Line Segment Intersection Using Sweep Line Algorithm

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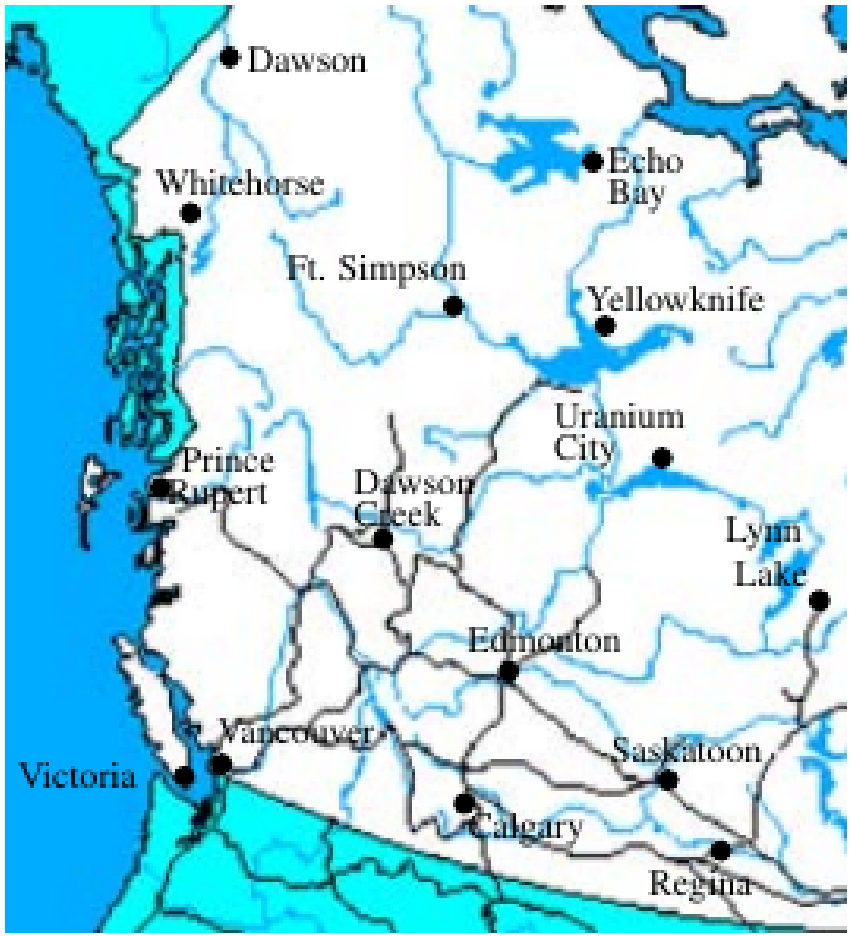
Thematic Map Overlay

- GIS allows users to create, manage, analyze, and map all types of data.



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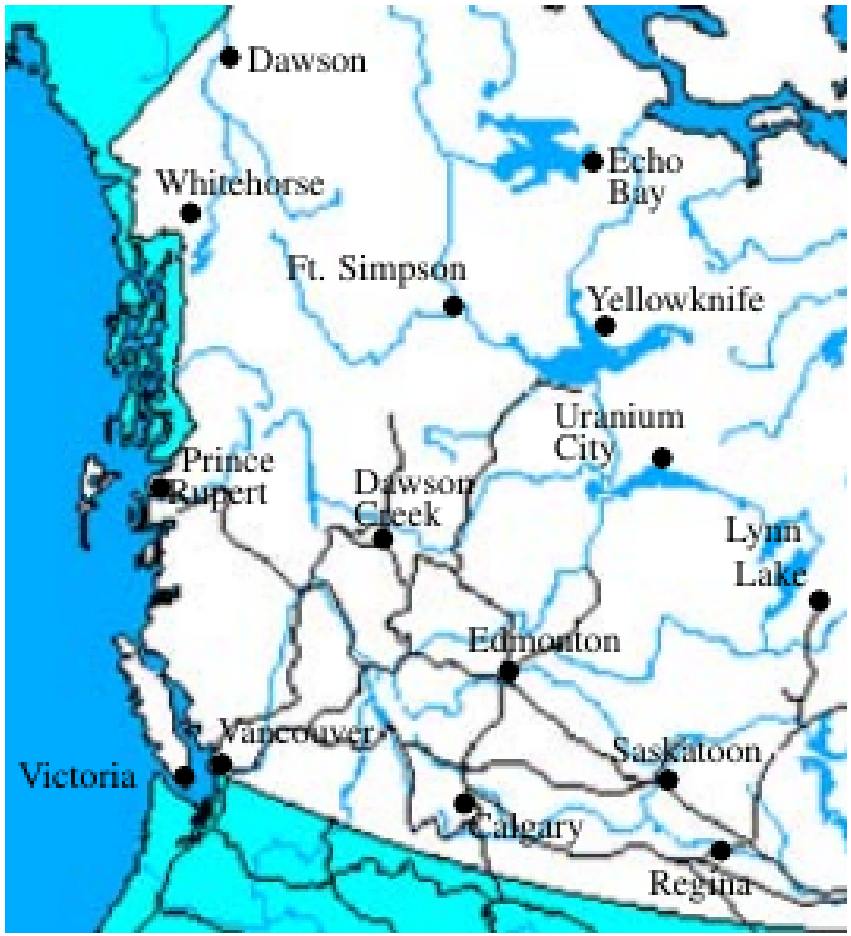
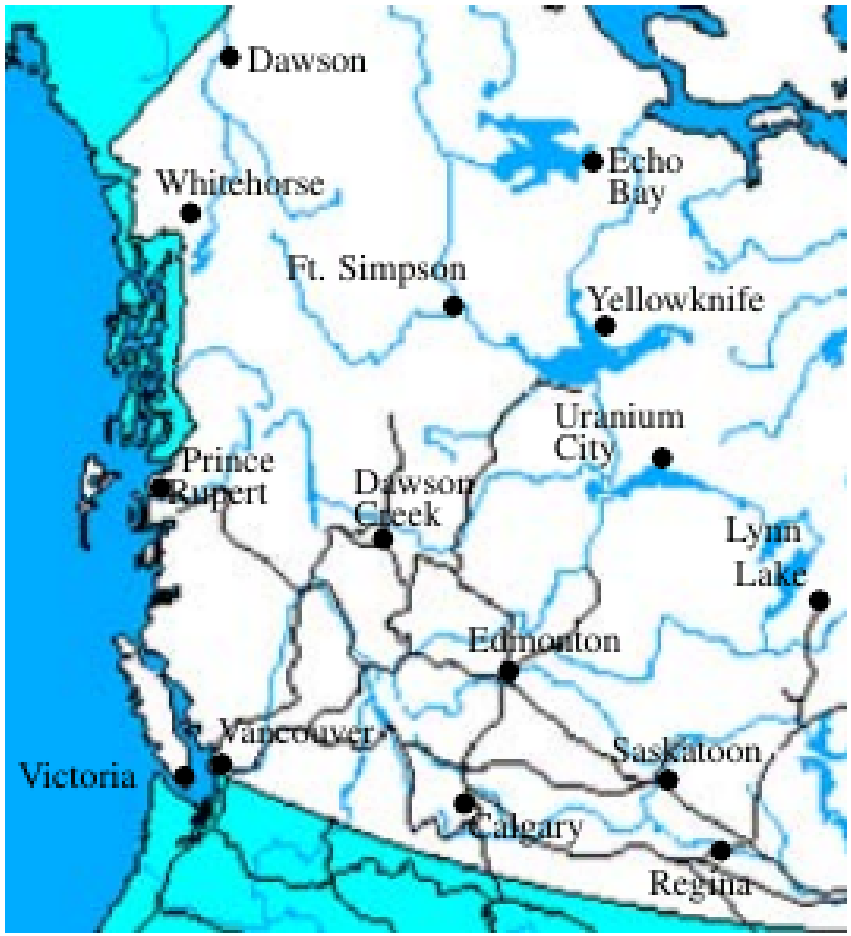
- GIS allows users to create, manage, analyze, and map all types of data.
- Each type of information (e.g., roads, rivers) is stored in separate layers.



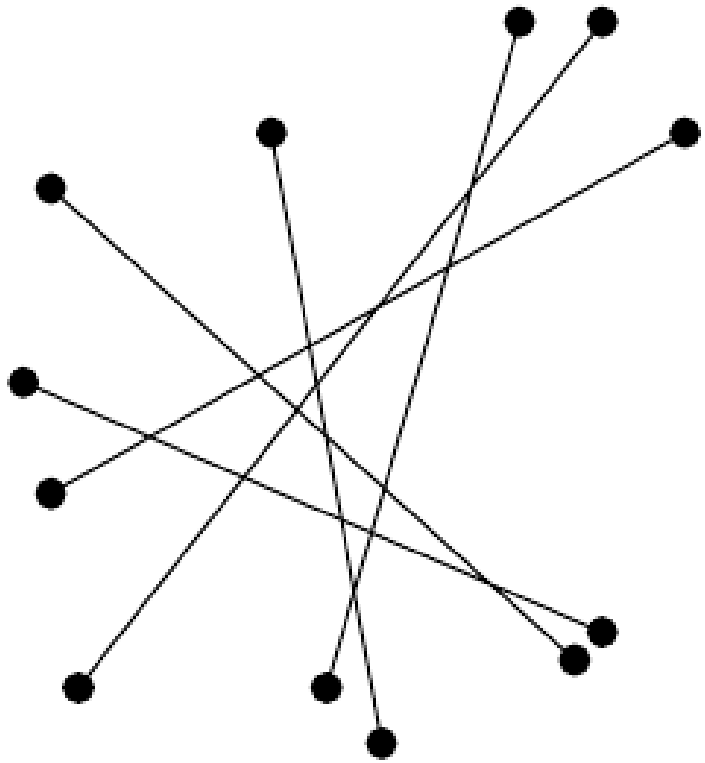
Thematic Map Overlay

- GIS allows users to create, manage, analyze, and map all types of data.
- Each type of information (e.g., roads, rivers) is stored in separate layers.
- Intersections are positions of special interest: For example, in the overlay of road and river layers.



Restatement of the Problem

- Given two sets of line segments, compute all intersections between a segment from one set and a segment from the other.



- **Input:** A set S of line segments in the plane.

