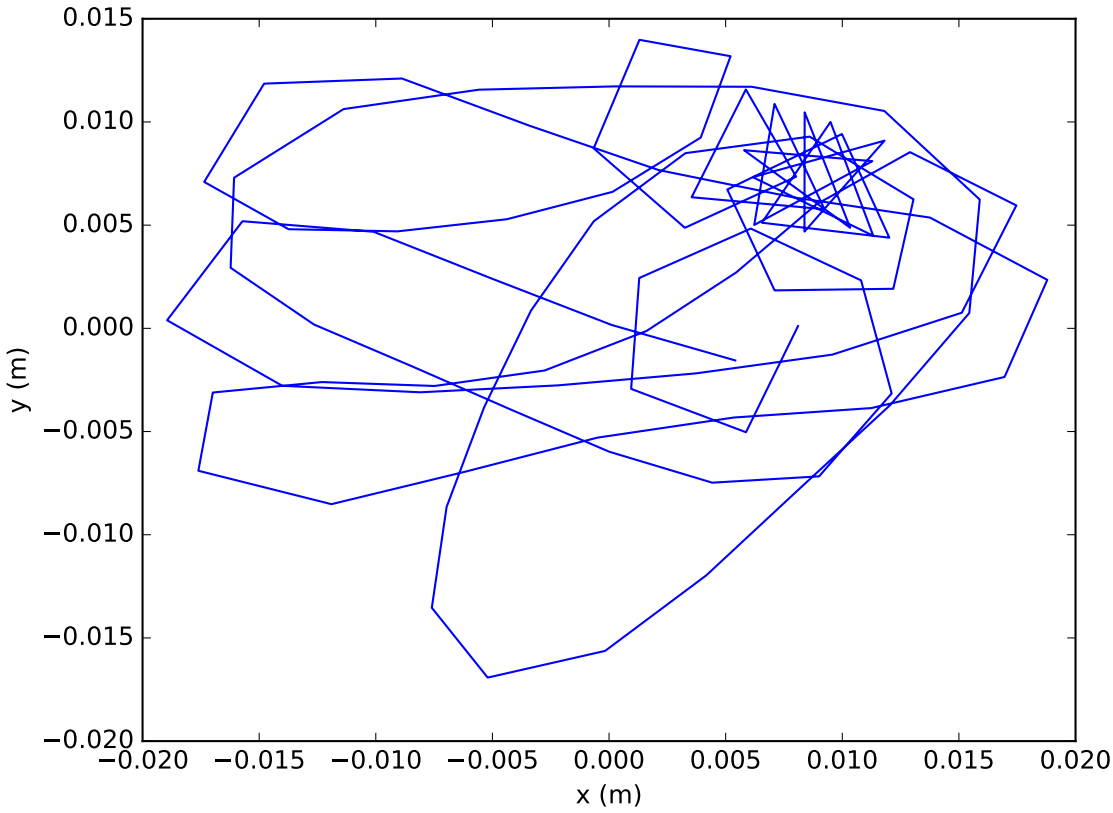

Figure 5: The trajectory of a particle with timestep $\Delta t = 10^{-10}\,\mathrm{s}$.

