Parallel Programming

Prefix sum (scan)

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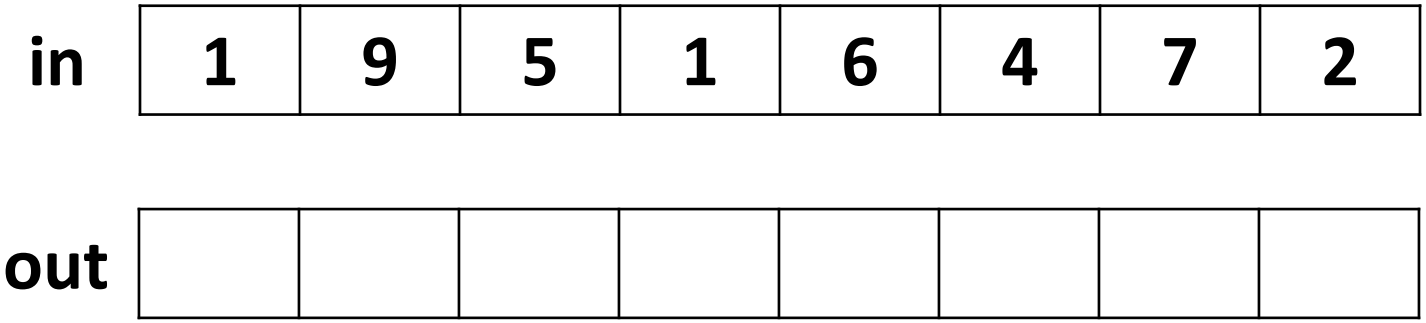
Overview

- The “scan” task
- Sequential implementation
- Parallel implementation
  - Kernel 1: work - inefficient
  - Kernel 2: work - efficient

The “scan” task



- **Inclusive scan**: $out[i] = \sum_{j=0}^{i} in[j]$

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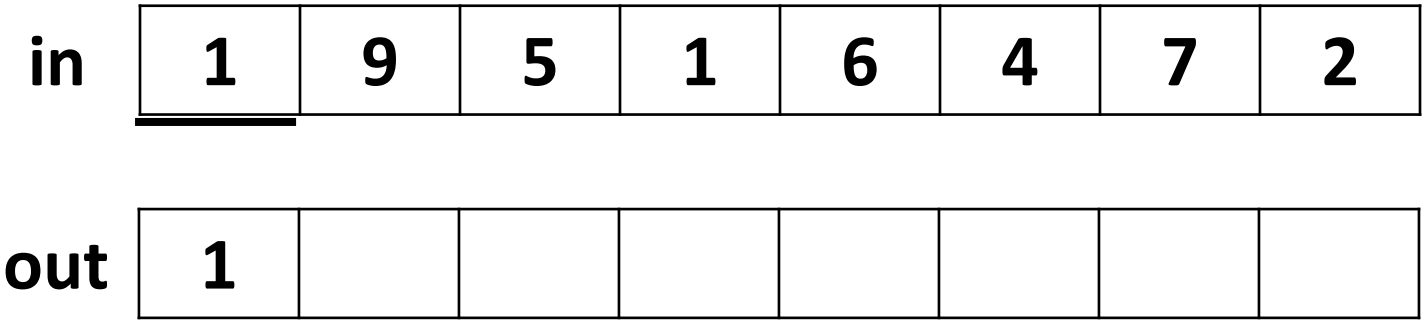

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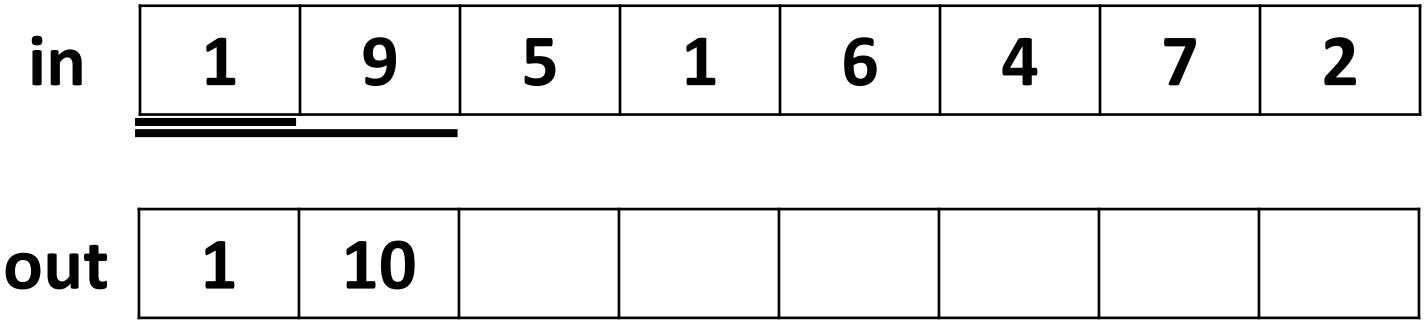

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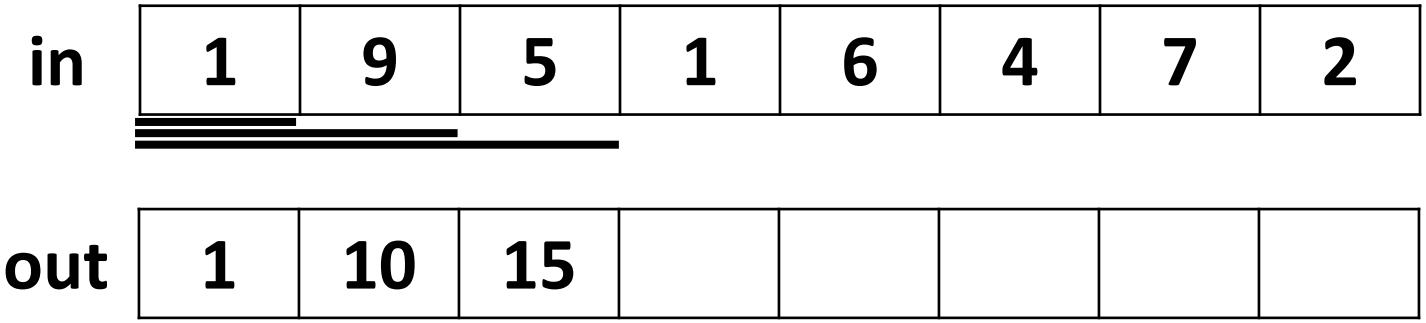

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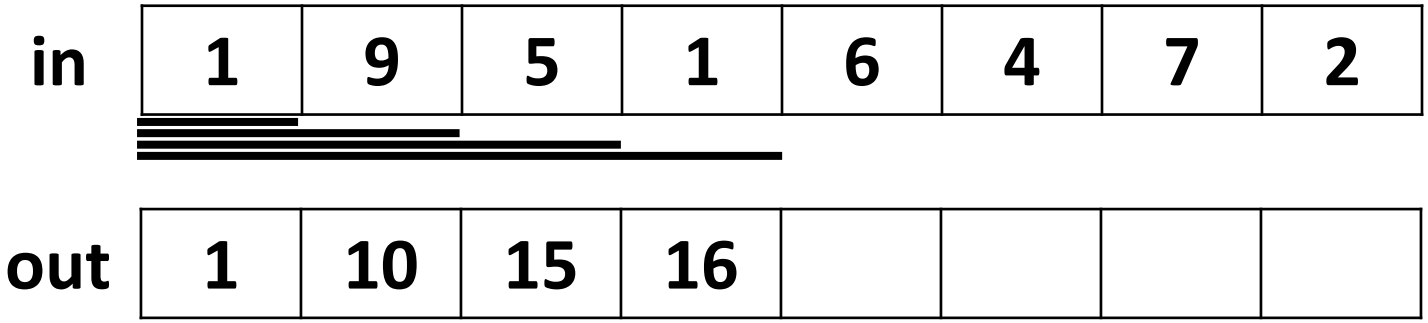

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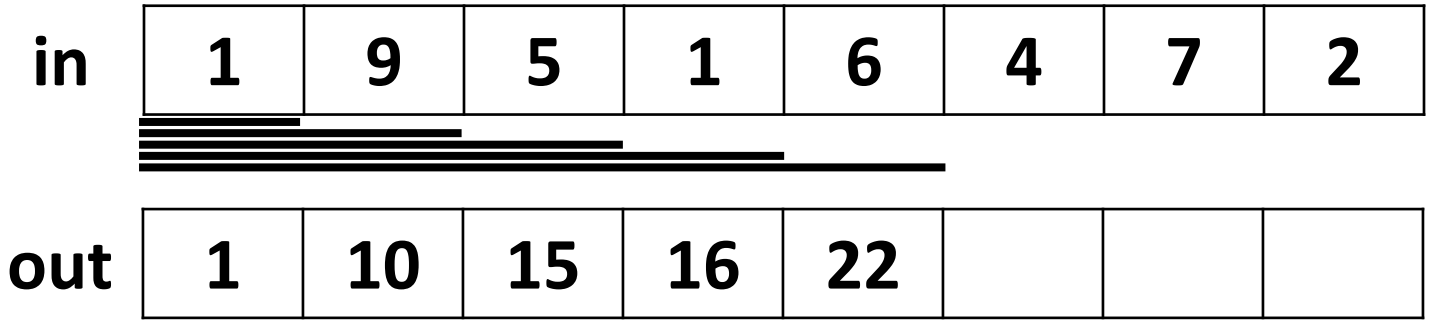

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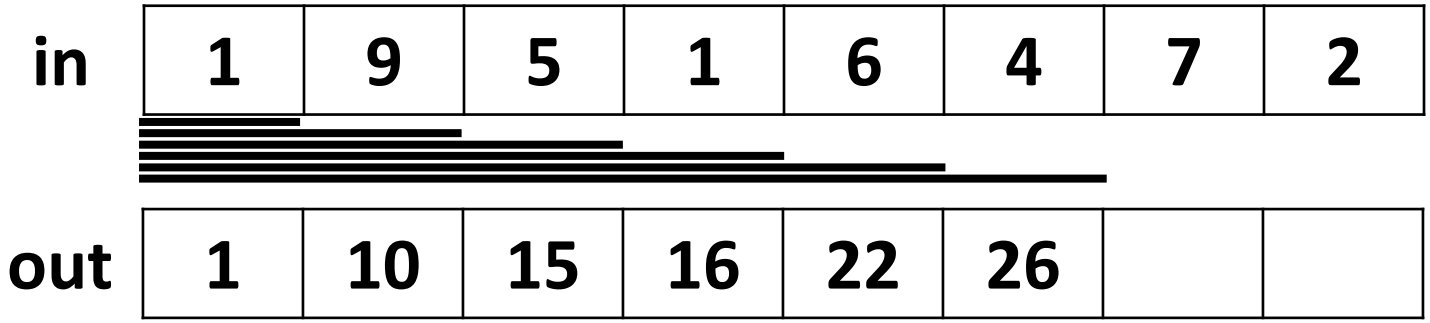

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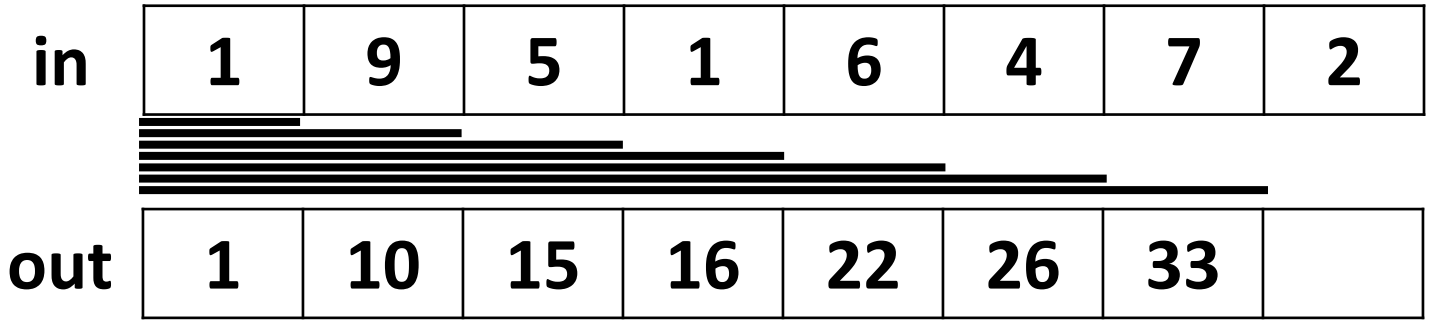