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- **Ouput:** The set of intersection points among the segments in S, with for each intersection point the segments that contain it

Ways of Solving the Line Segment Intersection Problem

- **Brute-force**: simply take each pair of segments, compute whether they intersect, and, if so, report their intersection point.($O(n^2)$)

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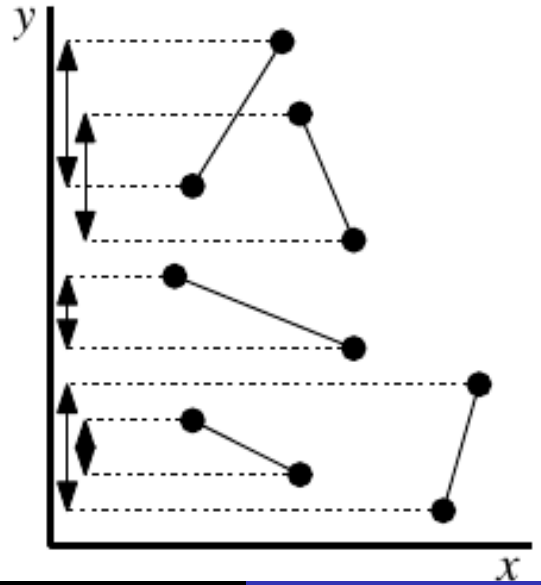
- **Brute-force**: simply take each pair of segments, compute whether they intersect, and, if so, report their intersection point.($O(n^2)$)
- **Sweep Line**: segments that are close together are candidates for intersection, unlike segments that are far apart. ($O((n+k)logn)$ where k is the size of the output)



Sweep Line Algorithm

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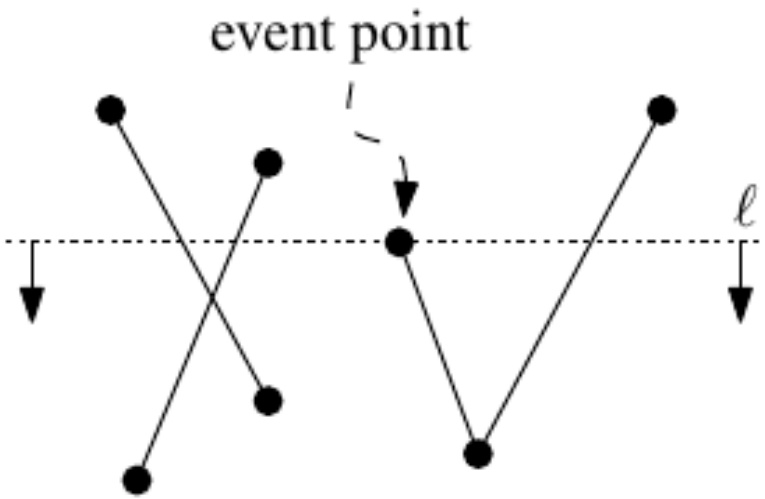

- After sweeping the entire plane and treating the last event point, all intersection points are computed.

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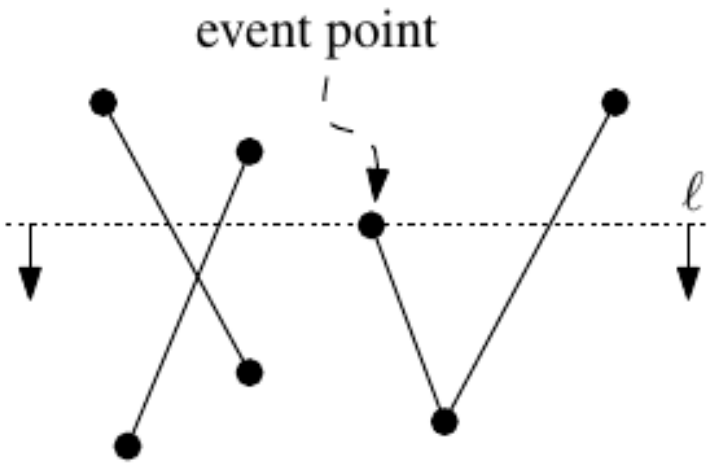
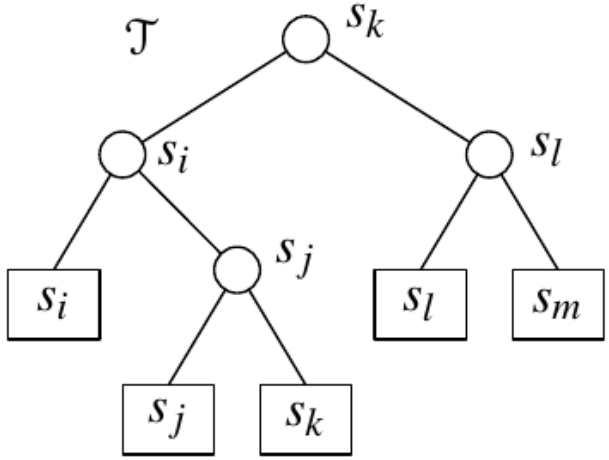
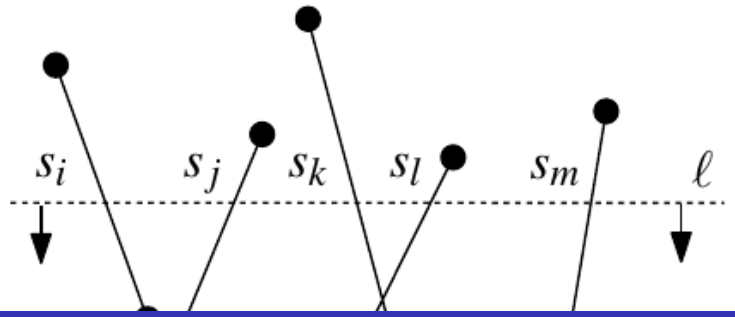
- Invariant: All intersection points above the sweep line are correctly computed at any given time during the plane sweep.



Data Structures

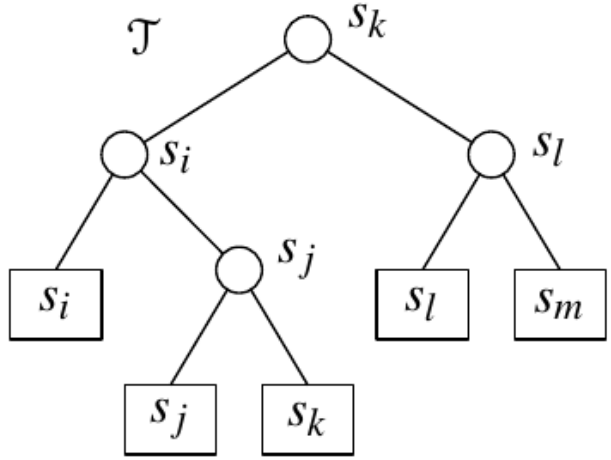
- **Event Queue (Q):**
  - Stores upcoming events (endpoints and intersections).
  - Sorted by y-coordinate.

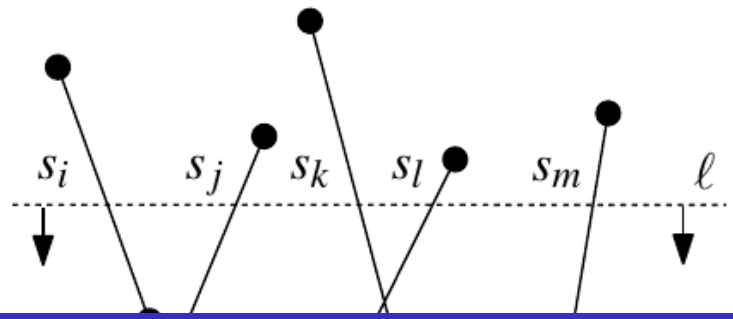




Data Structures

- **Event Queue (Q):**
  - Stores upcoming events (endpoints and intersections).
  - Sorted by y-coordinate.
- **Status Structure (T):**
  - Maintains current intersecting segments.
  - Sorted by x-coordinate at the sweep line.





Event Points in Plane Sweep

- **Upper Endpoint:**
  - New segment intersects the sweep line.
  - Test for intersections with adjacent segments.

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  - Segment stops intersecting the sweep line
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Event Points in Plane Sweep

- **Upper Endpoint:**
  - New segment intersects the sweep line.
  - Test for intersections with adjacent segments.
- **Lower Endpoint:**
  - Segment stops intersecting the sweep line
  - Test new adjacent segments for intersections.
- **Intersection Point:**
  - Two segments swap positions.
  - Test new adjacent segments for intersections.

Handling Event Points

- **Upper Endpoint:**
  - Add new segment to status.
  - Test for intersection with neighbors.

Handling Event Points

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- **Lower Endpoint:**
  - Remove segment from status.
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- **Intersection Point**
  - Swap segment order.
  - Test new neighbors for intersections.

Algorithm Steps

- **Initialization:**
  - Insert all segment endpoints into event queue Q.
  - Initialize empty status structure T.

