Challenger disaster

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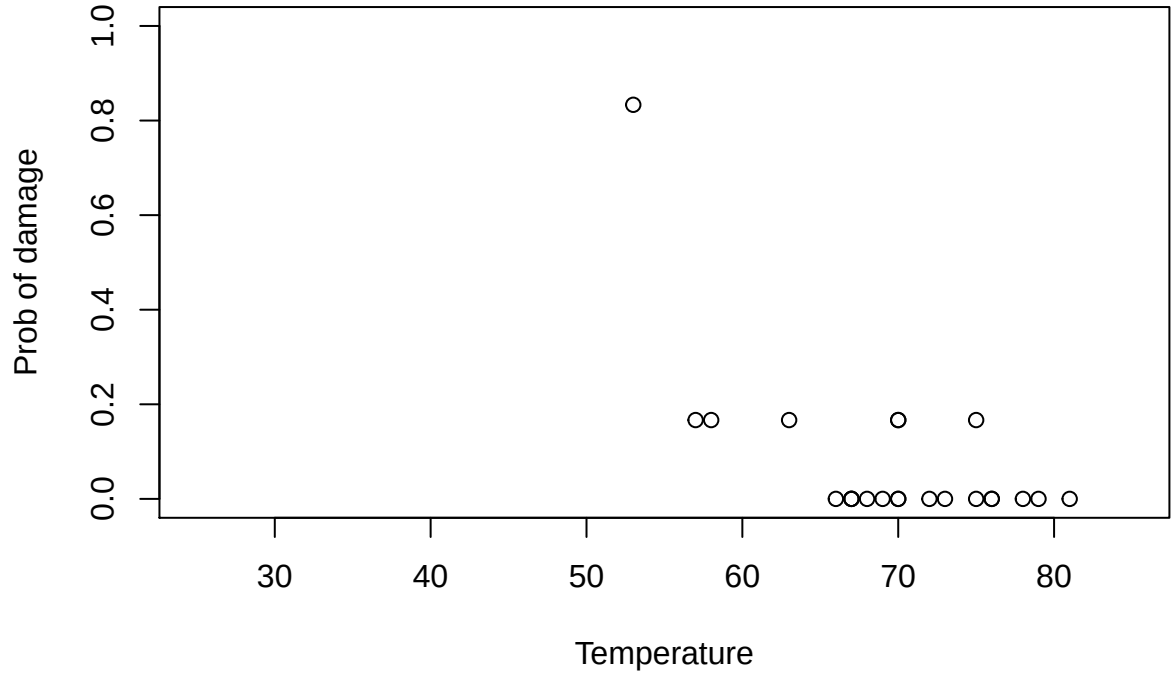
load and plot data

```
library(faraway)
data(orings)
str(orings)
```

```
## 'data.frame':    23 obs. of  2 variables:
##  $ temp  : num  53 57 58 63 66 67 67 67 68 69 ...
##  $ damage: num  5 1 1 1 0 0 0 0 0 0 ...
```