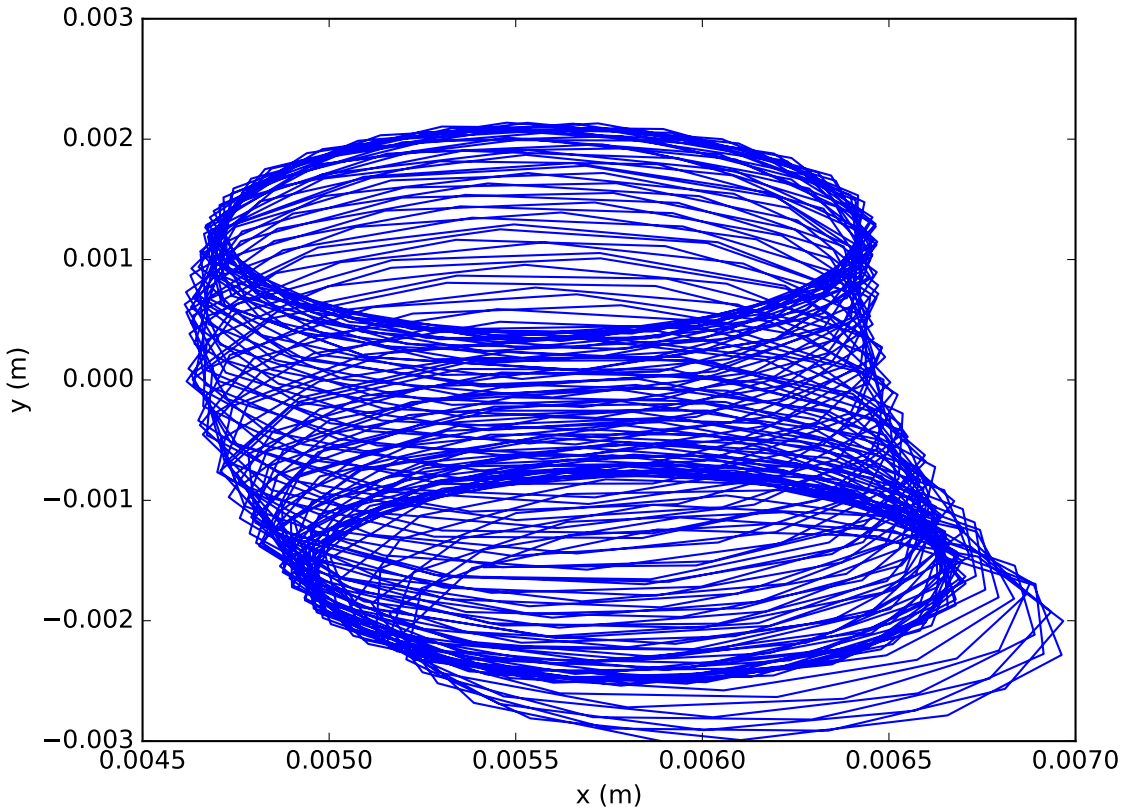

Figure 6: The trajectory of a particle with timestep $\Delta t = 10^{-11}\,\mathrm{s}$.

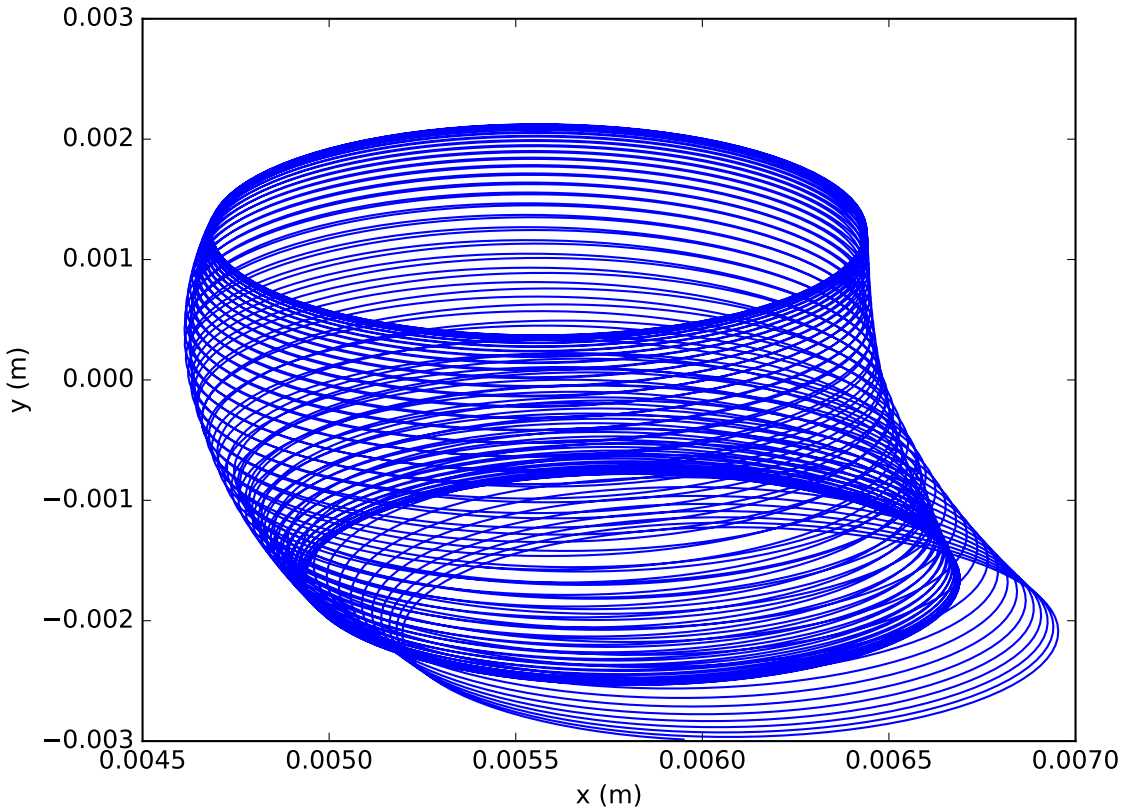

Figure 7: The trajectory of a particle with timestep $\Delta t = 10^{-12}\,\mathrm{s}$.