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- **Processing:** While Q is not empty
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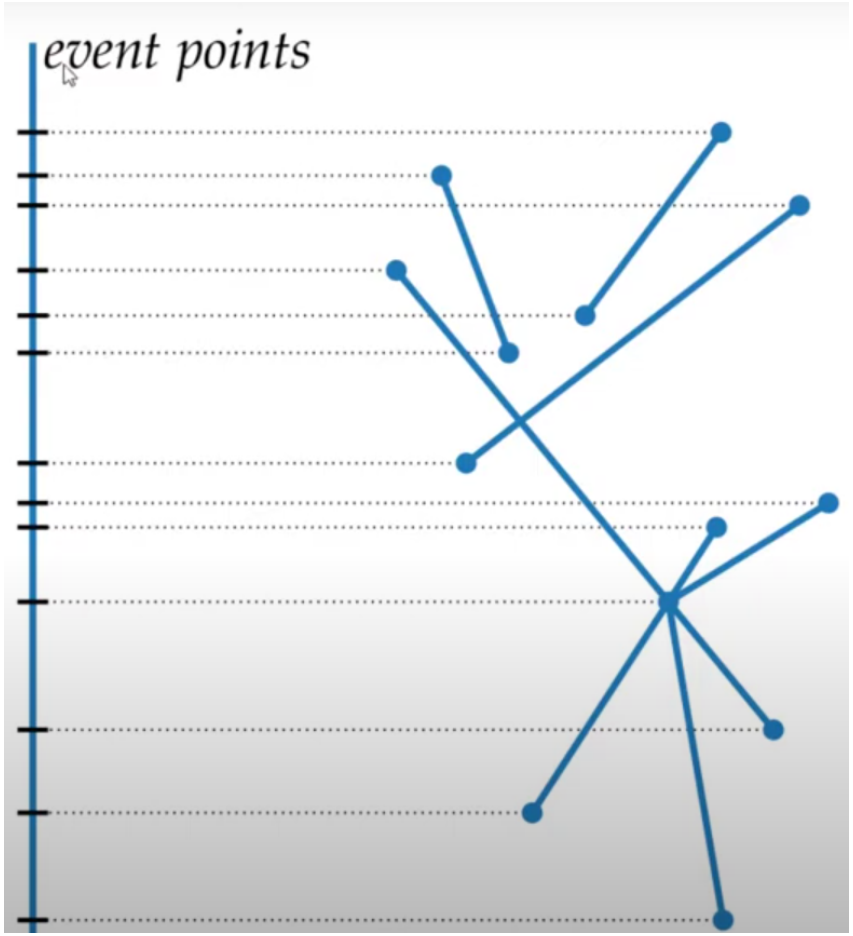
- **Initialization:**
  - Insert all segment endpoints into event queue Q.
  - Initialize empty status structure T.
- **Processing:** While Q is not empty
  - Extract the next event point p.
  - Handle event at p.
- **Handling Events:**
  - Update status structure T.
  - Test for new intersections.

An Example



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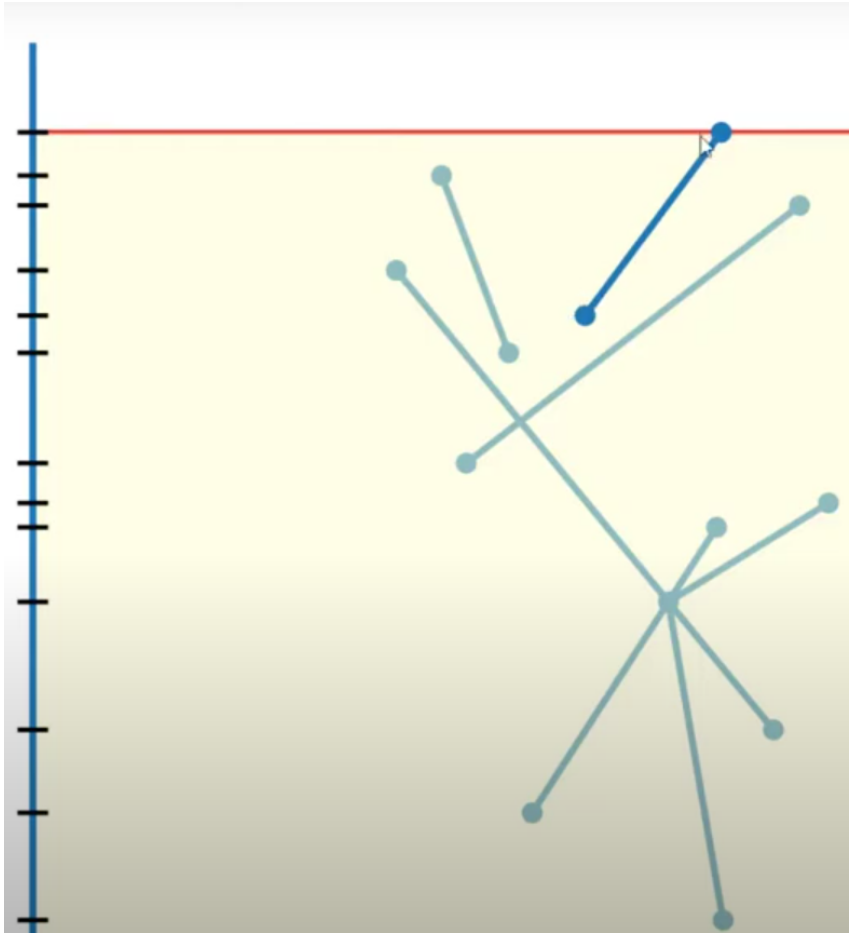

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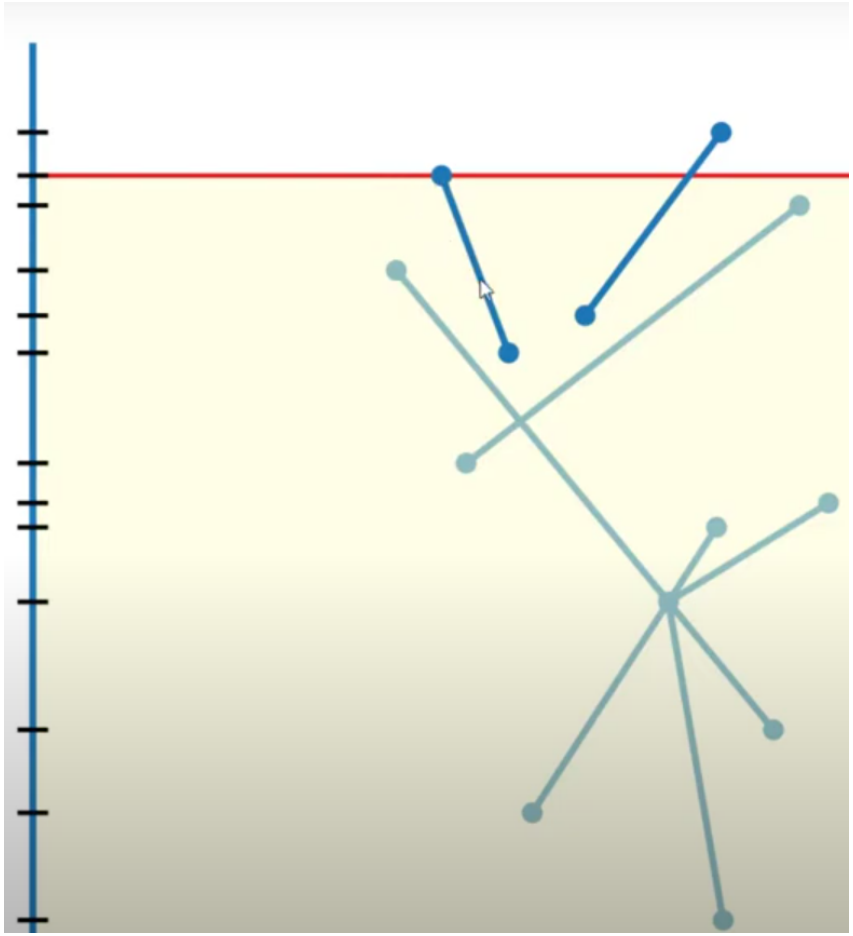

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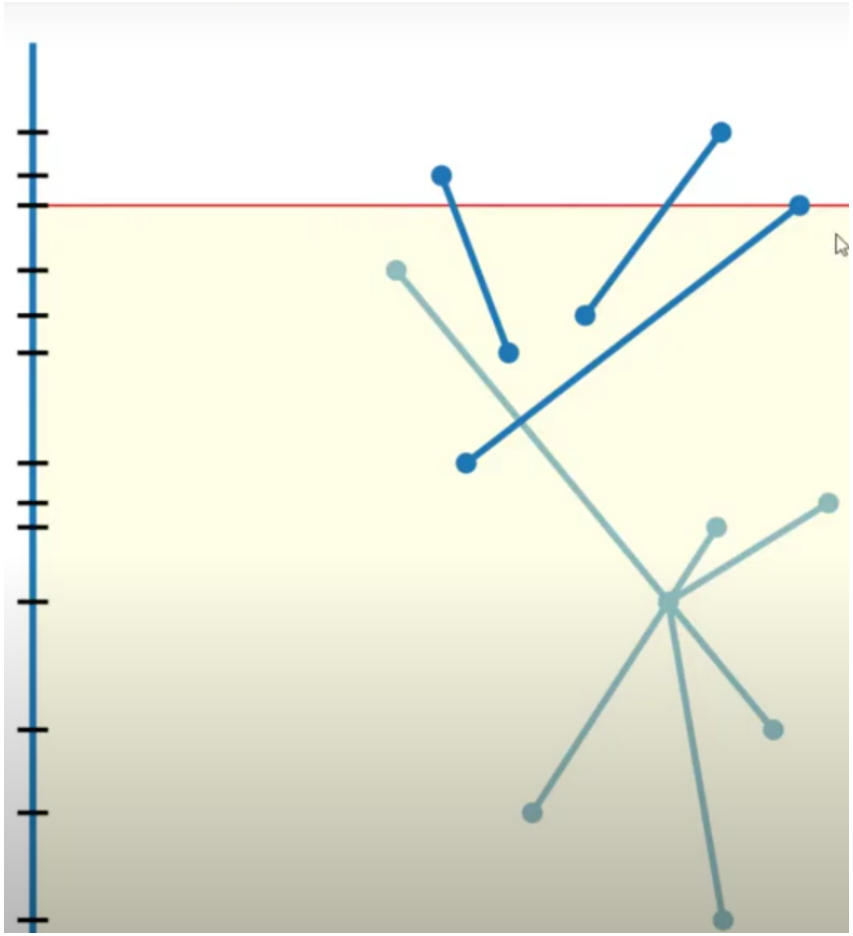