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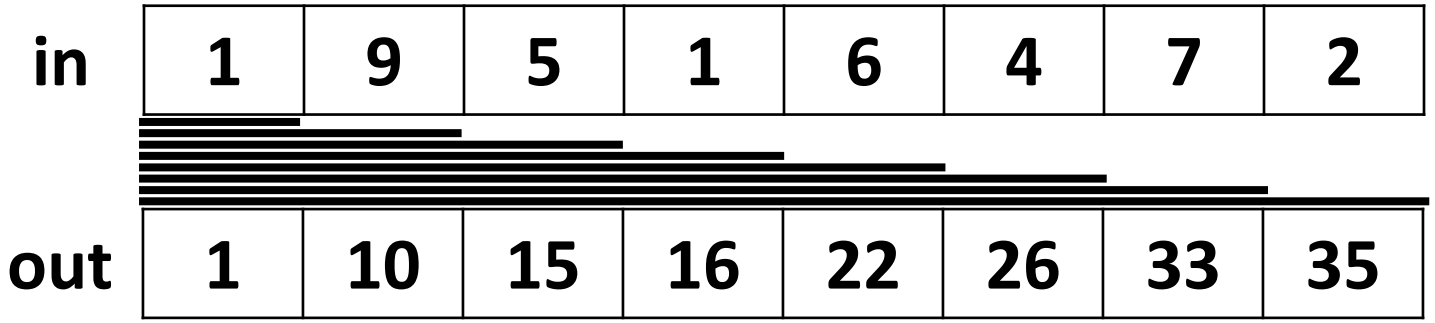

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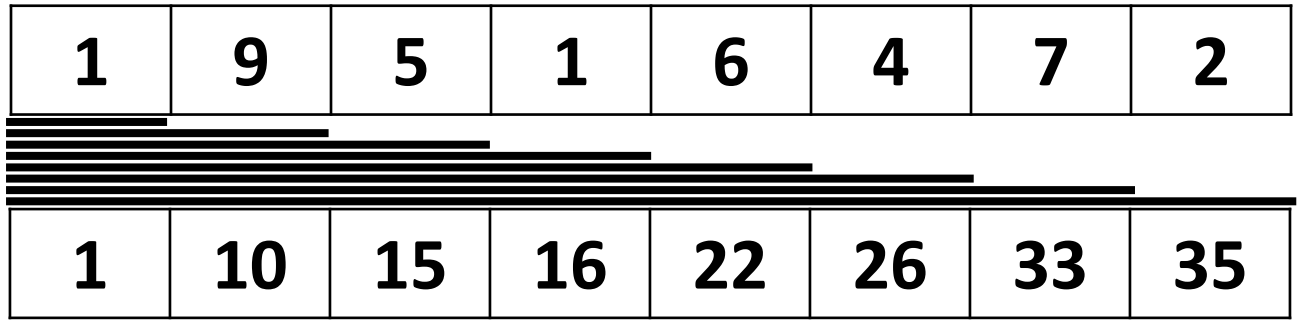
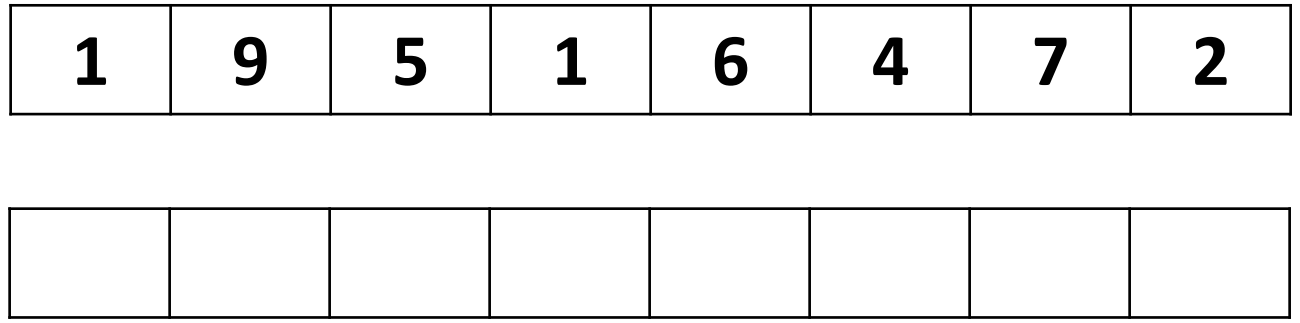


- **Exclusive scan**: $out[0] = 0, out[i] = \sum_{j=0}^{i-1} in[j] \; \forall i > 0$



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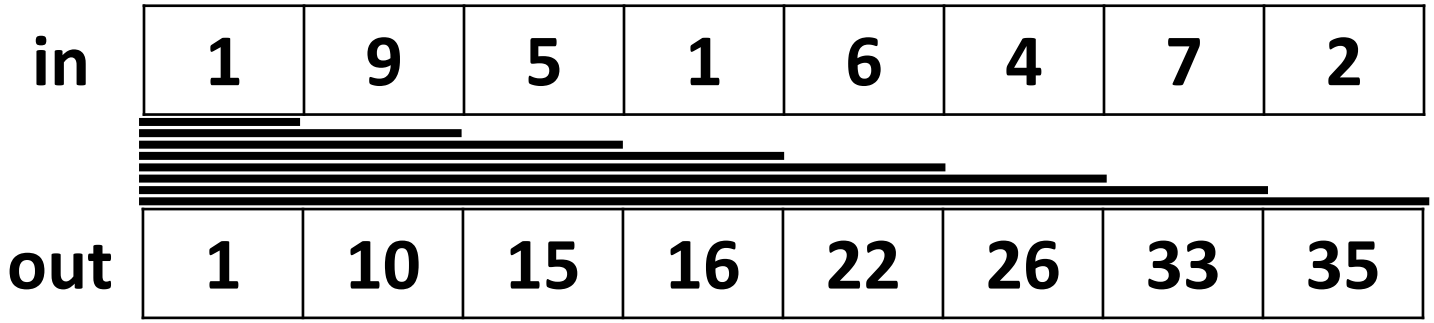

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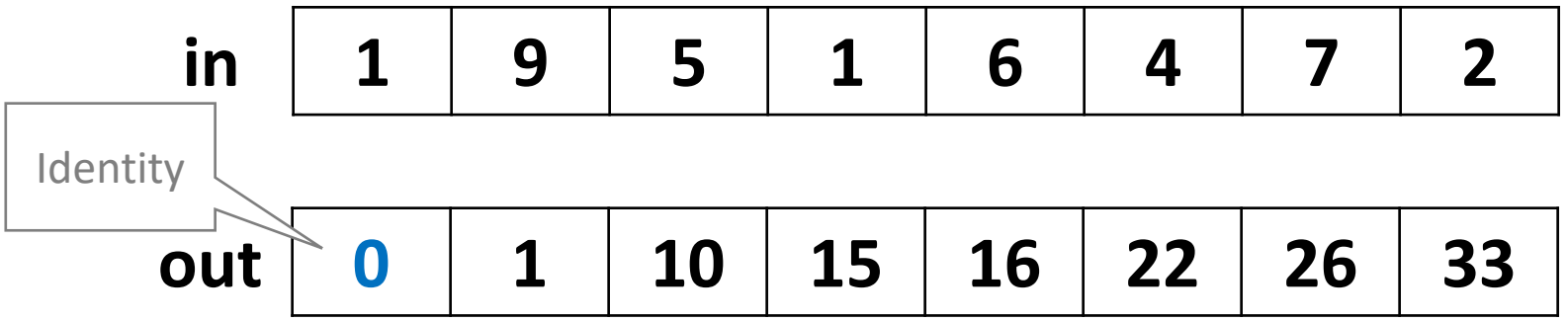


- In addition to plus operation, it can be applied for product, max, min, …
- Here we will focus on inclusive scan with plus operation

Introduction

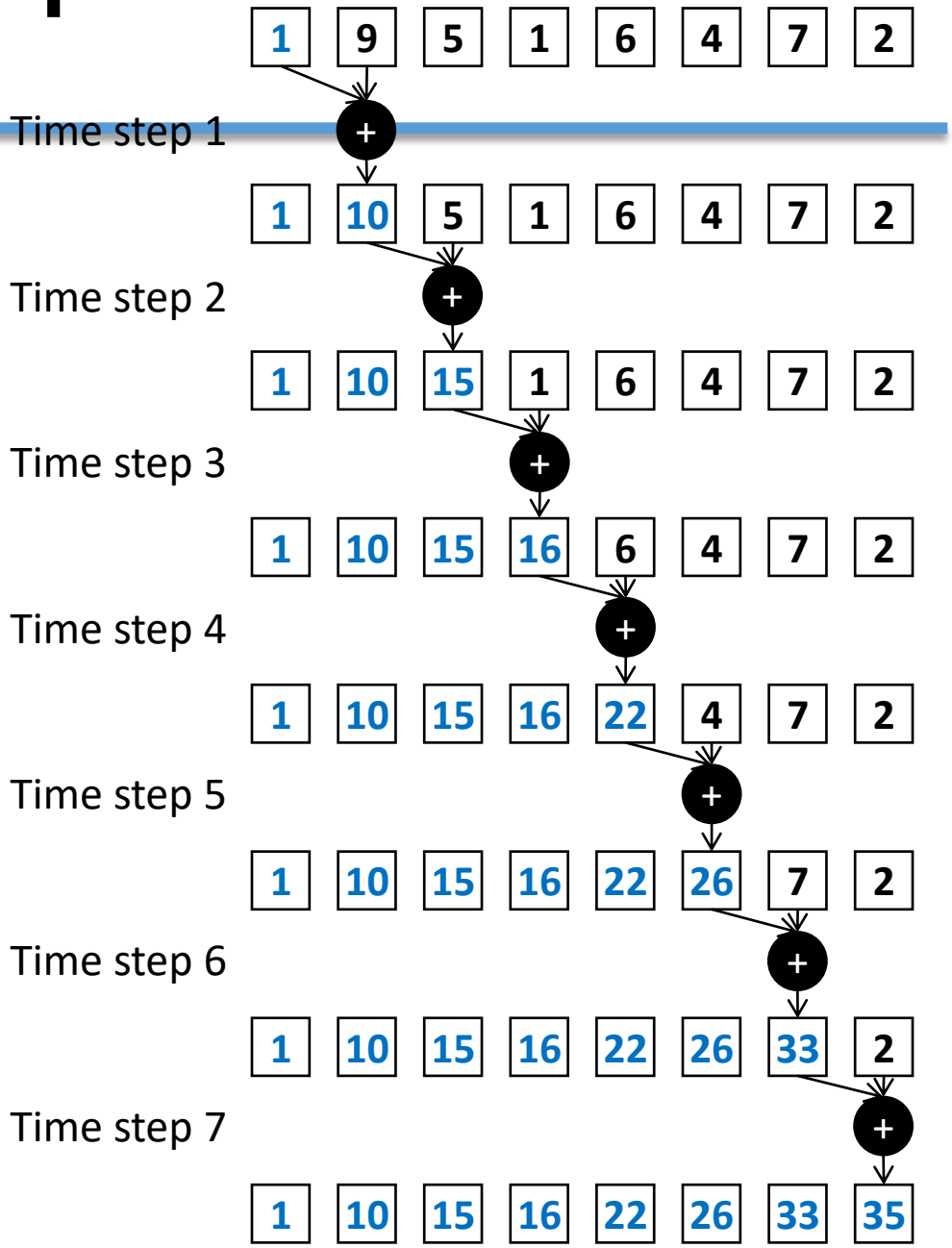
- **Parallel scan** is used to parallelize ***seemingly sequential operations***:
  - Resource allocation, work assignment, and polynomial evaluation
- A key primitive in many parallel algorithms to convert serial computation (recursion) into parallel computation
  - Radix sort, quick sort, histogram, string comparison,…
- Work efficiency in parallel code/algorithms
  - Parallel algorithms have higher complexity than a sequential algorithm

Sequential implementation

```
void scanOnHost(int *in, int *out, int n)
{
   out[0] = in[0];
   for (int i = 1; i < n; i++)
   {
            out[i] = out[i - 1] + in[i];
   }
}
```

Time (# time steps):

Work (# pluses):