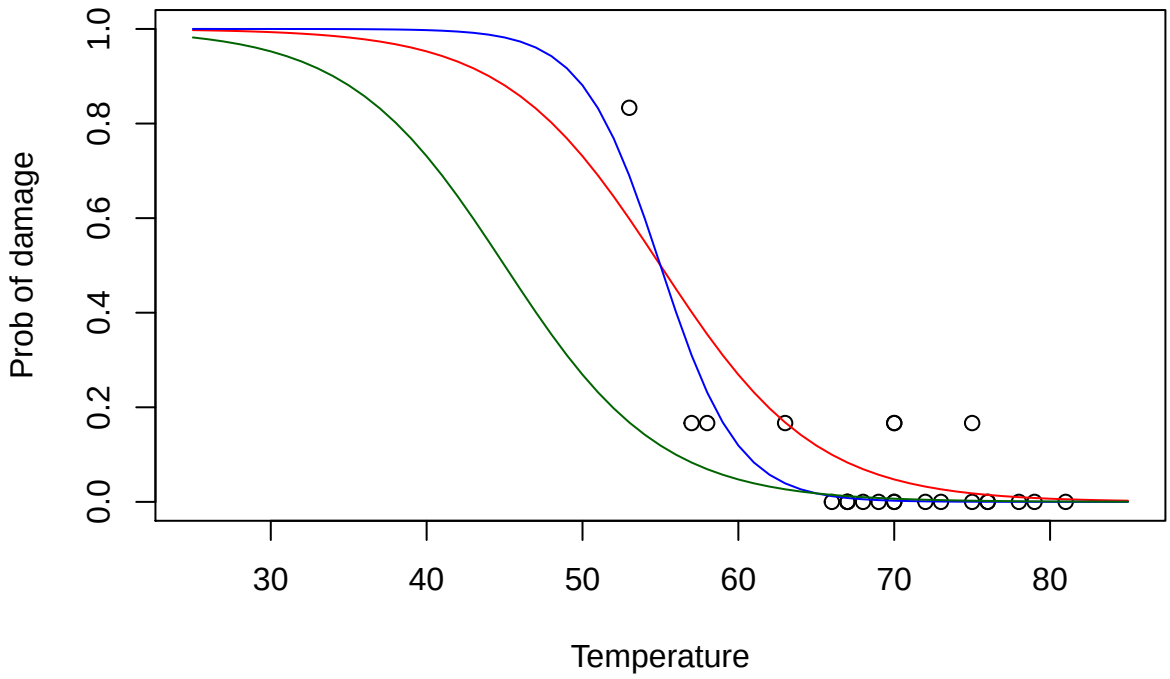
```
plot(damage/6 ~ temp, orings, xlim=c(25,85), ylim=c(0,1),
     xlab="Temperature", ylab="Prob of damage")
```



logistic function with different values for beta0 and beta1

```
try <- function(a, b, col = "red") {
  t <- seq(25, 85, 1)
  p <- 1/(1 + exp(-a - b*t))
  lines(t, p, col = col)
}
plot(damage/6 ~ temp, orings, xlim=c(25,85), ylim=c(0,1),
     xlab="Temperature", ylab="Prob of damage")
try(11, -0.2, col="red")
## Compared to red curve: same location, stronger steepness
```

```
try(22, -0.4, col="blue")
## Compared to red curve: shifted location, same steepness
try(9, -0.2, col="darkgreen")
```



maximum likelihood fitting

Define the log likelihood

```
logL <- function(beta, orings) {
  eta <- cbind(1, orings$temp) %*% beta
  return( sum( orings$damage*eta - 6*log(1 + exp(eta)) ) )
}
```

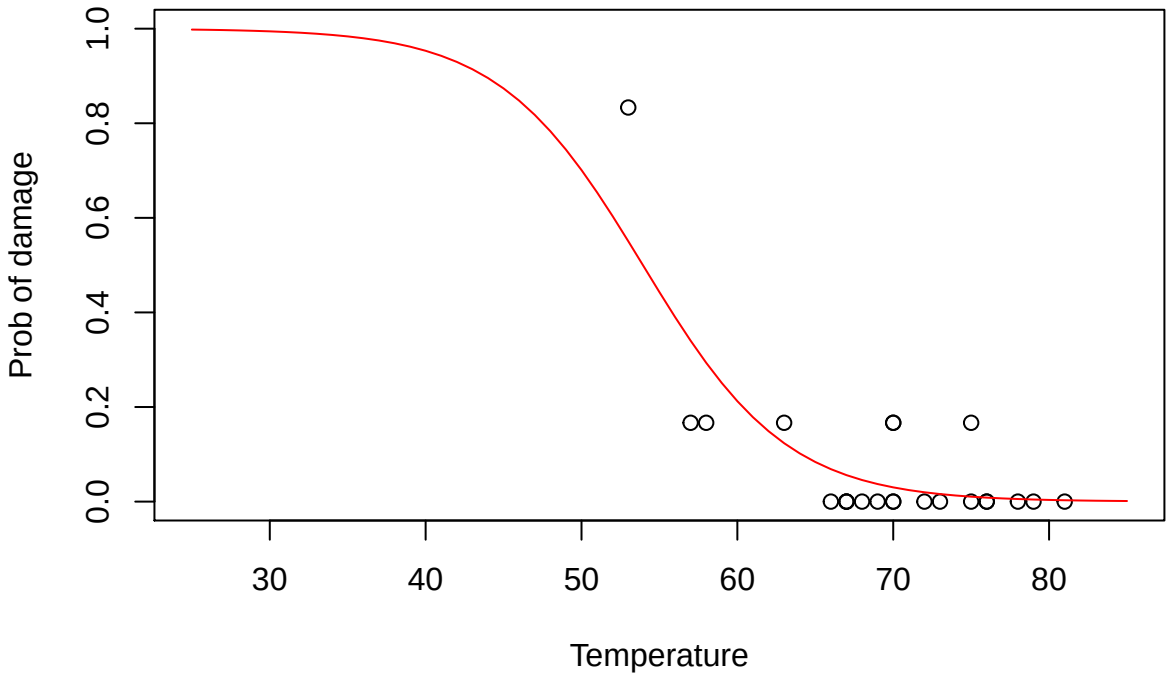
Find MLE using optim function

```
(betahat <- optim(c(10, -.1), logL, orings=orings, control=list(fnscale=-1))$par)
```

```
## [1] 11.6671414 -0.2162982
```