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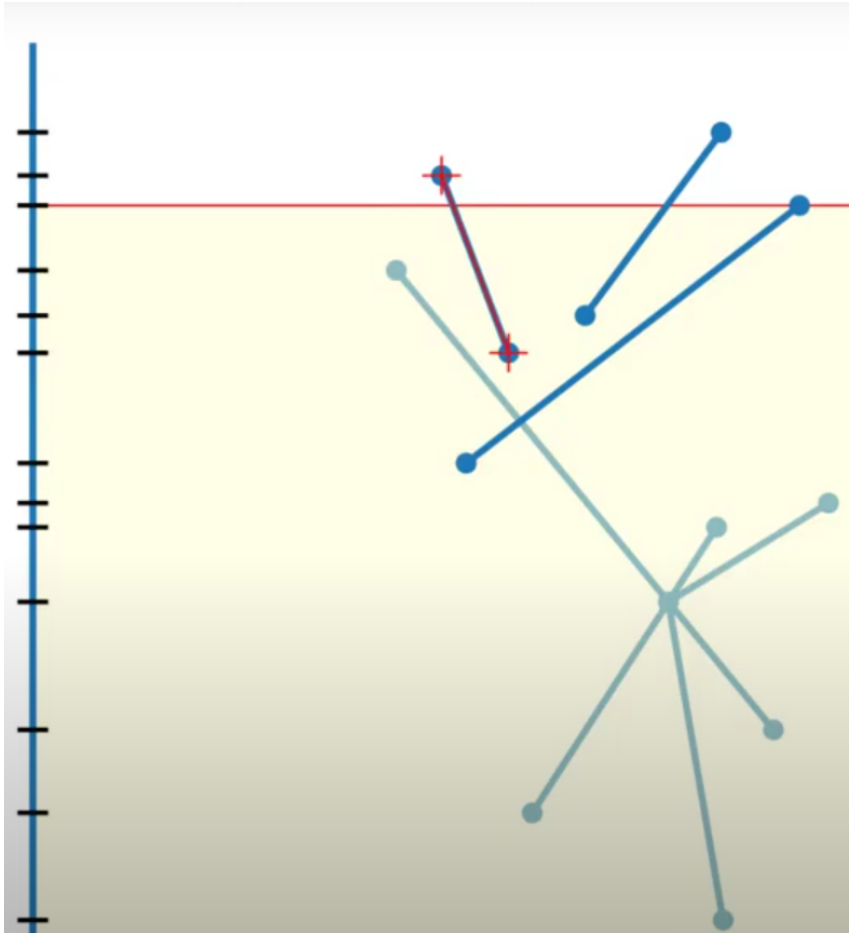

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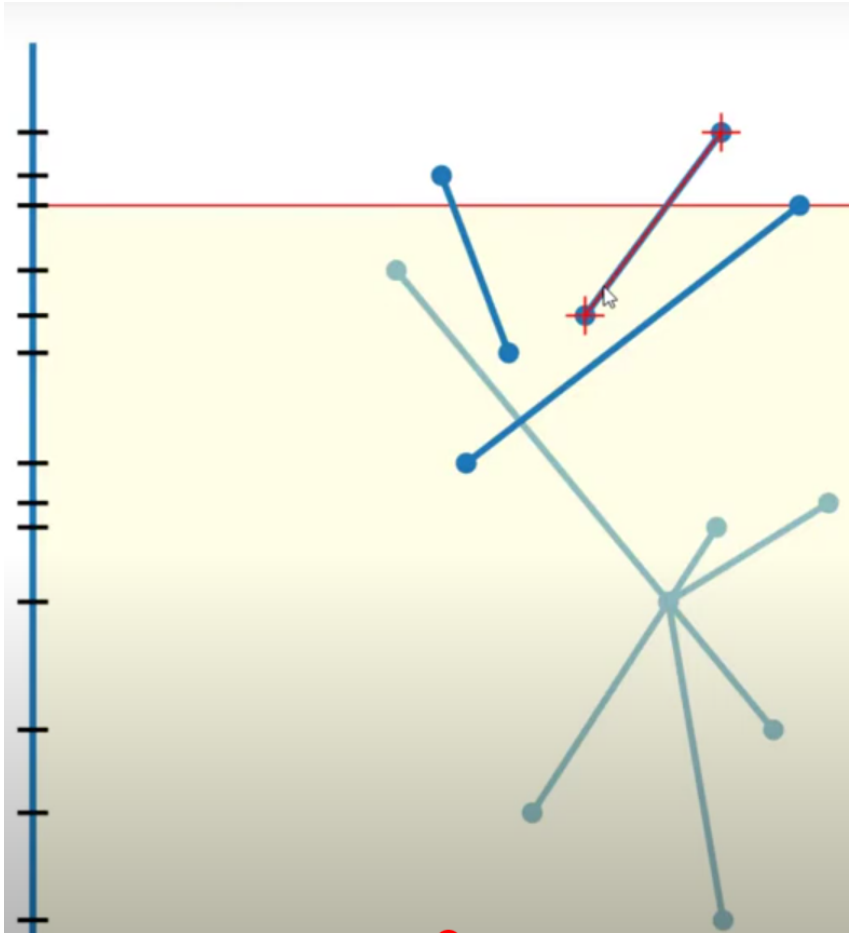

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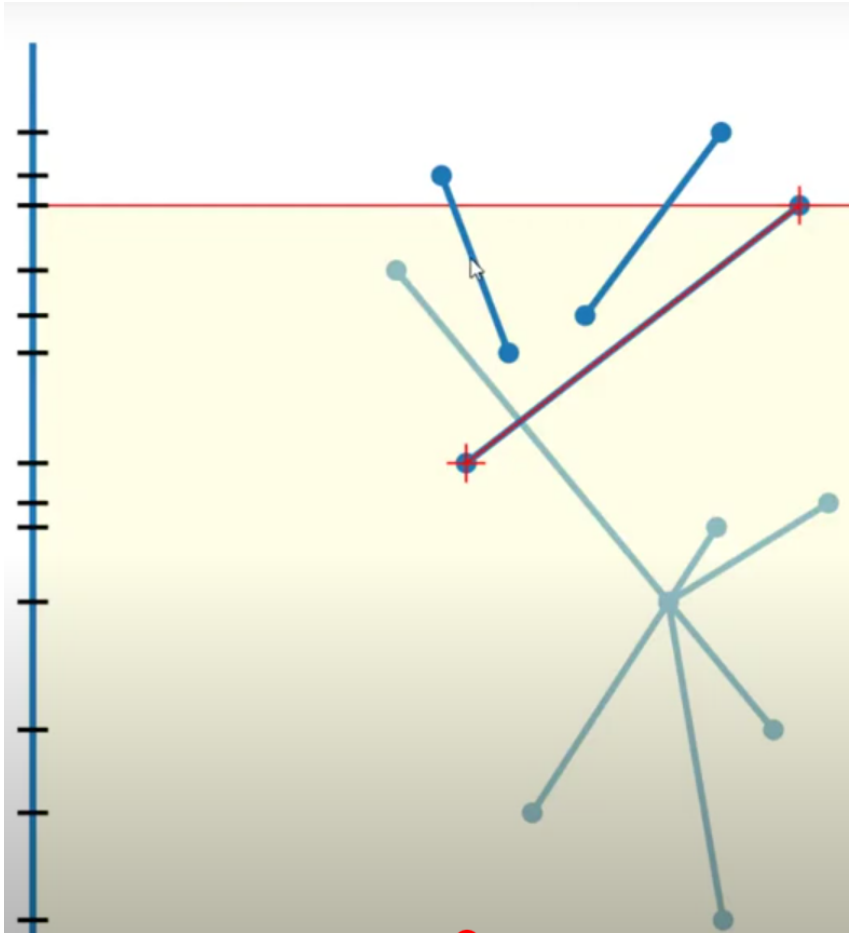

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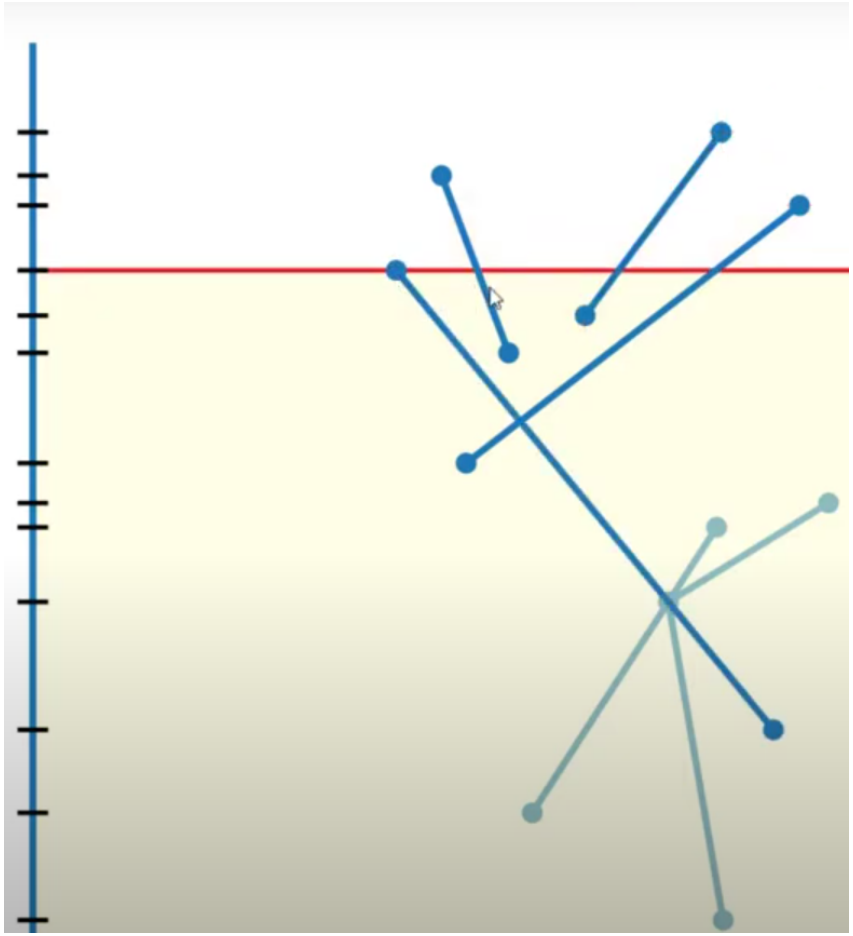