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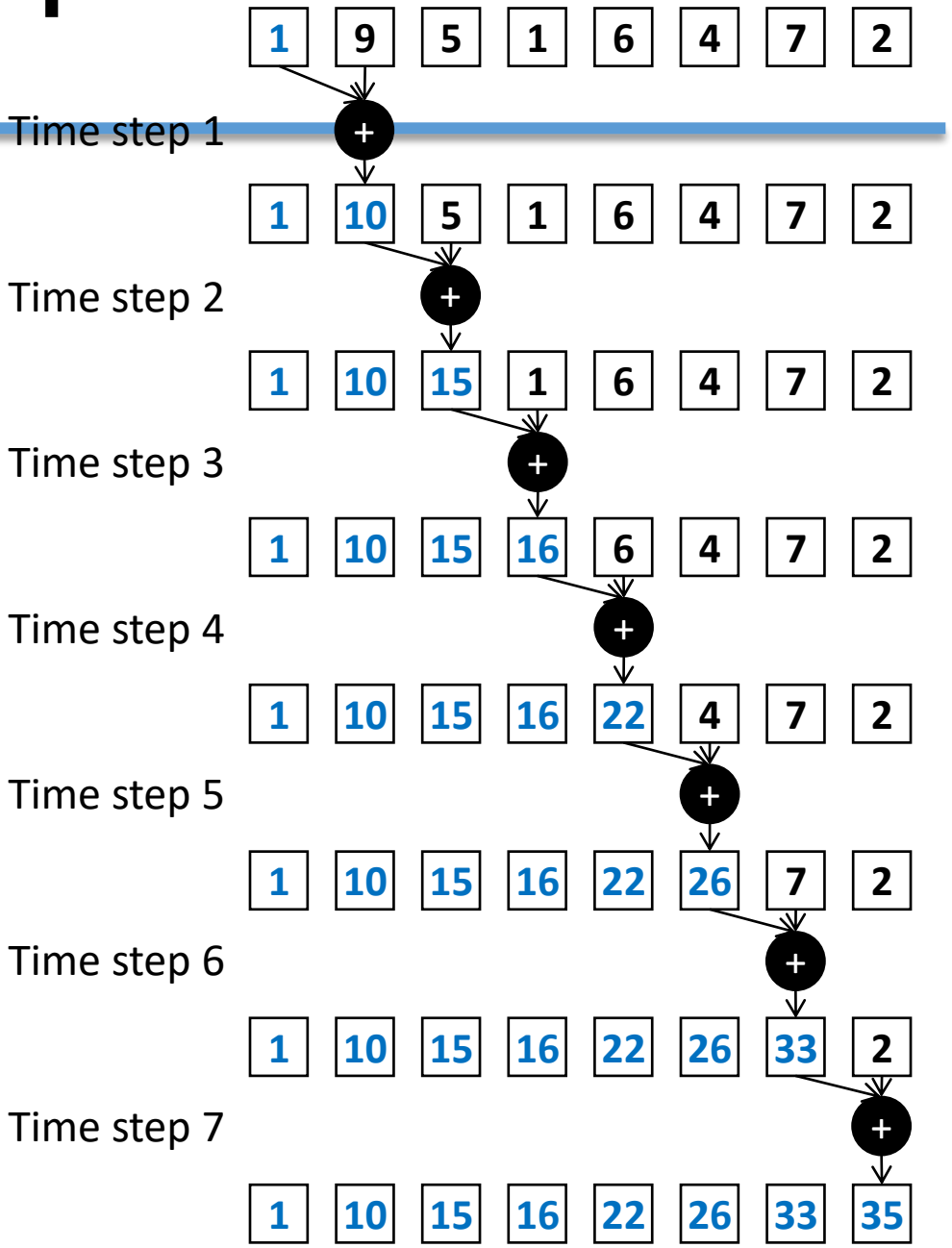
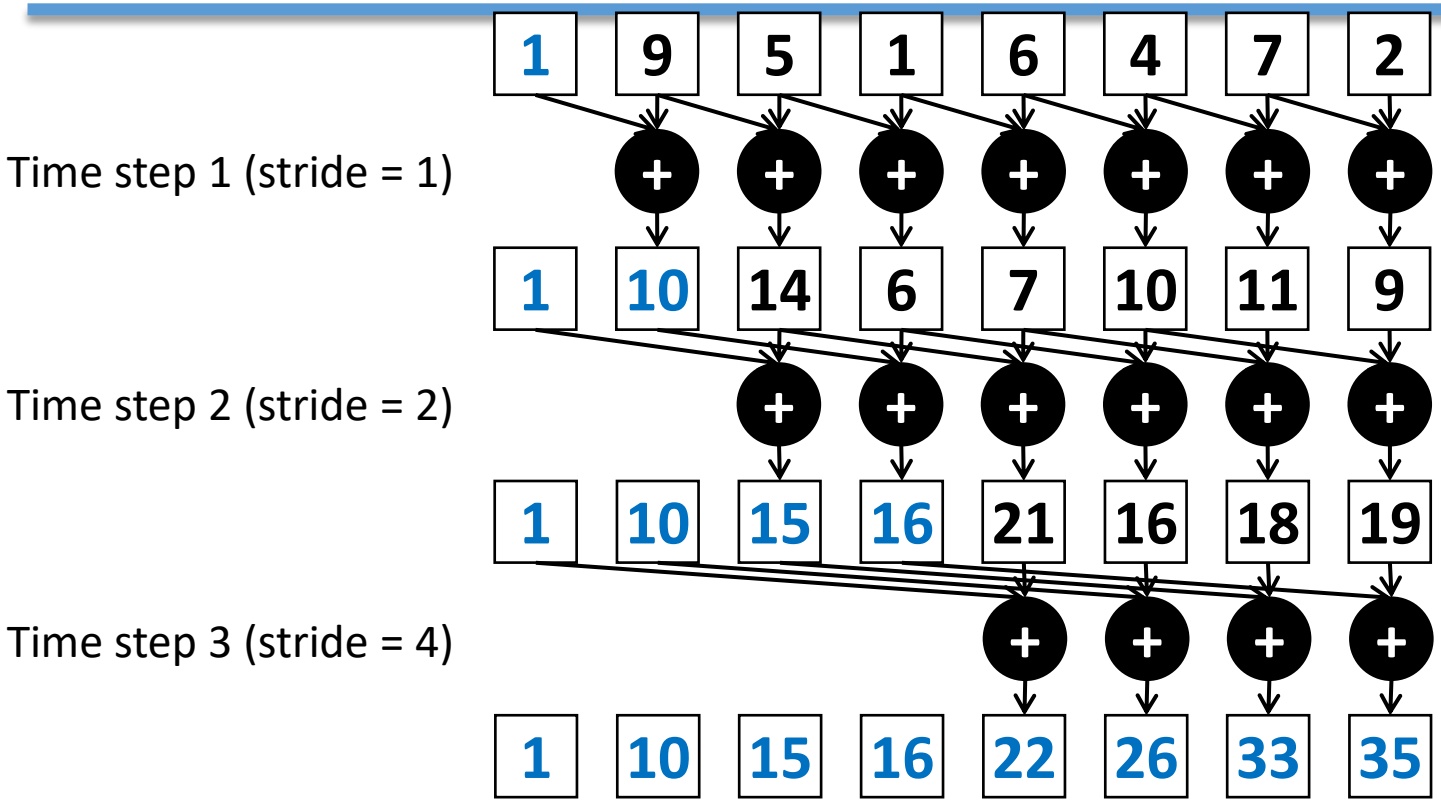
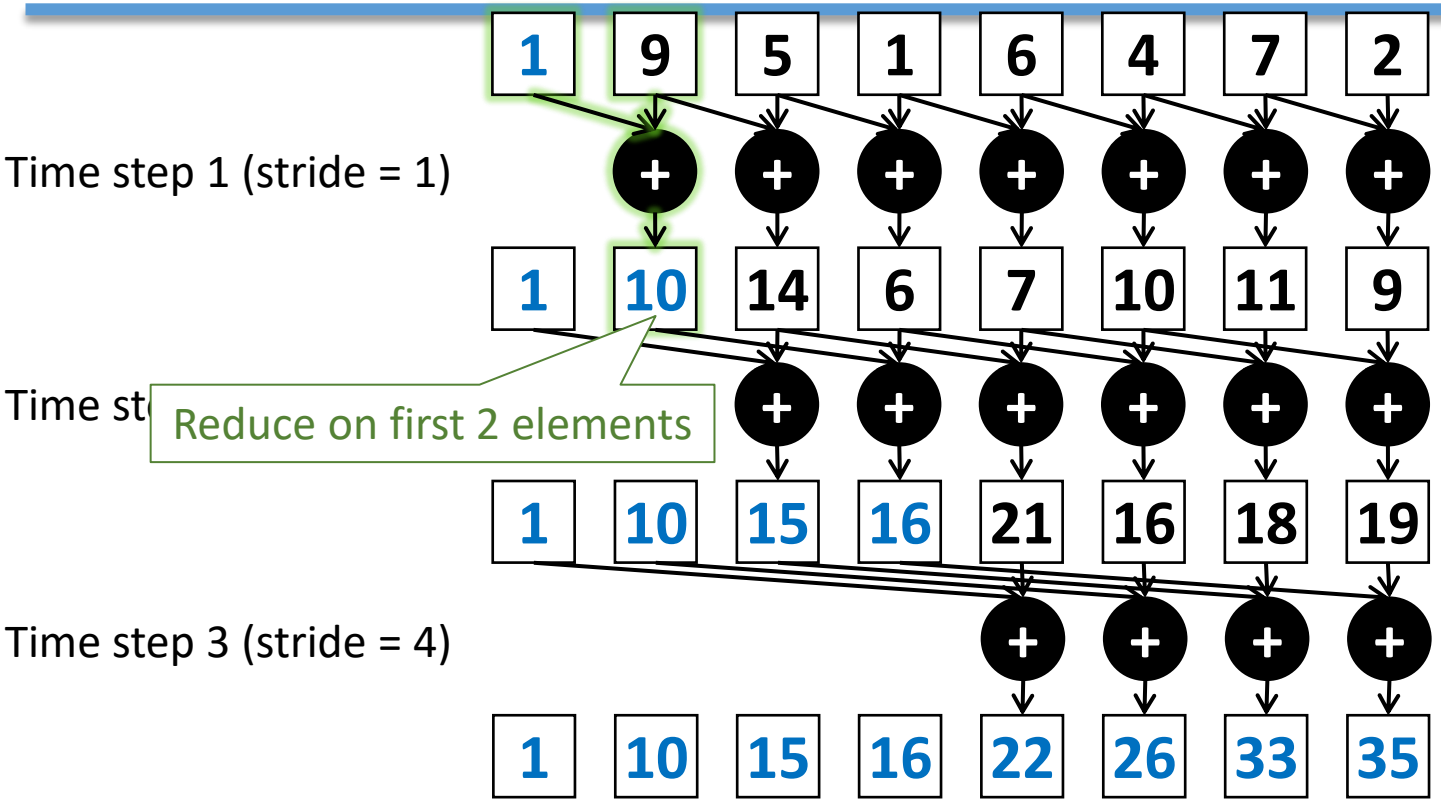


Time (# time steps): 7 = n-1 = O(n)
Work (# pluses): 7 = n-1 = O(n)

Parallel implementation



Parallel implementation



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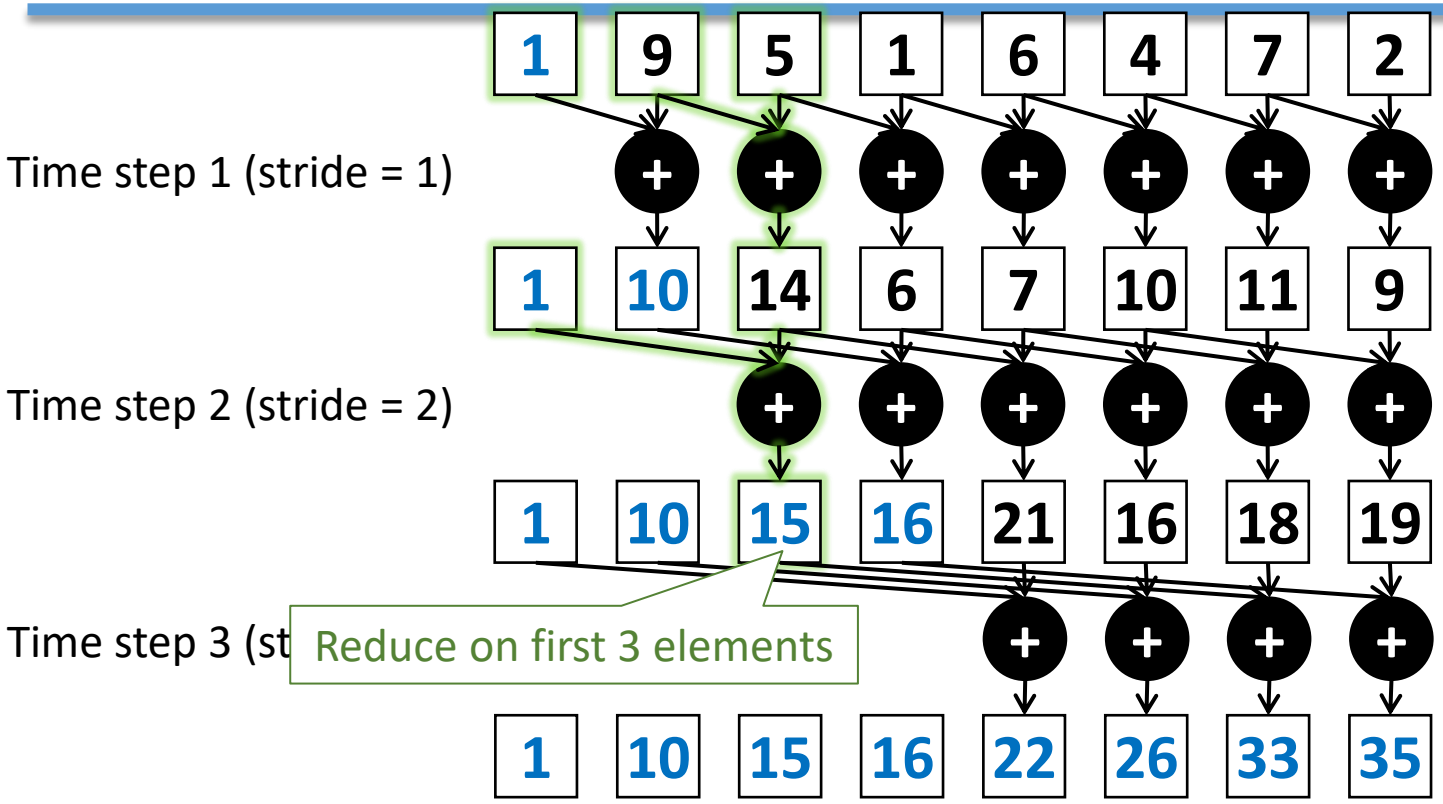

