MAST30027: Modern Applied Statistics

Week 11 Lab Sheet

1. Suppose that $X \sim \text{pois}(\lambda)$ and put $\mu = \mathbb{E}X$. Show that the Jeffreys prior for μ is $\propto \mu^{-1/2}$. Show this prior is improper. How would you approximate this (improper) prior using a gamma distribution?

   Solution: The Jeffreys prior is proportional to the square root of the Fisher information $I(\mu)$. We have

$$\begin{aligned} \frac{d^2}{d\mu^2} \log\left(\frac{e^{-\mu}\mu^x}{x!}\right) &= \frac{-x}{\mu^2} \\ I(\mu) = -\mathbb{E}\frac{d^2}{d\mu^2} \log L(\mu) &= \frac{1}{\mu} \end{aligned}$$

   Thus the Jeffreys prior is $\propto 1/\sqrt{\mu}$ as required.

   This prior is improper because $\int_0^\infty 1/\sqrt{\mu} d\mu = [2\sqrt{\mu}]_0^\infty = \infty$.

   A gamma(α, β) density is proportional to $x^{\alpha-1}e^{-\beta x}$, so taking $\alpha = 1/2$ and $\beta = 0.001$ (small) we get something $\approx cx^{-1/2}$, provided x is not too large.