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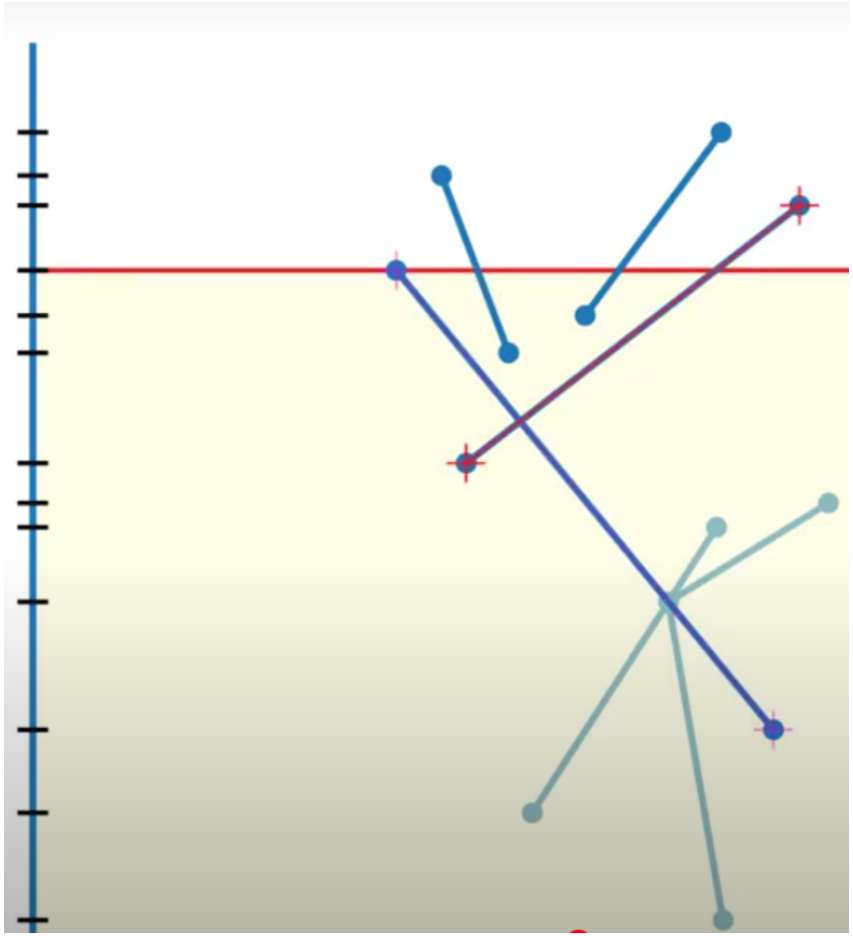

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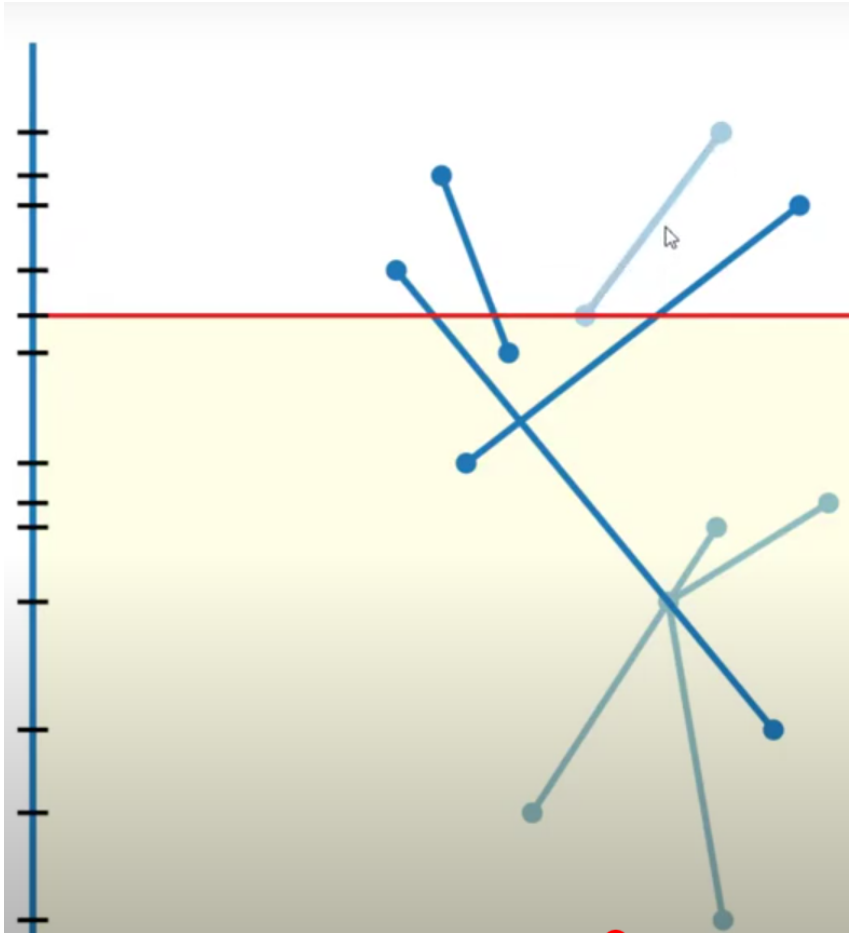

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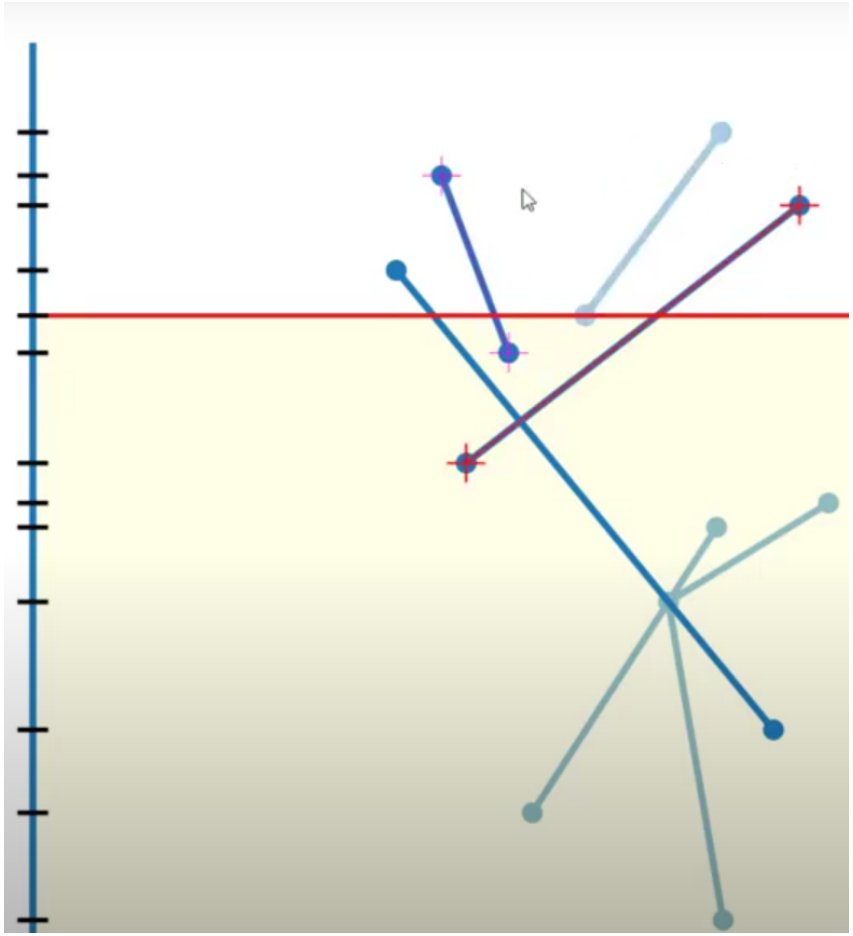

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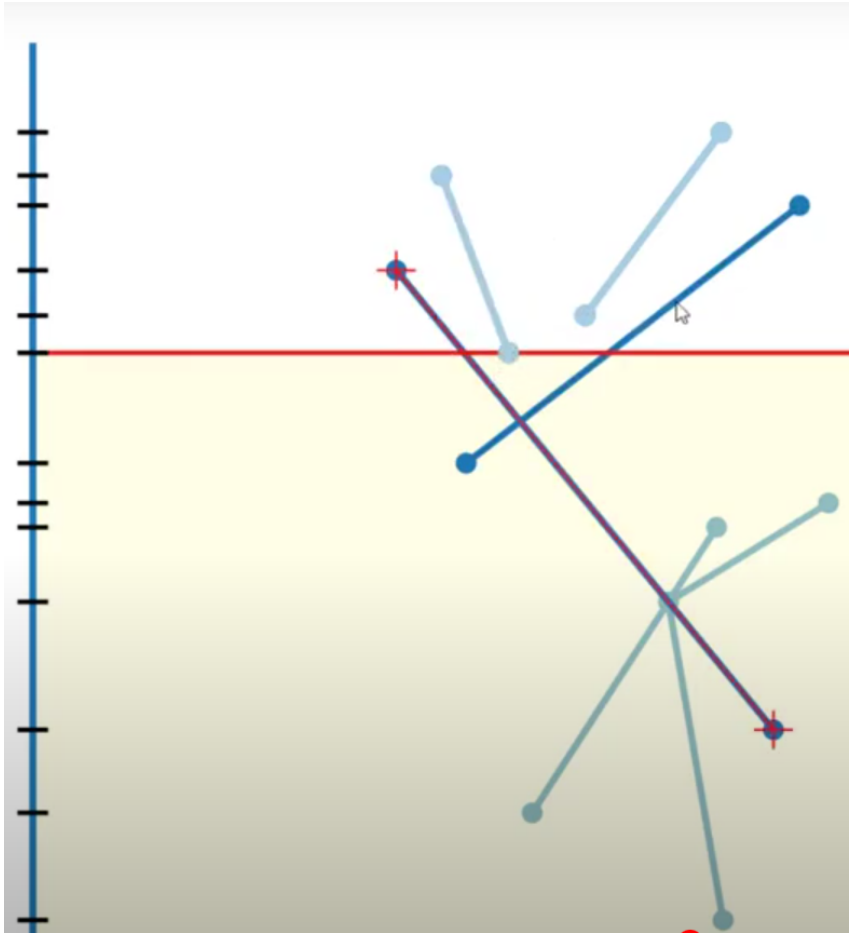