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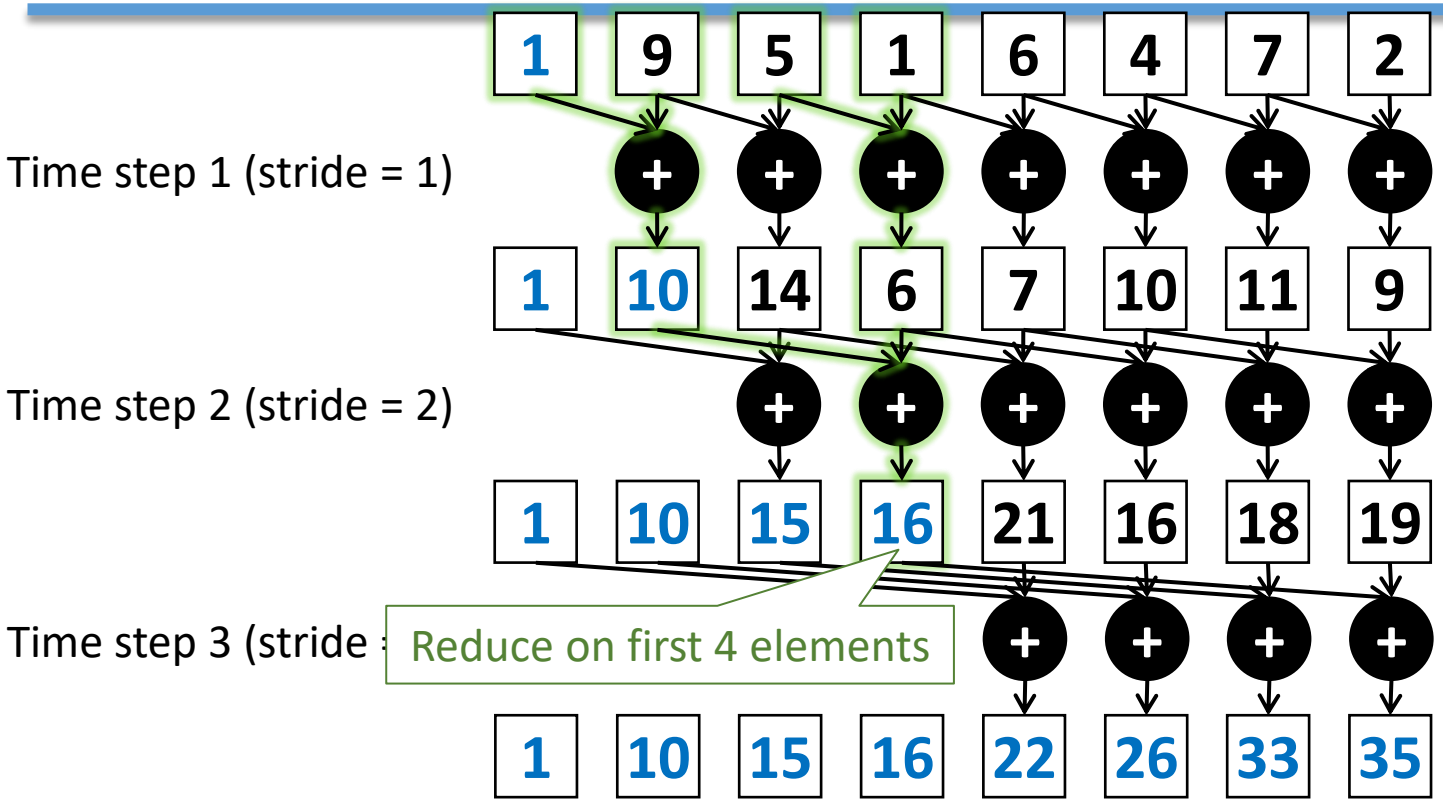

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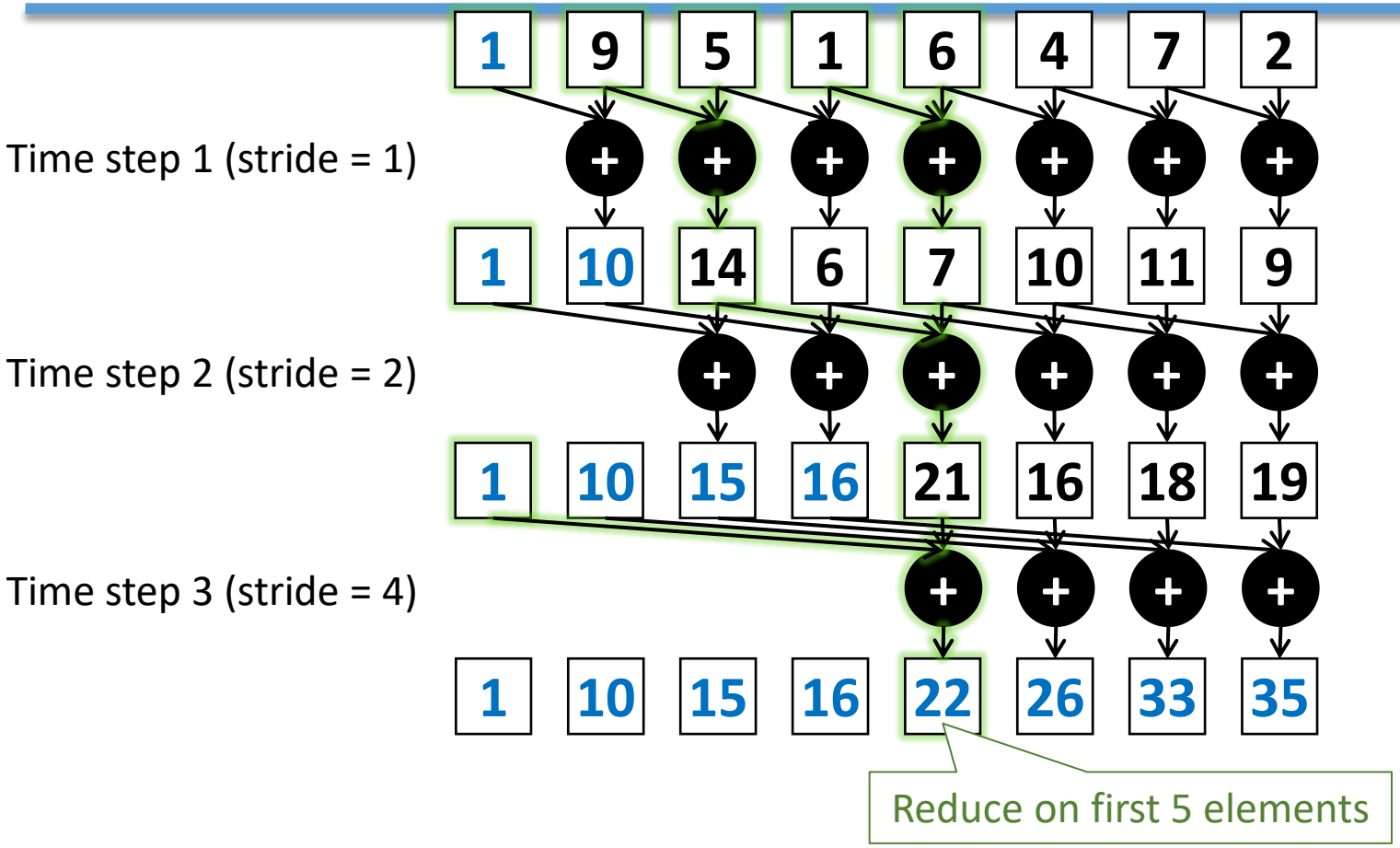

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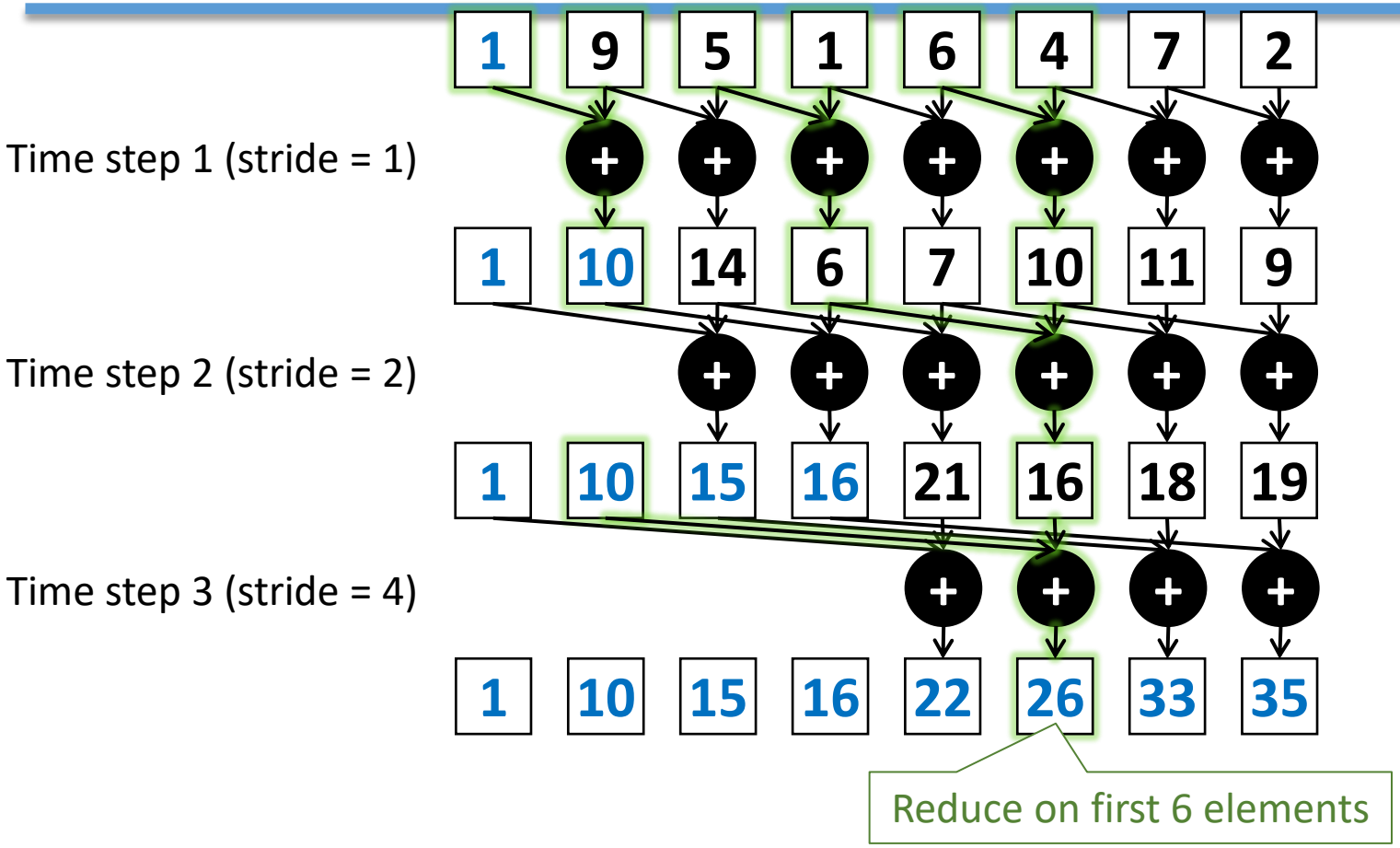

