Random-walk Metropolis-Hastings example from A simple Metropolis-Hastings MCMC in R, Florian Hartig, September 17 2010, copied from http://theoreticalecology.wordpress.com/2010/09/17/metropolis-hastings-mcmc-in-r/

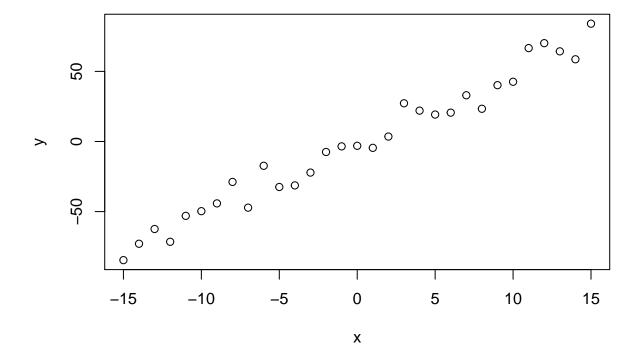
Creating test data

```
trueA <- 5
trueB <- 0
trueSd <- 10
sampleSize <- 31

set.seed(3)
# create independent x-values
x <- (-(sampleSize-1)/2):((sampleSize-1)/2)
# create dependent values according to ax + b + N(0,sd)
y <- trueA * x + trueB + rnorm(n=sampleSize,mean=0,sd=trueSd)

plot(x,y, main="Test Data")</pre>
```

Test Data

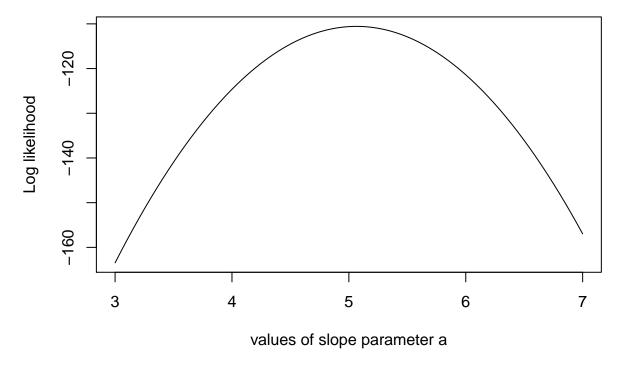


Defining the statistical model and Derive the likelihood function from the model

```
likelihood <- function(param){
    a = param[1]
    b = param[2]
    sd = param[3]

pred = a*x + b
    singlelikelihoods = dnorm(y, mean = pred, sd = sd, log = T)
    suml1 = sum(singlelikelihoods)
    return(suml1)
}

# Example: plot the likelihood profile of the slope a
slopevalues <- function(x){return(likelihood(c(x, trueB, trueSd)))}
slopelikelihoods <- lapply(seq(3, 7, by=.05), slopevalues)
plot (seq(3, 7, by=.05), slopelikelihoods, type="l", xlab = "values of slope parameter a", ylab = "Log</pre>
```



Defining the prior

```
# Prior distribution
prior <- function(param){
    a = param[1]
    b = param[2]
    sd = param[3]
    aprior = dunif(a, min=0, max=10, log = T)</pre>
```

```
bprior = dnorm(b, sd = 5, log = T)
sdprior = dunif(sd, min=0, max=30, log = T)
return(aprior+bprior+sdprior)
}
```

The posterior

```
posterior <- function(param){
   return (likelihood(param) + prior(param))
}</pre>
```

The MCMC

```
####### Metropolis algorithm ###############
proposalfunction <- function(param){</pre>
    return(rnorm(3, mean = param, sd= c(0.1, 0.5, 0.3)))
run_metropolis_MCMC <- function(startvalue, iterations){</pre>
    chain = array(dim = c(iterations+1,3))
    chain[1,] = startvalue
    for (i in 1:iterations){
        proposal = proposalfunction(chain[i,])
        probab = exp(posterior(proposal) - posterior(chain[i,]))
        if (runif(1) < probab){</pre>
            chain[i+1,] = proposal
        }else{
            chain[i+1,] = chain[i,]
        }
    }
    return(chain)
}
set.seed(1)
startvalue = c(4,0,10)
chain = run_metropolis_MCMC(startvalue, 10000)
burnIn = 5000
acceptance = 1-mean(duplicated(chain[-(1:burnIn),]))
acceptance
```

[1] 0.7546491

```
par(mfrow = c(2,3))
hist(chain[-(1:burnIn),1],nclass=30, , main="Posterior of a", xlab="True value = red line")
abline(v = mean(chain[-(1:burnIn),1]), col="green")
abline(v = trueA, col="red" )
hist(chain[-(1:burnIn),2],nclass=30, main="Posterior of b", xlab="True value = red line")
abline(v = mean(chain[-(1:burnIn),2]), col="green")
abline(v = trueB, col="red" )
hist(chain[-(1:burnIn),3],nclass=30, main="Posterior of sd", xlab="True value = red line")
abline(v = mean(chain[-(1:burnIn),3]), col="green" )
abline(v = trueSd, col="red" )
plot(chain[-(1:burnIn),1], type = "l", xlab="True value = red line", main = "Chain values of a", )
abline(h = trueA, col="red" )
plot(chain[-(1:burnIn),2], type = "l", xlab="True value = red line", main = "Chain values of b", )
abline(h = trueB, col="red" )
plot(chain[-(1:burnIn),3], type = "l", xlab="True value = red line", main = "Chain values of sd",)
abline(h = trueSd, col="red" )
            Posterior of a
                                              Posterior of b
                                                                                Posterior of sd
    009
                                       9
    400
                                  Frequency
                                                                     Frequency
Frequency
    200
                                      200
    0
                                       0
                                                                         0
                                                                                 7 8
         4.5
                5.0
                       5.5
                                           -6
                                               -4
                                                    -2
                                                        0
                                                                                      9
           True value = red line
                                             True value = red line
                                                                               True value = red line
          Chain values of a
                                            Chain values of b
                                                                              Chain values of sd
chain[-(1:burnIn), 1]
                                  chain[-(1:burnIn), 2]
                                                                     chain[-(1:burnln), 3]
                                                                         10
    8.
                                       ဖှ
    4.4
                                                                               1000
        0 1000
                   3000
                           5000
                                           0 1000
                                                     3000
                                                             5000
                                                                             0
                                                                                        3000
                                                                                               5000
           True value = red line
                                             True value = red line
                                                                               True value = red line
# for comparison:
summary(lm(y~x))
##
## Call:
## lm(formula = y \sim x)
##
## Residuals:
```

4.6336 15.0324

Min

1Q

-15.2214 -6.3852 -0.5285

Median