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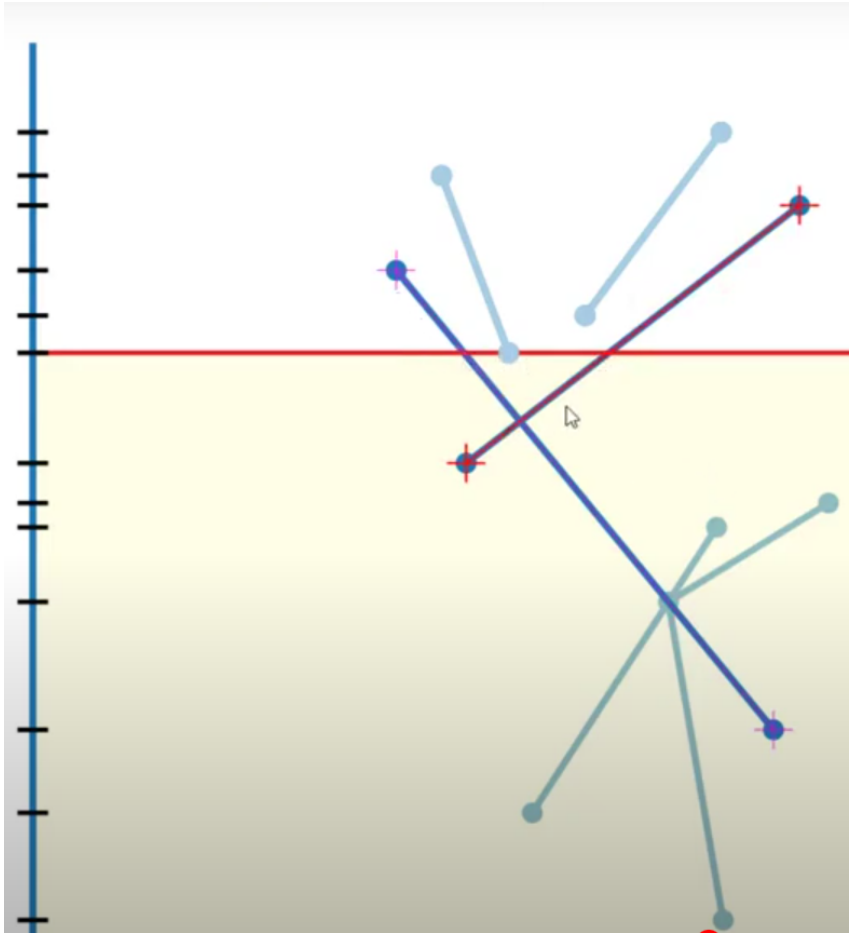

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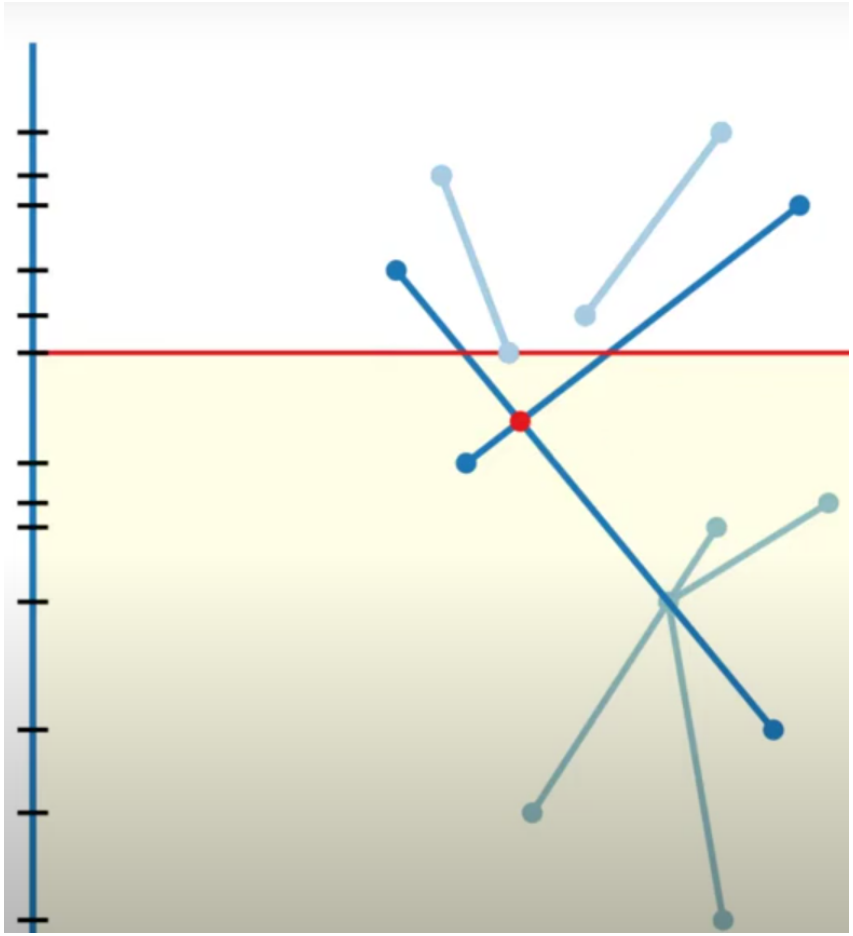

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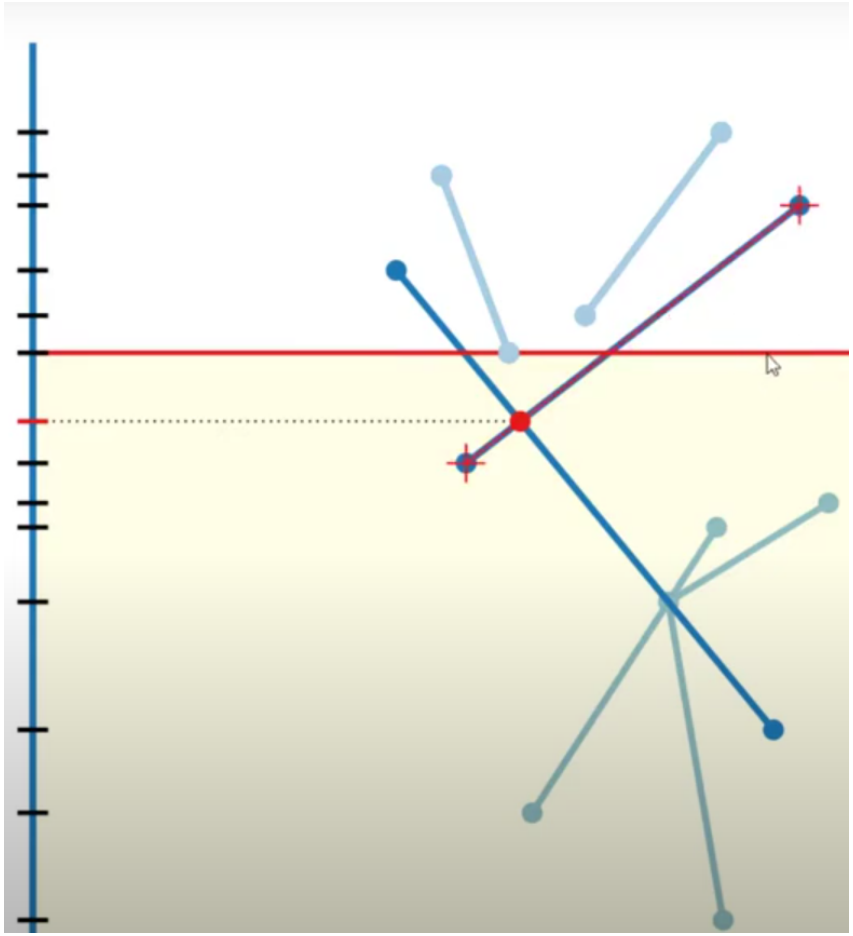

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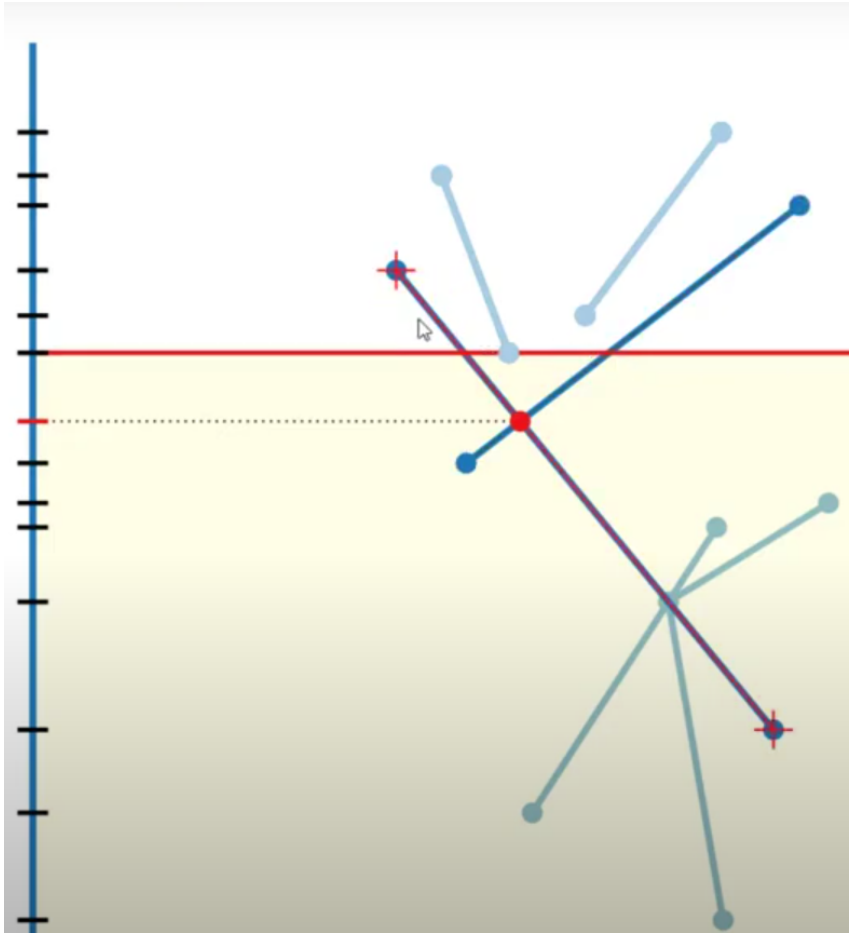