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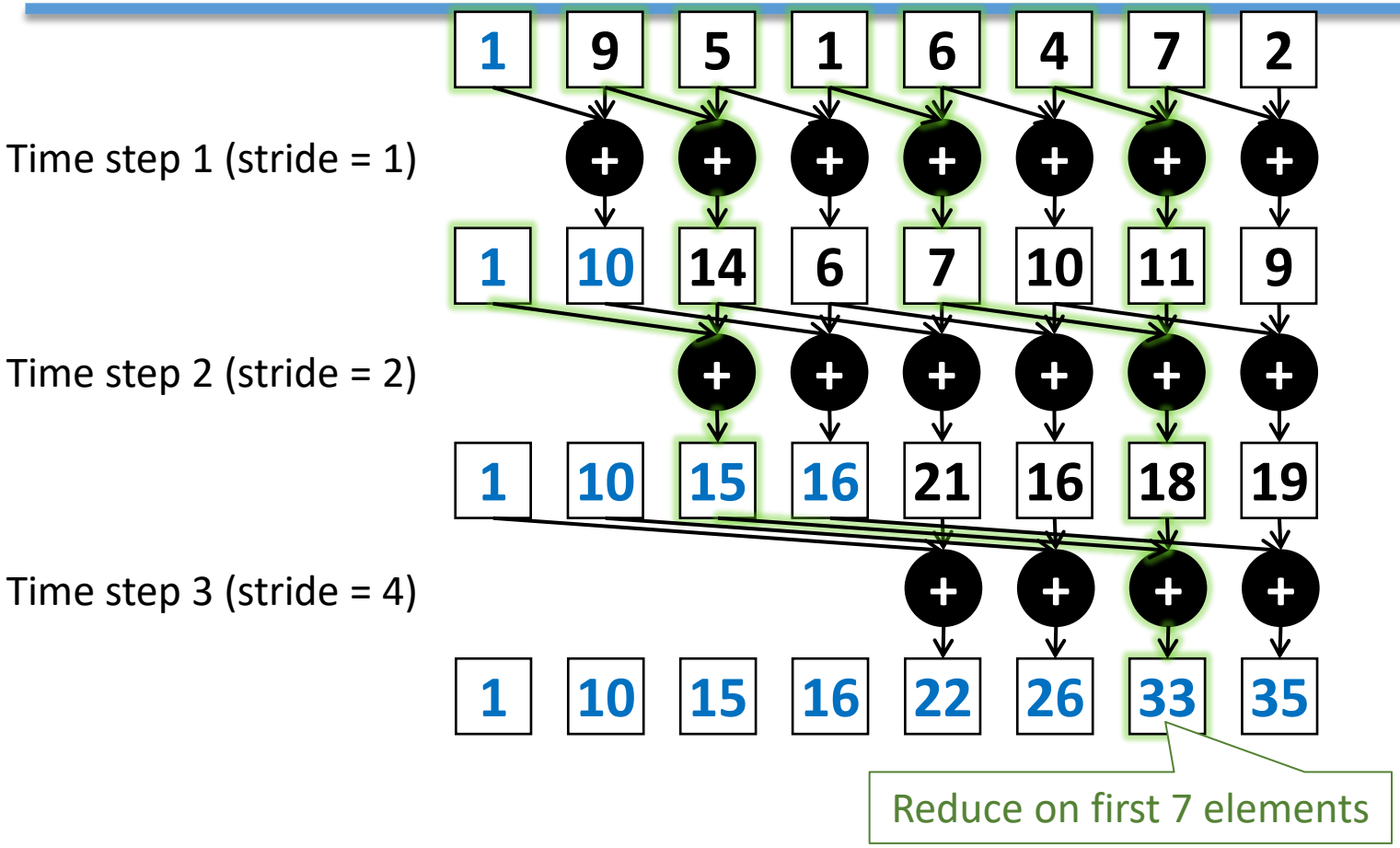

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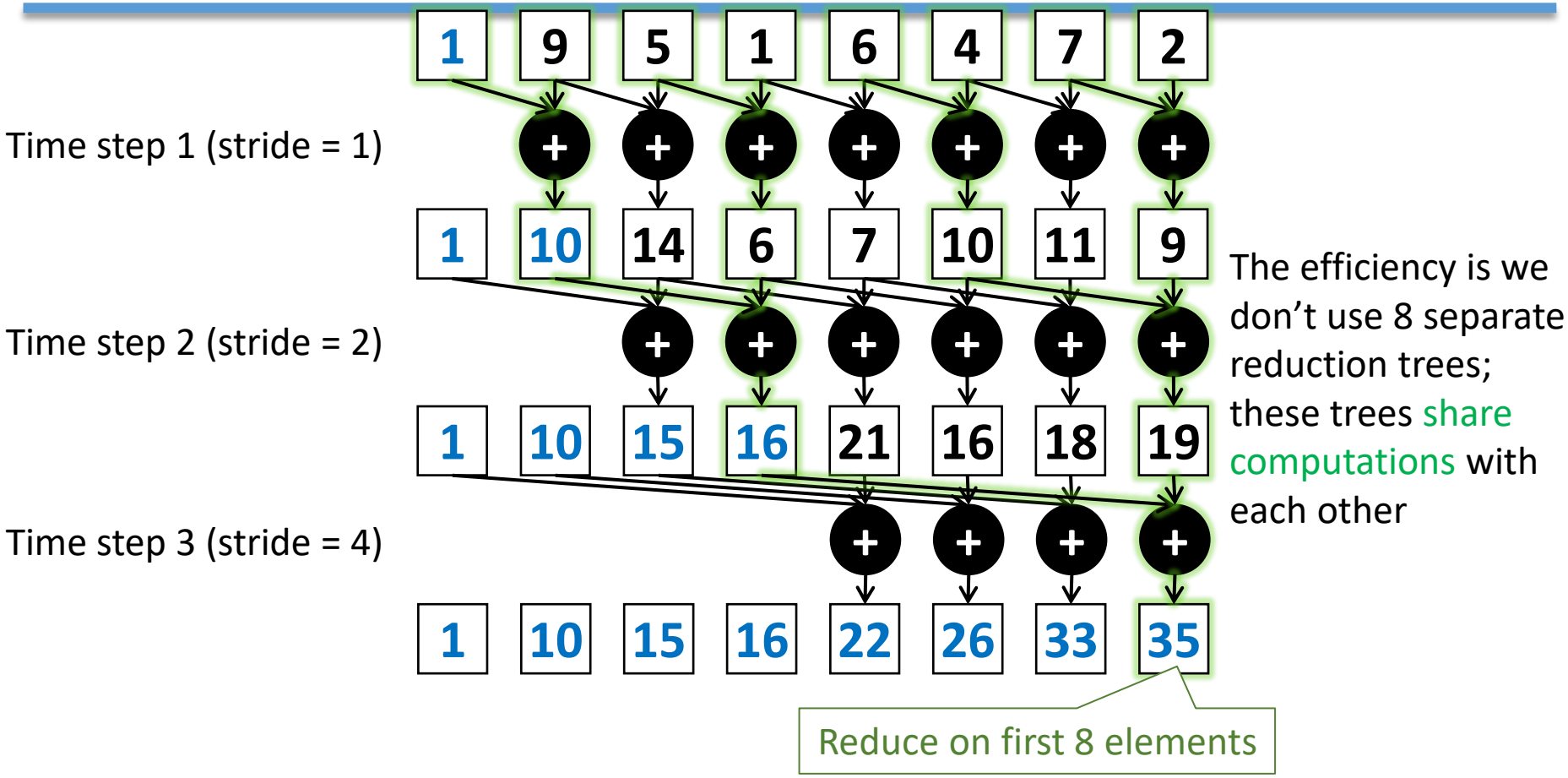
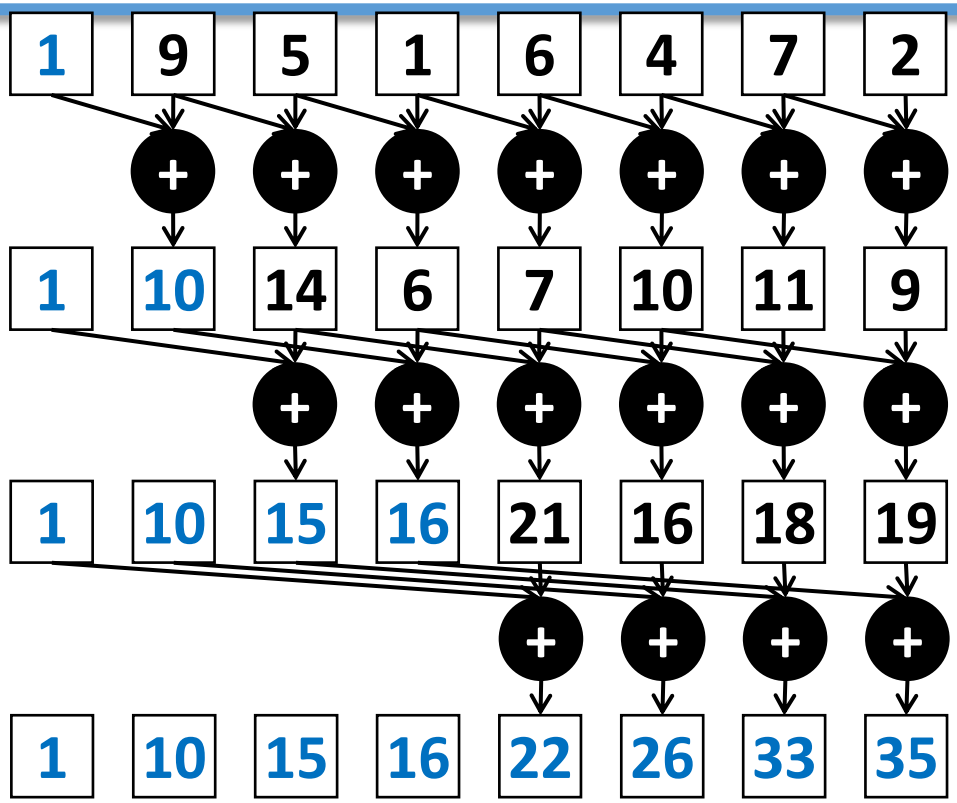


The efficiency is we don't use 8 separate reduction trees; these trees share computations with each other

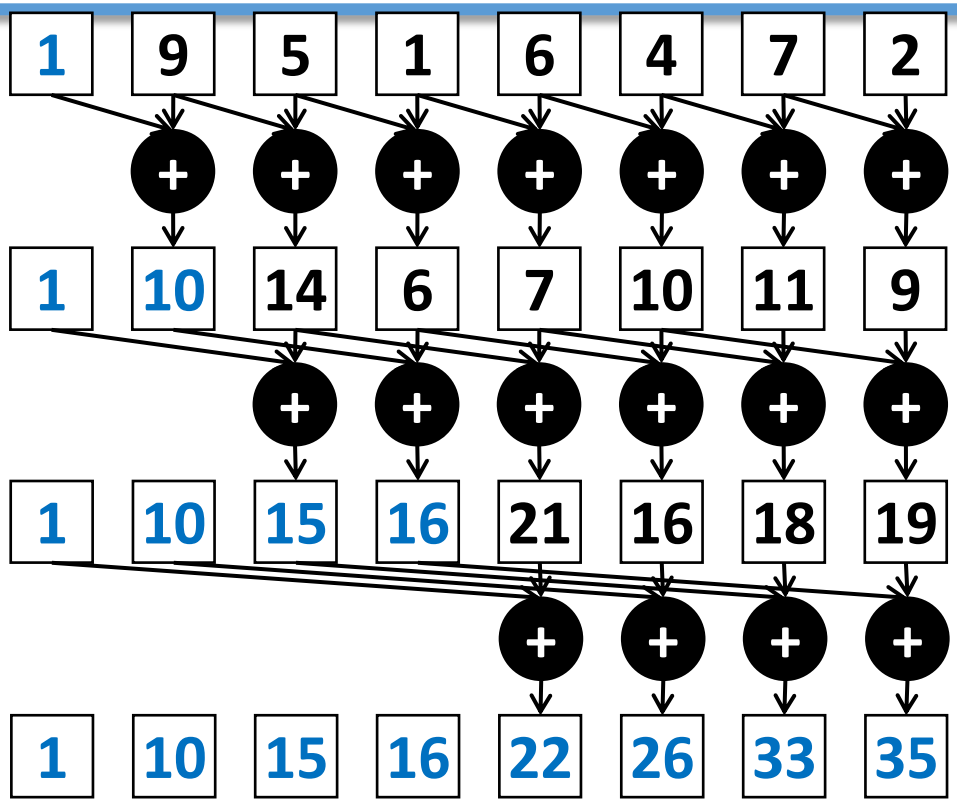
Parallel implementation



Time:

Work:

Parallel implementation



Time: 3 = $\log_2 n$ = $O(\log_2 n)$

Work: 17 = (n-1) + (n-2) + (n-4) + …+ (n-n/2)

= $n\log_2 n$ - (1 + 2 + … + n/2) = $O(n\log_2 n)$

n - 1

Work-inefficient

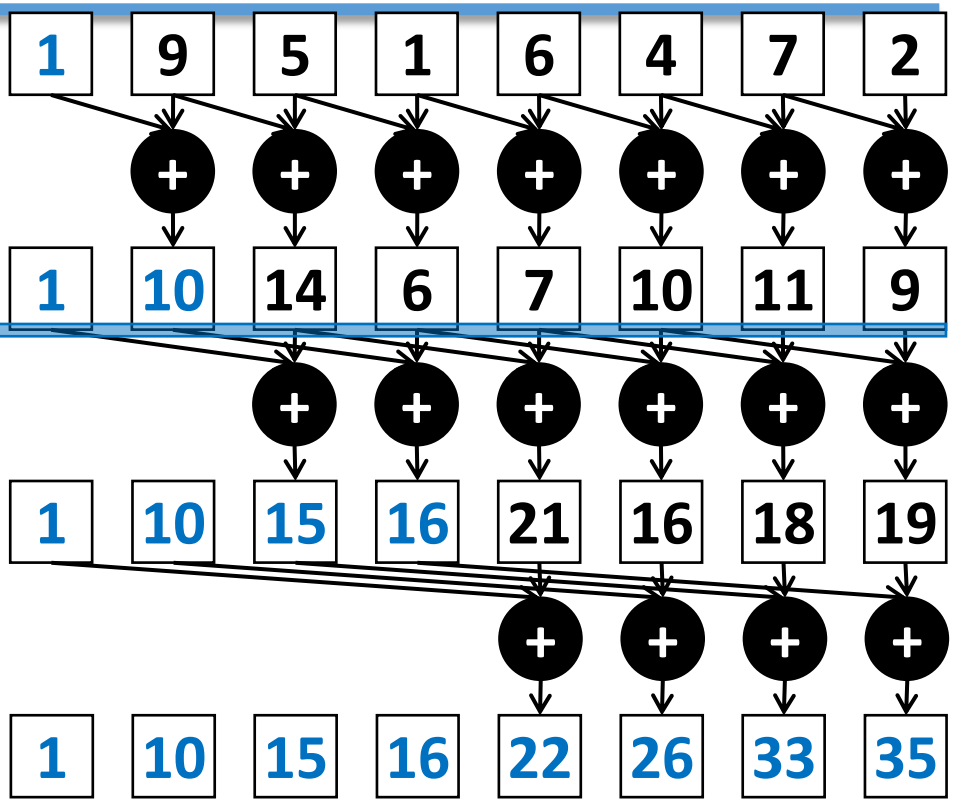
Parallel implementation

Need **synchronize** before next step?