plot fitted model

```
plot(damage/6 ~ temp, orings, xlim=c(25,85), ylim=c(0,1),
     xlab="Temperature", ylab="Prob of damage")
x <- seq(25,85,1)
ilogit <- function(x) exp(x)/(1+exp(x))
lines(x, ilogit(betahat[1] + betahat[2]*x), col="red")
```



prediction for temp of 29

```
ilogit (betahat[1] + betahat[2]*29)
```

```
## [1] 0.995479
```

Using the glm command instead

```
logitmod <- glm(cbind(damage,6-damage) ~ temp, family=binomial, orings)
summary(logitmod)
```

```
##
## Call:
## glm(formula = cbind(damage, 6 - damage) ~ temp, family = binomial,
##     data = orings)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.9529  -0.7345  -0.4393  -0.2079   1.9565
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) 11.66299    3.29626   3.538 0.000403 ***
## temp        -0.21623    0.05318  -4.066 4.78e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 38.898  on 22  degrees of freedom
## Residual deviance: 16.912  on 21  degrees of freedom
```

```
## AIC: 33.675
##
## Number of Fisher Scoring iterations: 6
```

```
predict(logitmod, newdata=data.frame(temp=29), type="response")
```

```
##         1
## 0.9954687
```

Confidence Interval for p

Compute standard errors

```
phat <- ilogit(betahat[1] + orings$temp*betahat[2])
I11 <- sum(6*phat*(1 - phat))
I12 <- sum(6*orings$temp*phat*(1 - phat))
I22 <- sum(6*orings$temp^2*phat*(1 - phat))
Iinv <- solve(matrix(c(I11, I12, I12, I22), 2, 2))
sqrt(Iinv[1,1])
```

```
## [1] 3.296634
```

```
sqrt(Iinv[2,2])
```

```
## [1] 0.05318407
```

Compute CI for eta = beta0 + beta1 29

```
si2 <- matrix(c(1, 29), 1, 2) %*% Iinv %*% matrix(c(1, 29), 2, 1)
etahat = betahat[1] + betahat[2]*29
eta_l = etahat - 2*sqrt(si2)
eta_r = etahat + 2*sqrt(si2)
etahat
```

```
## [1] 5.394493
```

```
c(eta_l, eta_r)
```

```
## [1] 1.851533 8.937452
```

Compute CI for p

```
ilogit (etahat)
```

```
## [1] 0.995479
```

```
c(ilogit (eta_l), ilogit (eta_r))
```

```
## [1] 0.8643070 0.9998686
```

Wald Test

Compute MLE

```
library(faraway)
data(orings)
logL <- function(beta, orings) {
  eta <- cbind(1, orings$temp) %*% beta
```

```
  return( sum( orings$damage*eta - 6*log(1 + exp(eta)) ) )
}
(betahat <- optim(c(10, -.1), logL, orings=orings, control=list(fnscale=-1))$par)
```

```
## [1] 11.6671414 -0.2162982
```