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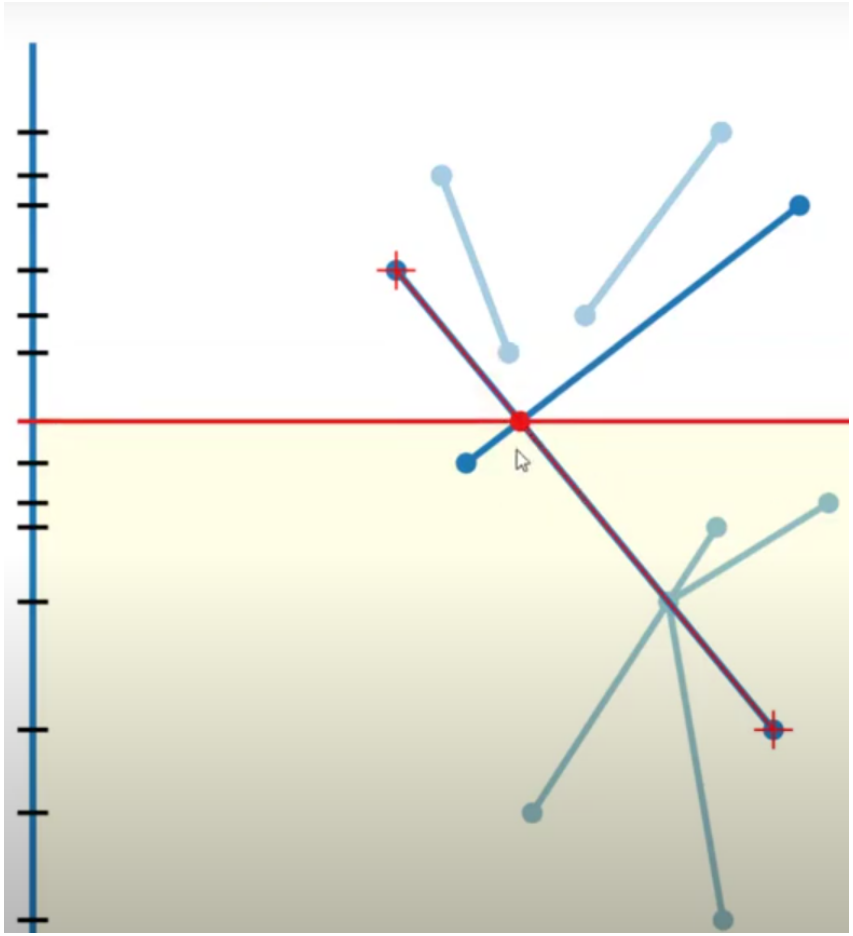

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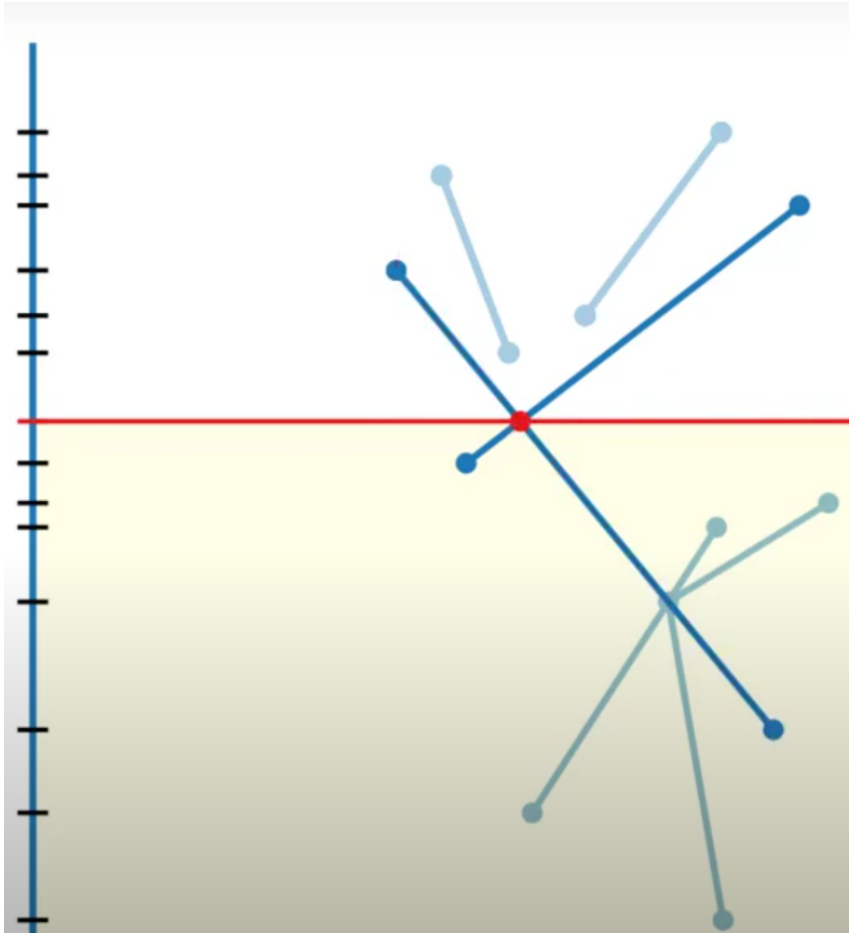

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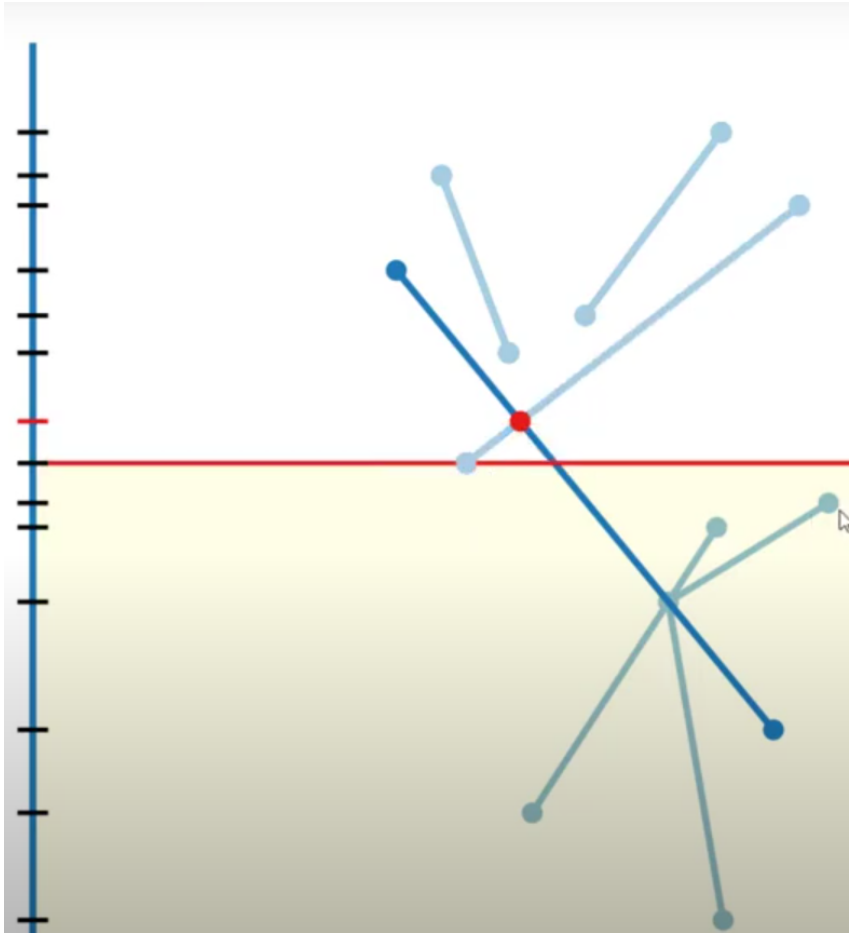

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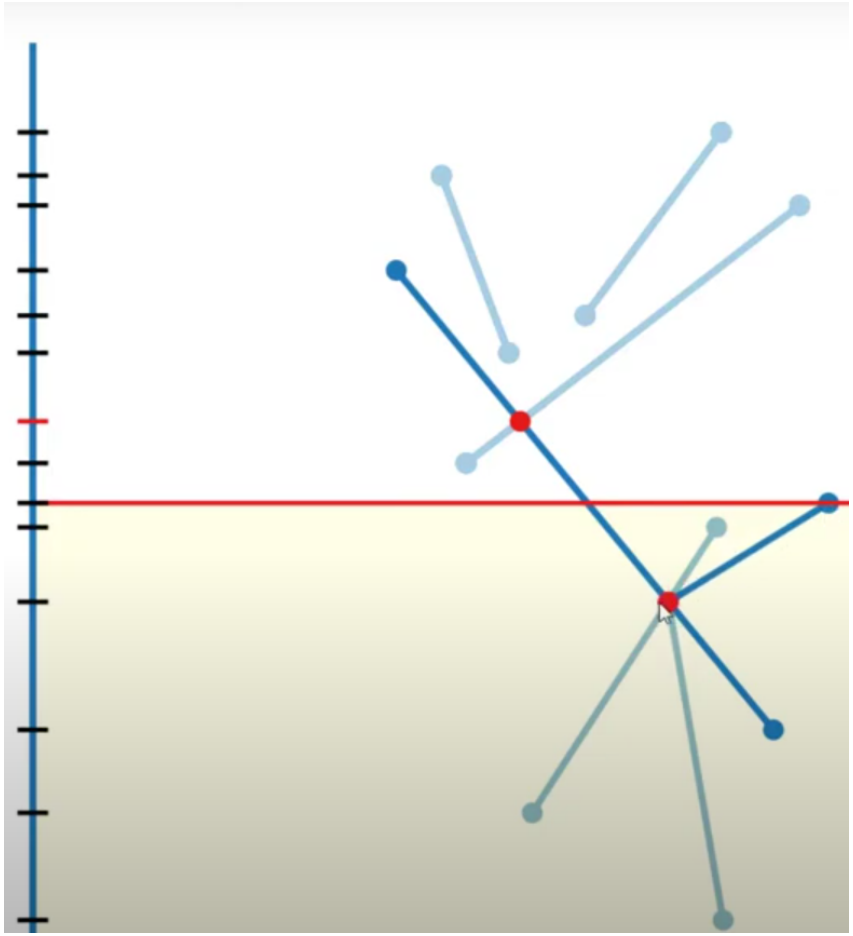