But we can only synchronize threads in the same block

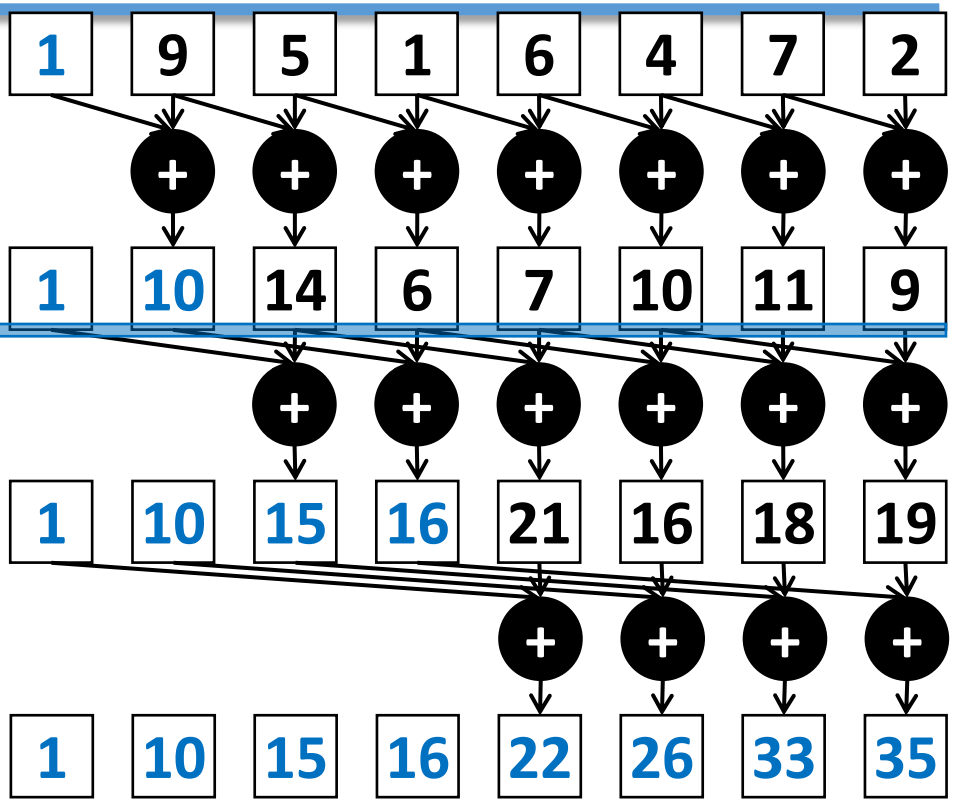
If n ≤ block-data-size, we can use a kernel with one block

If n > block-data-size, what will we do?



Parallel implementation

During every iteration, each thread can overwrite the input of another thread



- Barrier synchronization to ensure all inputs have been properly generated
- All threads secure input operand that can be overwritten by another thread
- Barrier synchronization is required to ensure that all threads have secured their inputs
- All threads perform addition and write output

Parallel implementation

If n > block-data-size, what will we do?

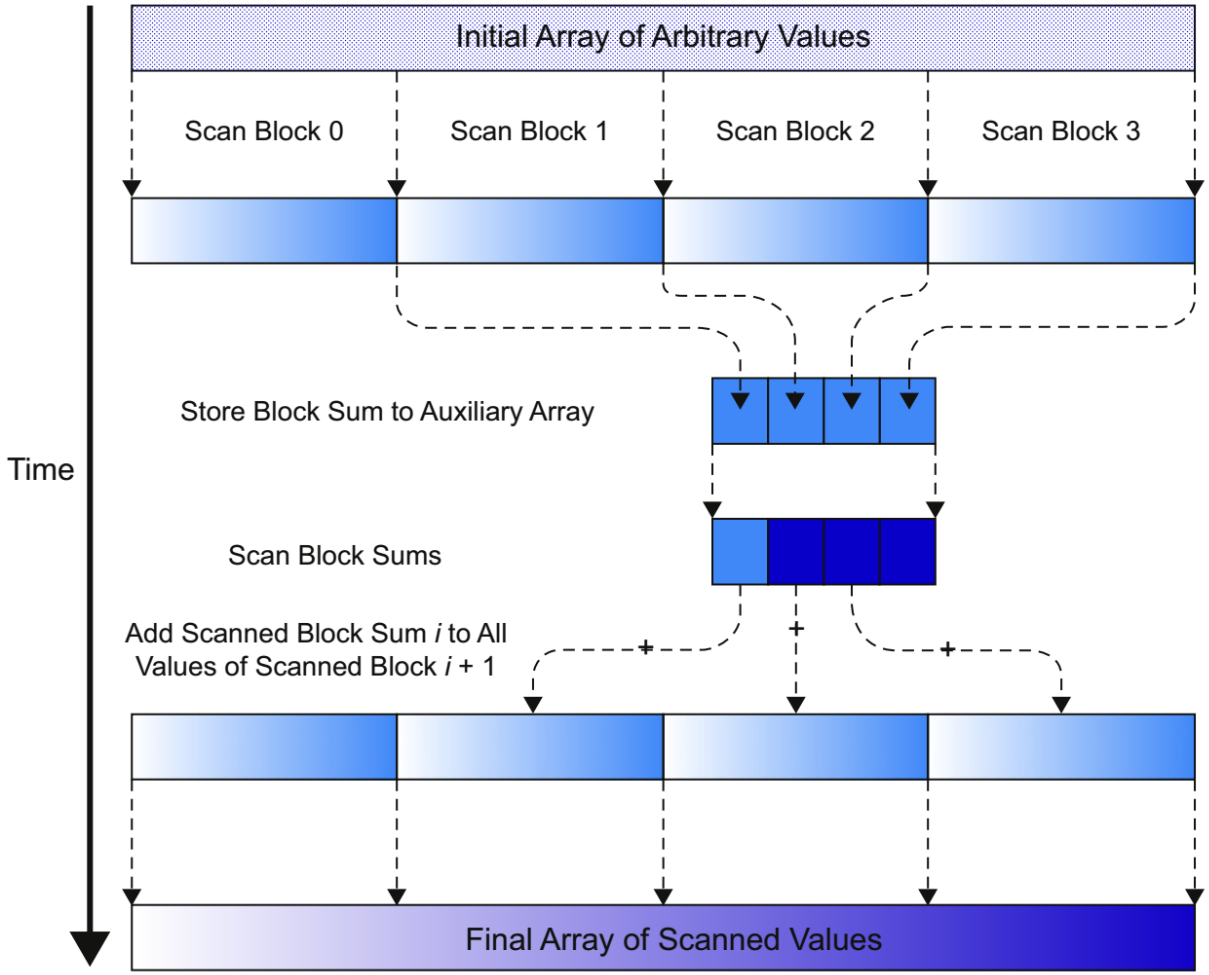
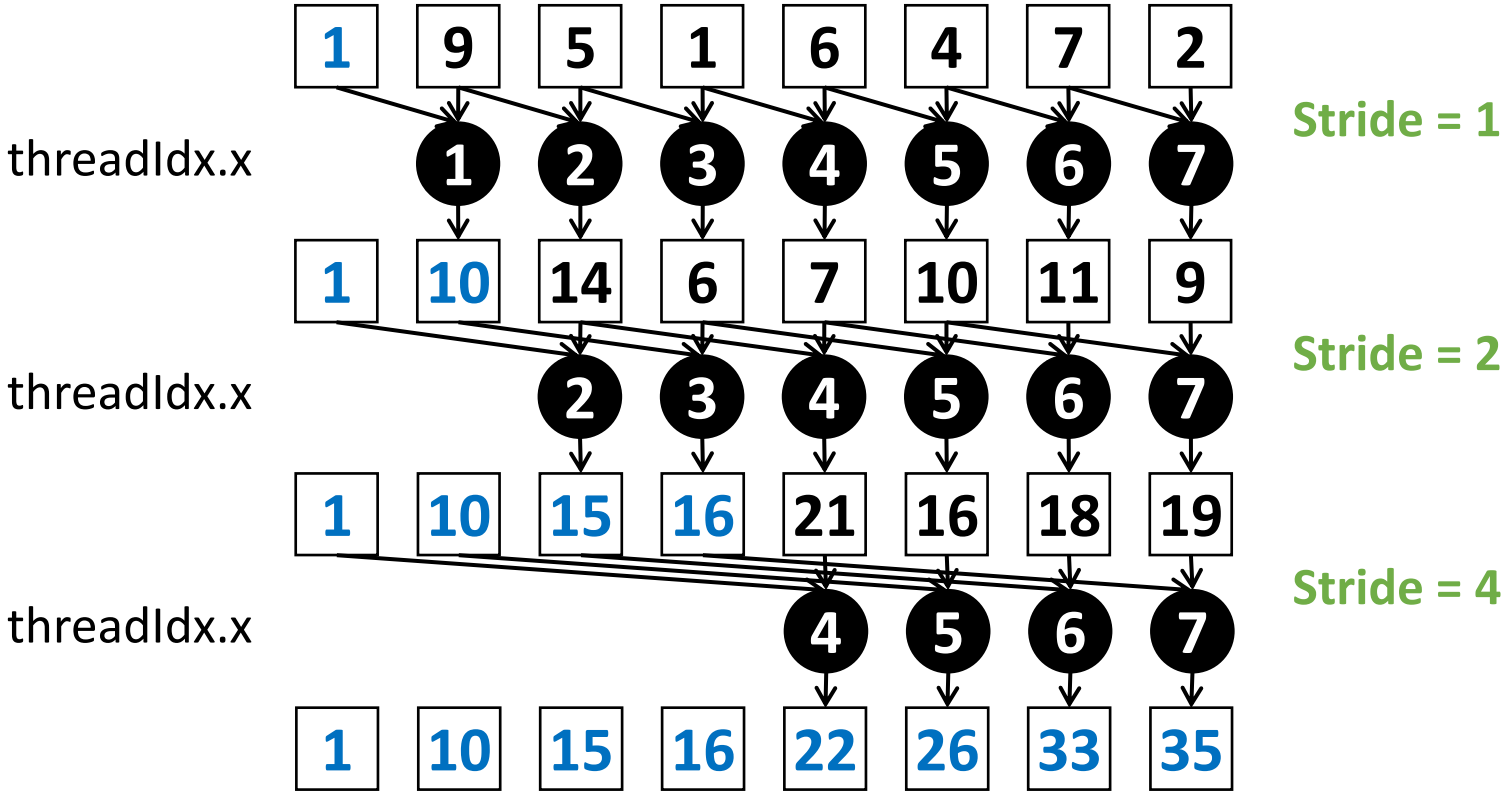