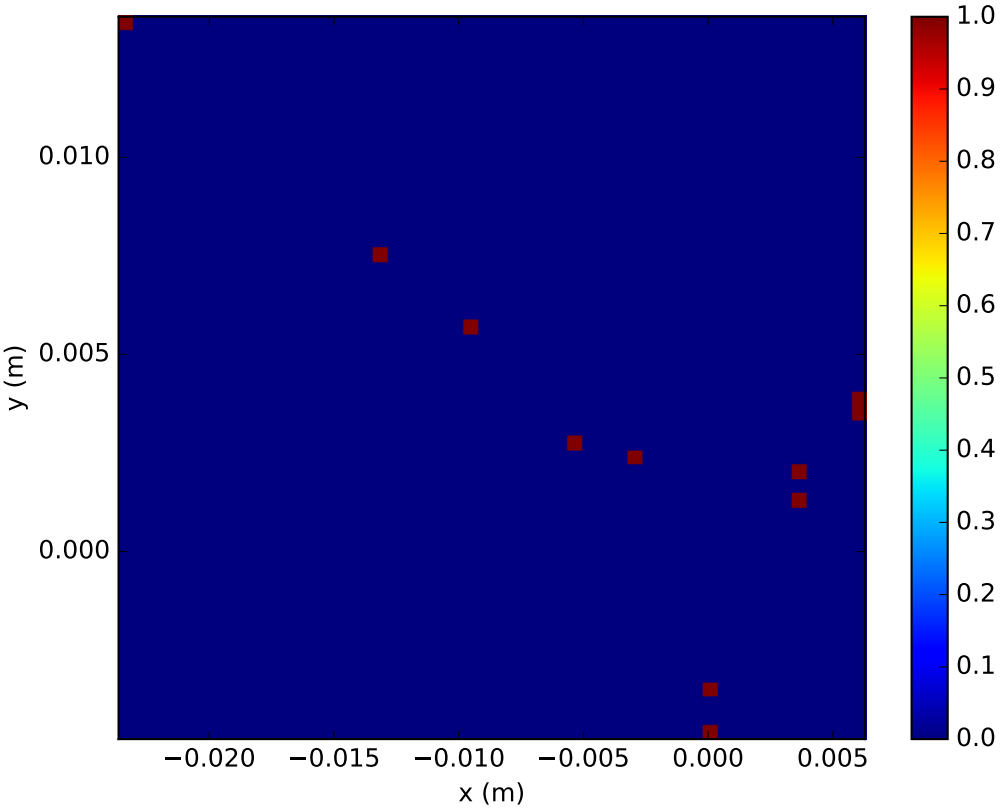

Figure 8: Histogram of electron collision locations at the end circle $z_1 = -160\,\mathrm{mm}$.