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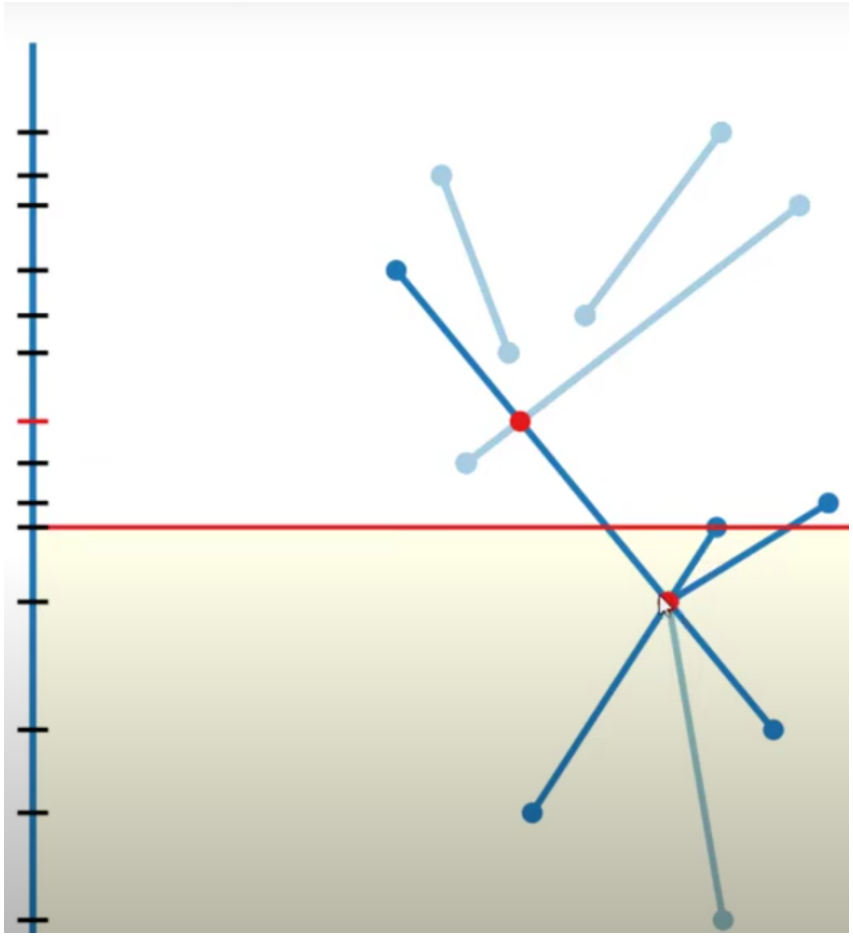

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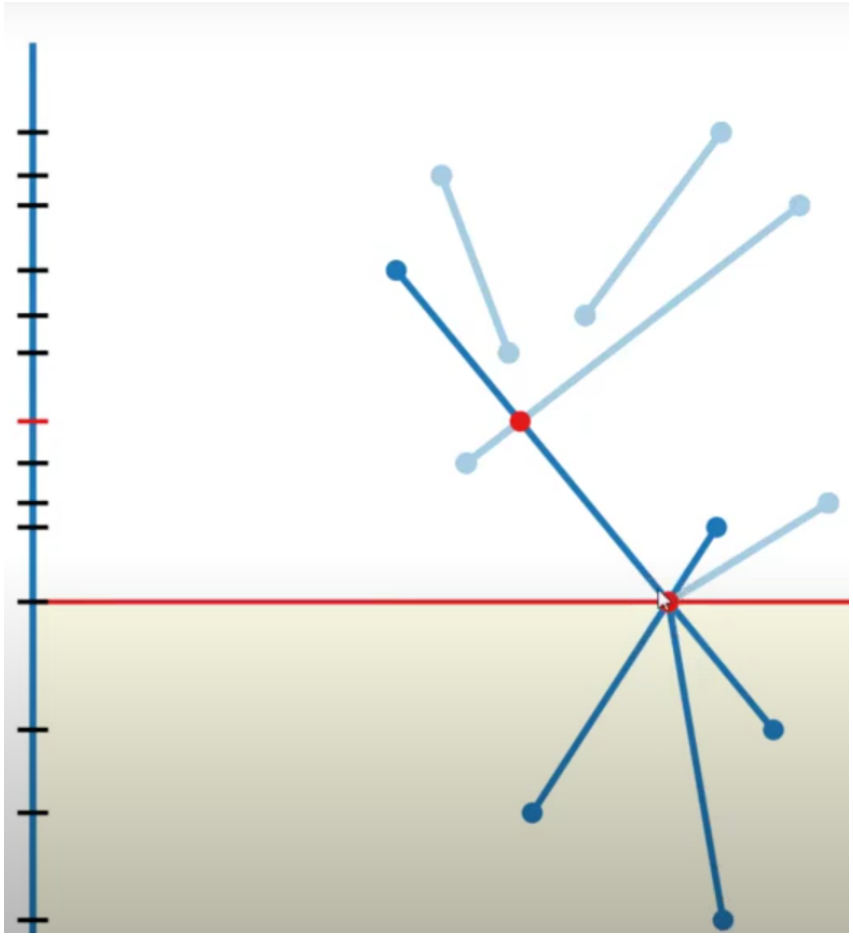

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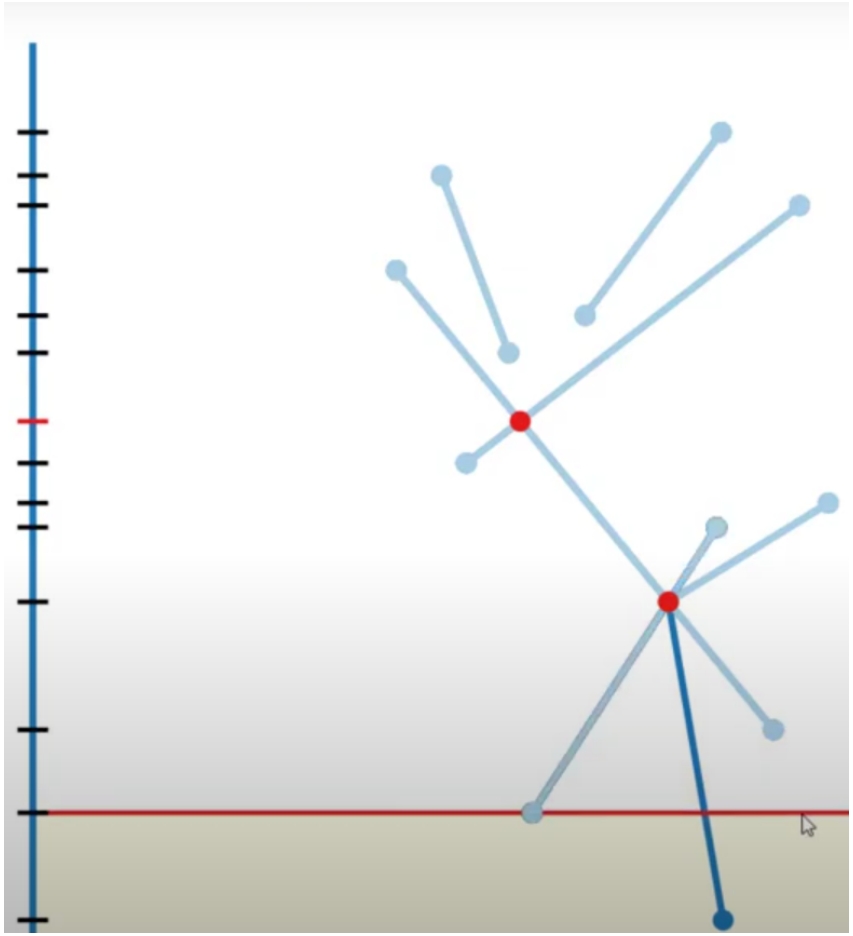

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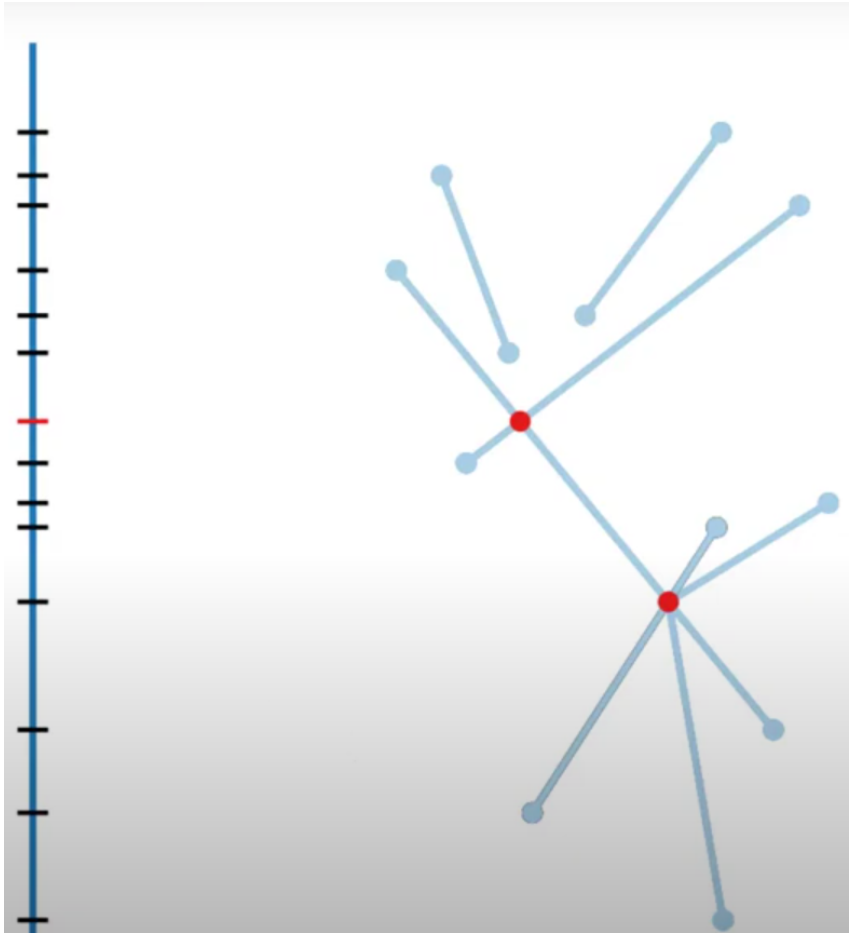

References


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Bartuschka, Ulrike and Mehlhorn, Kurt and Nn, Stefan, *A Robust and Efficient Implementation of a Sweep Line Algorithm for the Straight Line Segment Intersection Problem*, 1997


Sweep-Line Algorithm for Line Segment Intersection


Plane Sweep Algorithm for finding Line Segment Intersections