Image source: David B. Kirk et al. Programming Massively Parallel Processors

Scan in each block

Consider a block of 8 threads

1. Block reads data from GMEM to SMEM
2. Block scans with data on SMEM



3. Block writes result from SMEM to GMEM

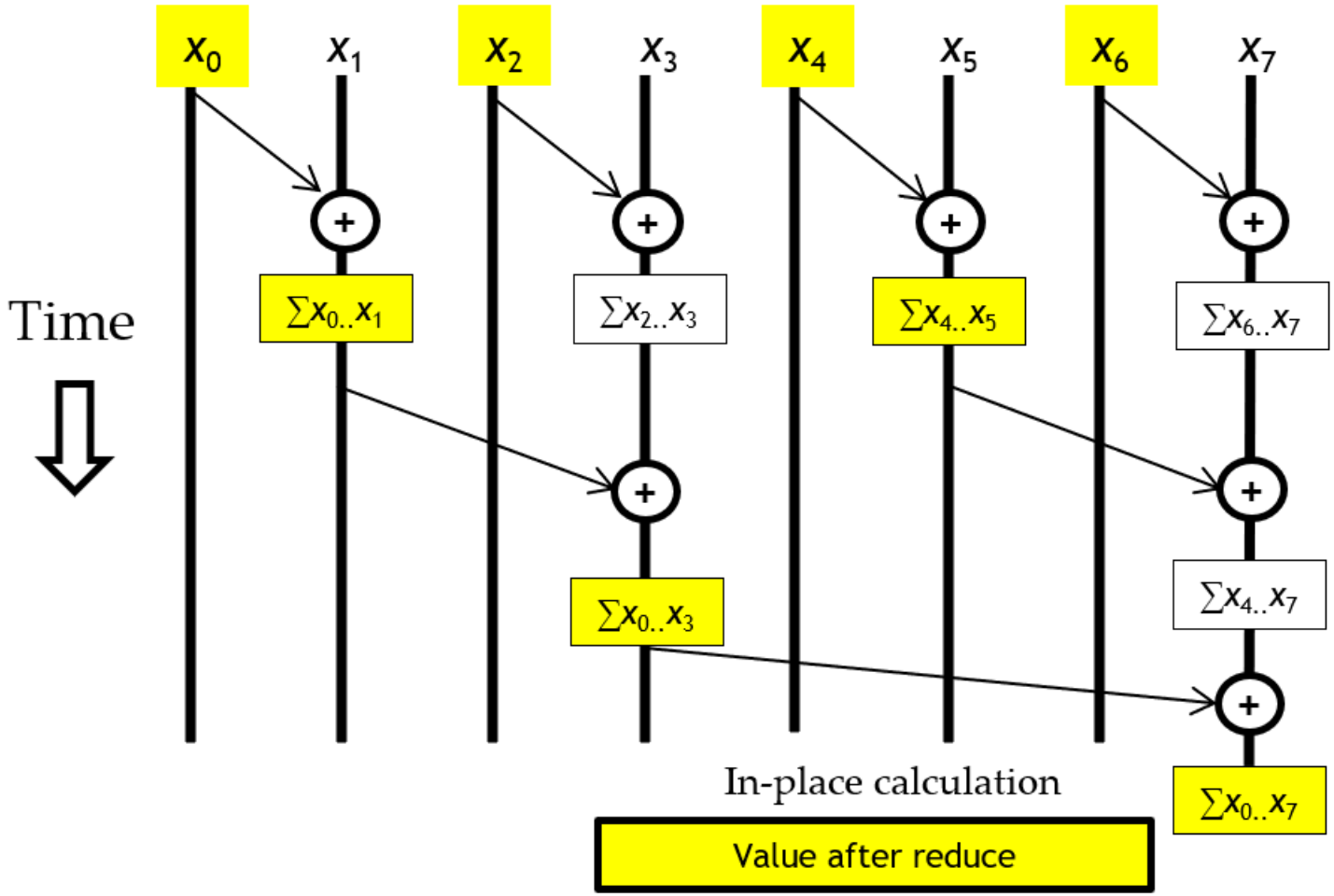
Live coding

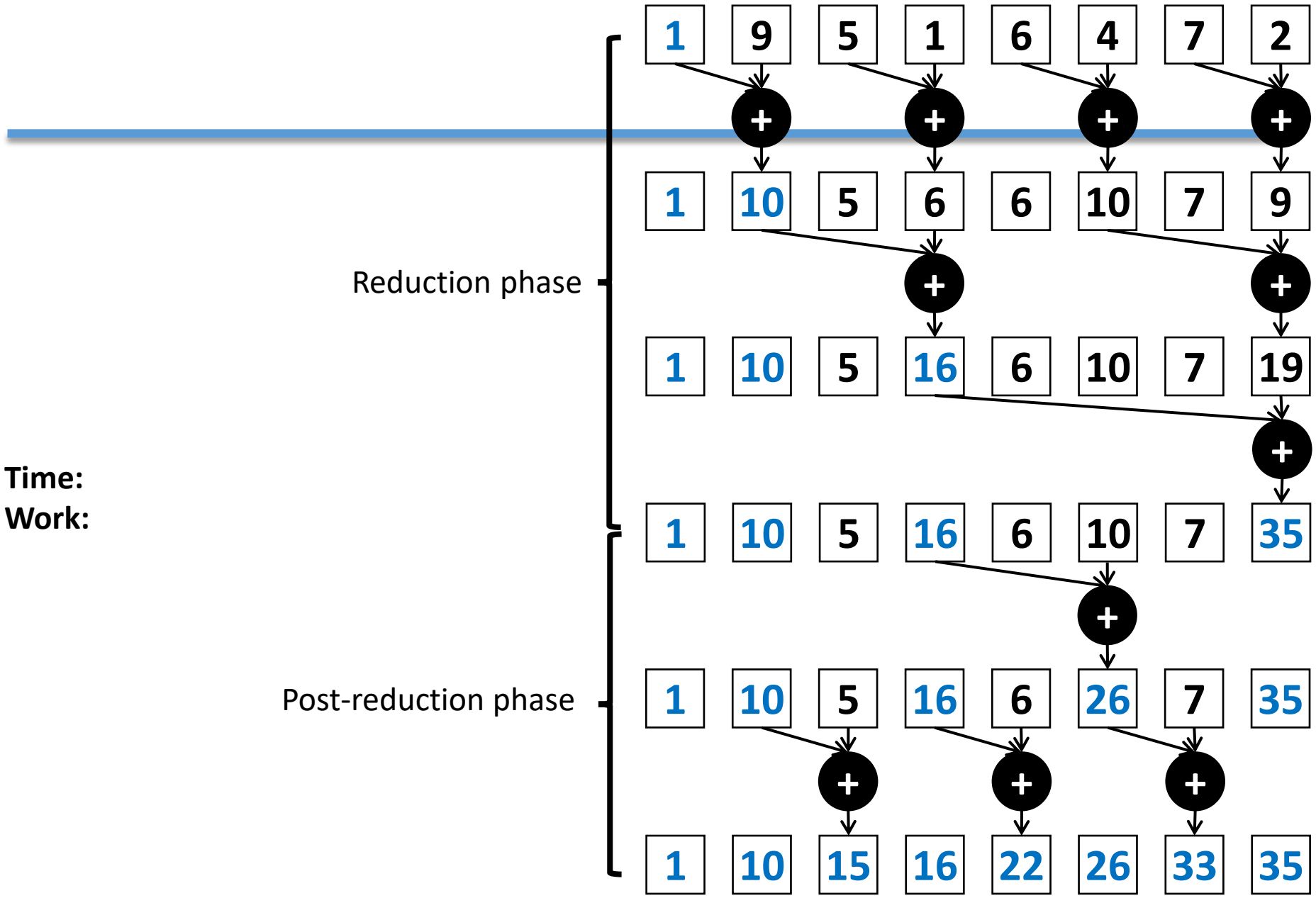
Kernel 2 – “work-efficient”

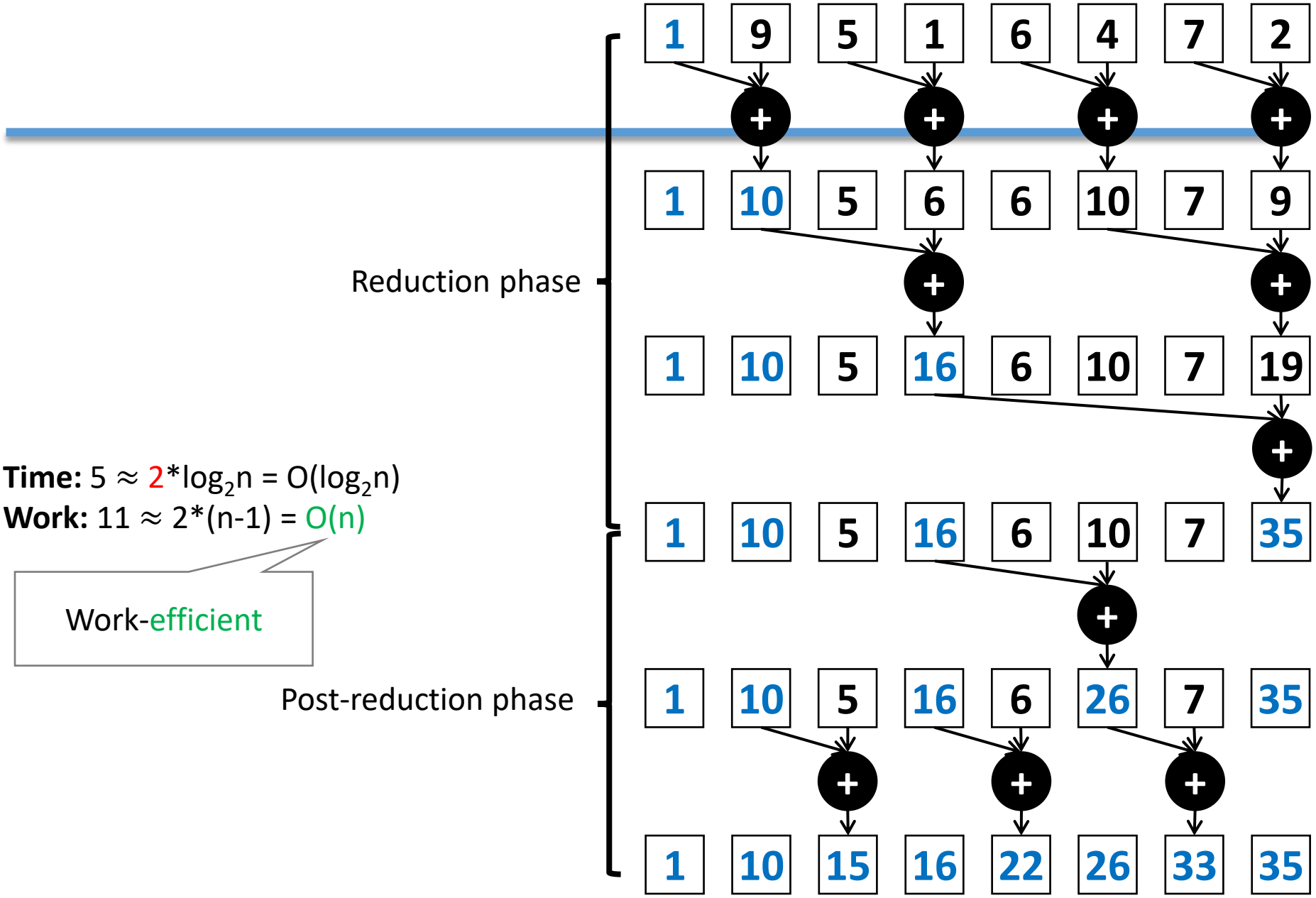
Idea: reduce # pluses by reusing results more

- Balanced Trees
  - Form a balanced binary tree on the input data and sweep it to and from the root
  - Tree is not an actual data structure, but a concept to determine what each thread does at each step
- For scan:
  - Traverse down from leaves to the root building partial sums at internal nodes in the tree
  - The root holds the sum of all leaves
  - Traverse back up the tree building the output from the partial sums

Kernel 2 – “work-efficient”