Compute standard errors of MLE

```
ilogit <- function(x) exp(x)/(1+exp(x))
phat <- ilogit(betahat[1] + orings$temp*betahat[2])
I11 <- sum(6*phat*(1 - phat))
I12 <- sum(6*orings$temp*phat*(1 - phat))
I22 <- sum(6*orings$temp^2*phat*(1 - phat))
Iinv <- solve(matrix(c(I11, I12, I12, I22), 2, 2))
sqrt(Iinv[1,1])
```

```
## [1] 3.296634
```

```
sqrt(Iinv[2,2])
```

```
## [1] 0.05318407
```

Wald test statistic

```
betahat[2]/sqrt(Iinv[2,2])
```

```
## [1] -4.066974
```

p-value from Wald test statistic

```
2*pnorm(abs(betahat[2]/sqrt(Iinv[2,2])), 0, 1, lower=FALSE)
```

```
## [1] 4.762755e-05
```

Likelihood Ratio test

Compute maximum log likelihood from the full model

```
(MaxlogL.F = logL(betahat,orings))
```

```
## [1] -27.37971
```

Compute maximum log likelihood from the reduced model

```
y <- orings$damage
n <- rep(6, length(y))
phatN <- sum(y)/sum(n)
(MaxlogL.R = sum(orings$damage)*log(phatN) + sum(6-orings$damage)*log(1-phatN))
```

```
## [1] -38.3724
```

LR test statistic

```
(LR = -2*(MaxlogL.R - MaxlogL.F))
```

```
## [1] 21.98538
```

p-value from LR test statistic

```
pchisq(LR, df=1,lower=FALSE)
```

```
## [1] 2.747354e-06
```

Wald Test vs Likelihood Ratio test

Square of Wald test statistic

```
(betahat[2]/sqrt(Iinv[2,2]))^2
```

```
## [1] 16.54028
```

LR test statistic

```
(LR = -2*(MaxlogL.R - MaxlogL.F))
```

```
## [1] 21.98538
```

Deviance

Deviance and df for the fitted model

```
y <- orings$damage
n <- rep(6, length(y))
ylogxy <- function(x, y) ifelse(y == 0, 0, y*log(x/y))
(D <- -2*sum(ylogxy(n*phat, y) + ylogxy(n*(1-phat), n - y)))
```

```
## [1] 16.91228
```

```
(df <- length(y) - length(betahat))
```

```
## [1] 21
```

```
pchisq(D, df,lower=FALSE)
```

```
## [1] 0.7164098
```

Deviance and df for the fitted model using the glm command

```
logitmod <- glm(cbind(damage,6-damage) ~ temp, family=binomial, orings)
deviance(logitmod)
```

```
## [1] 16.91228
```

```
df.residual(logitmod)
```

```
## [1] 21
```

Deviance and df for the null model

```
(phatN <- sum(y)/sum(n))
```

```
## [1] 0.07971014
```

```
(DN <- -2*sum(ylogxy(n*phatN, y) + ylogxy(n*(1-phatN), n - y)))
```

```
## [1] 38.89766
```

```
(dfN <- length(y) - 1)
```

```
## [1] 22
```

```
pchisq(DN, dfN,lower=FALSE)
```

```
## [1] 0.0144977
```

Deviance and df for the null model using the glm command

```
logitnull <- glm(cbind(y, n - y) ~ 1, family=binomial)
summary(logitnull)
```

```
##
## Call:
## glm(formula = cbind(y, n - y) ~ 1, family = binomial)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.9984  -0.9984  -0.9984   0.6947   4.4781
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -2.4463     0.3143  -7.783 7.06e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 38.898  on 22  degrees of freedom
## Residual deviance: 38.898  on 22  degrees of freedom
## AIC: 53.66
##
## Number of Fisher Scoring iterations: 4
```

```
ilogit(-2.4463)
```

```
## [1] 0.07970954
```

LRT using deviance

```
DN-D
```

```
## [1] 21.98538
```

```
pchisq(DN - D, dfN - df, lower=FALSE)
```

```
## [1] 2.747354e-06
```