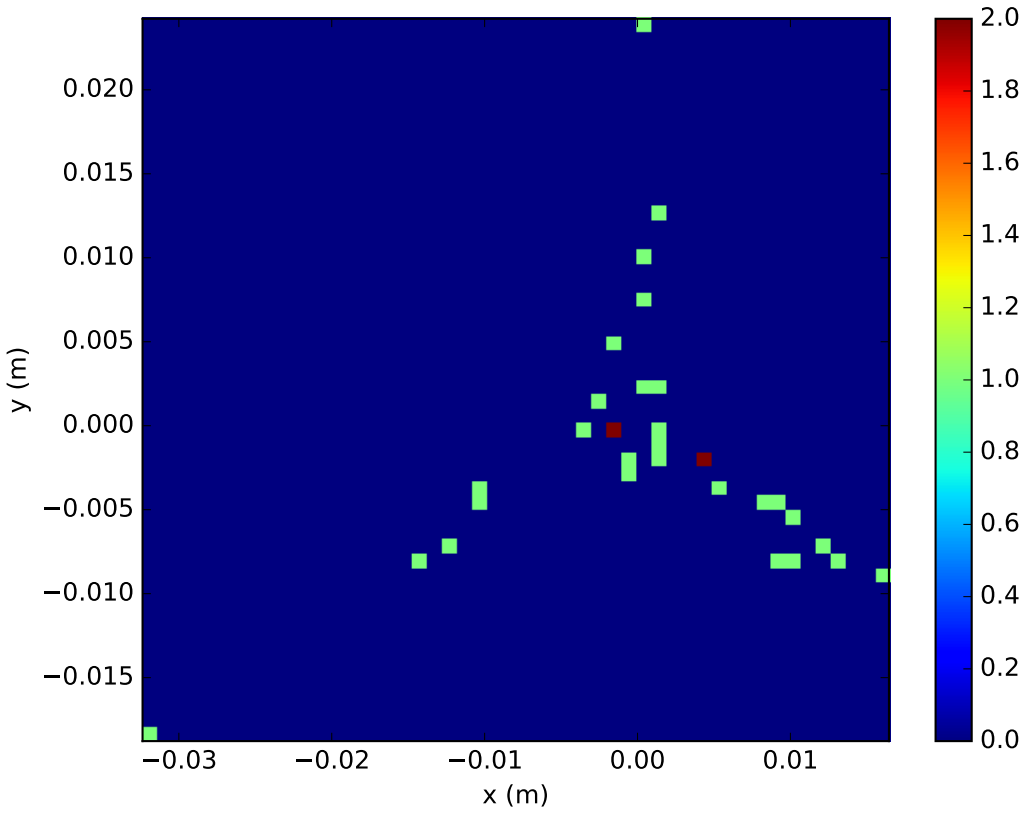

Figure 9: Histogram of electron collision locations at the end circle $z_2 = 170\,\mathrm{mm}$.

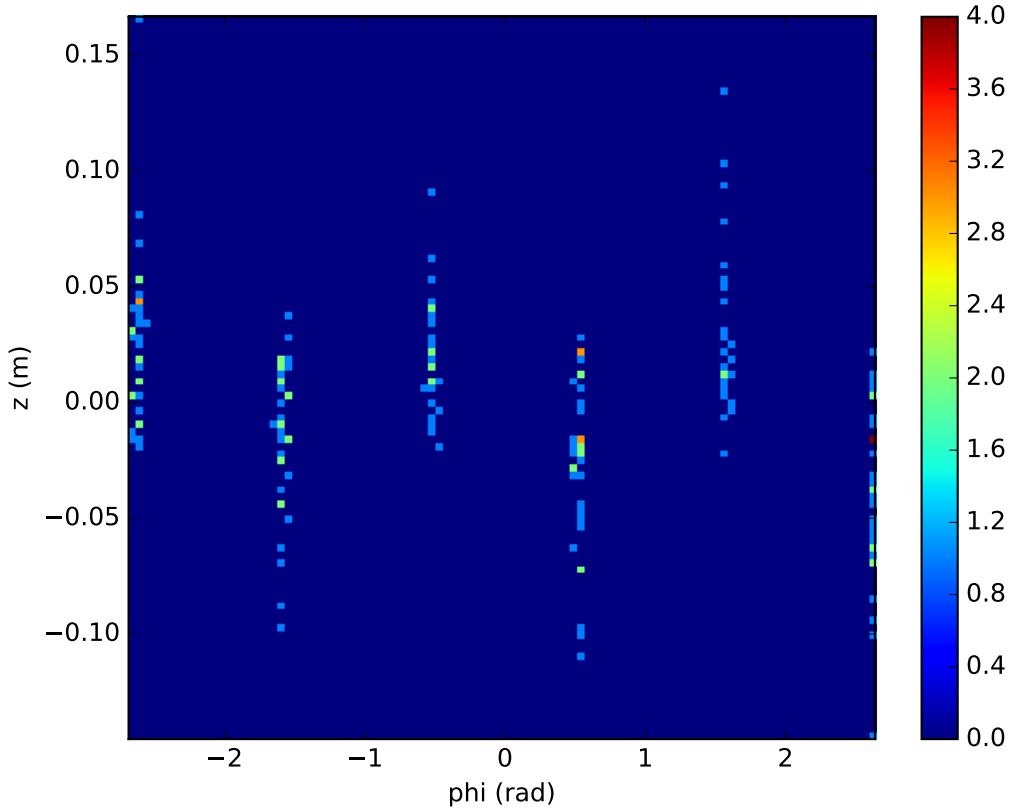

Figure 10: Histogram of electron collision locations at the wall of the cylinder.