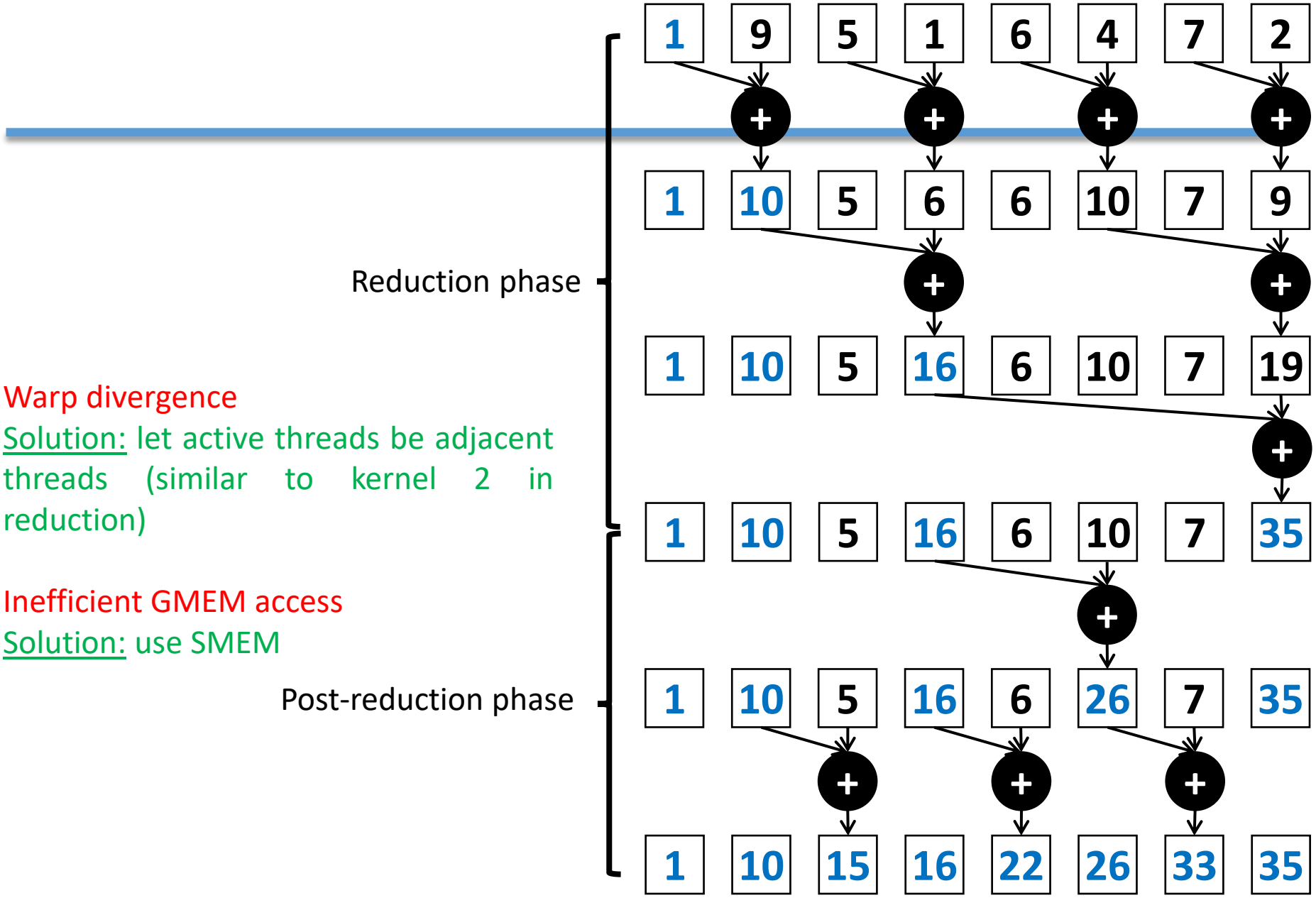



1 9 5 1 6 4 7 2
1 10 5 6 6 10 7 9
1 10 5 16 6 10 7 19
1 10 5 16 6 10 7 35
1 10 5 16 6 26 7 35
1 10 15 16 22 26 33 35
Reduction phase
Post-reduction phase
Time:
Work:



Time: 5 ≈ 2*$\log_2$n = O($\log_2$n)
Work: 11 ≈ 2*(n-1) = O(n)

Work-efficient



Warp divergence

Solution: let active threads be adjacent threads (similar to kernel 2 in reduction)

Inefficient GMEM access

Solution: use SMEM

Illustration for 16 elements

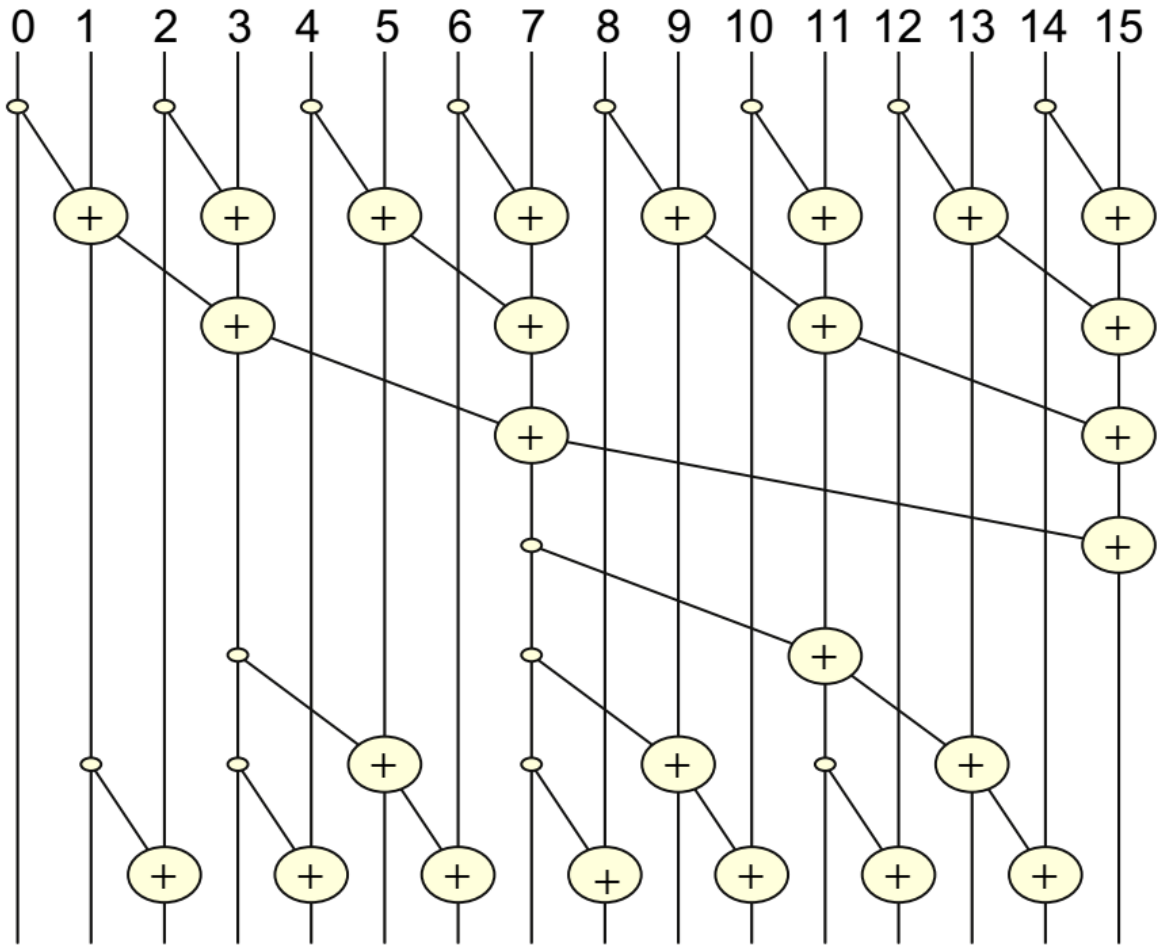
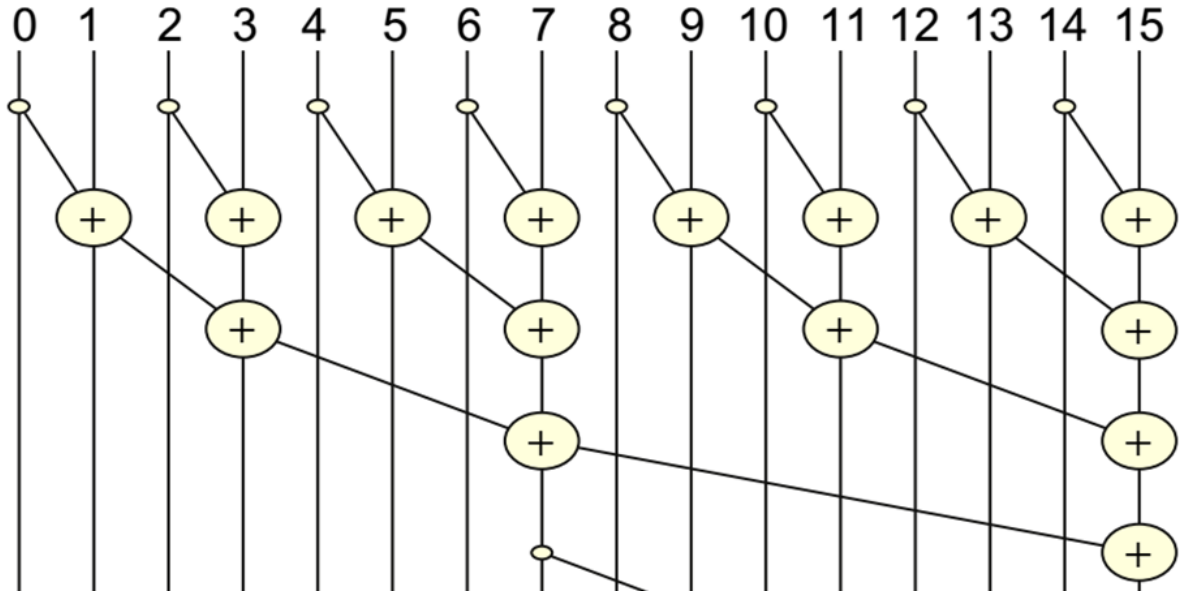


Image source: David B. Kirk et al. Programming Massively Parallel Processors

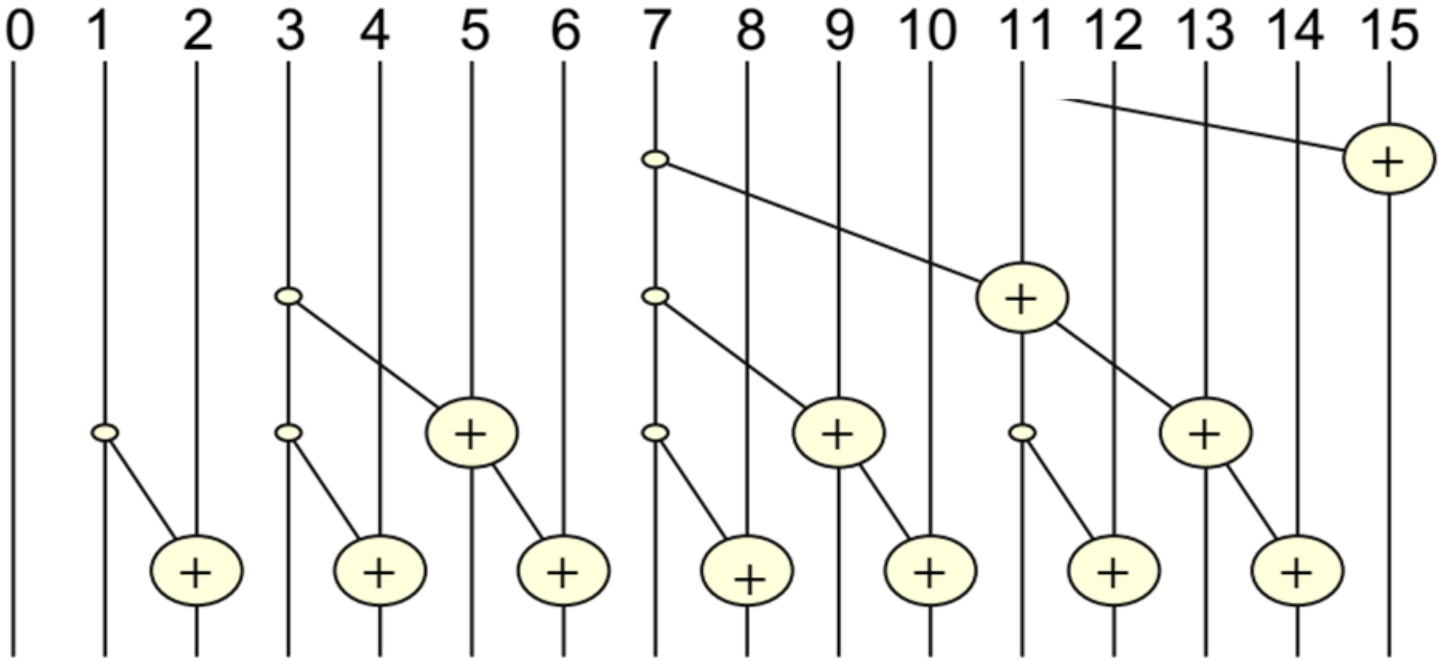
Reducion phase

```
for (int stride = 1; stride < 2 * blockDim.x; stride *= 2)
{
        int s_dataIdx = (threadIdx.x + 1) * 2 * stride - 1;
        if (s_dataIdx < 2 * blockDim.x)
                s_data[s_dataIdx] += s_data[s_dataIdx - stride];
        __syncthreads();
}
```



Post-reduction phase

```
for (int stride = blockDim.x / 2; stride > 0; stride /= 2)
{
	int s_dataIdx = (threadIdx.x + 1) * 2 * stride - 1 +
stride;
	if (s_dataIdx < 2 * blockDim.x)
		s_data[s_dataIdx] += s_data[s_dataIdx - stride];
	__syncthreads();
}
```



Reference

- [1] Wen-Mei, W. Hwu, David B. Kirk, and Izzat El Hajj. *Programming Massively Parallel Processors: A Hands-on Approach*. Morgan Kaufmann, 2022
- [2] Cheng John, Max Grossman, and Ty McKercher. *Professional Cuda C Programming*. John Wiley & Sons, 2014
- [3] Illinois GPU course

https://wiki.illinois.edu/wiki/display/ECE408/ECE408+Home

THE END