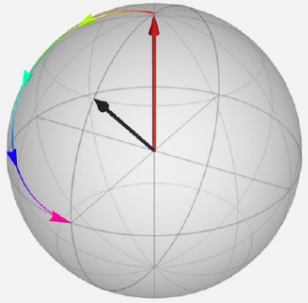
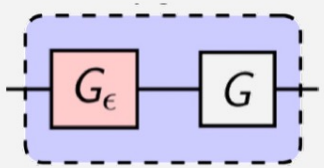
Bonus content

Bonus content

Coherent noise

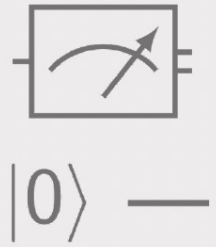




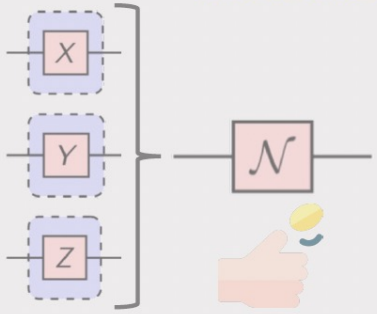
Measurement in a nutshell

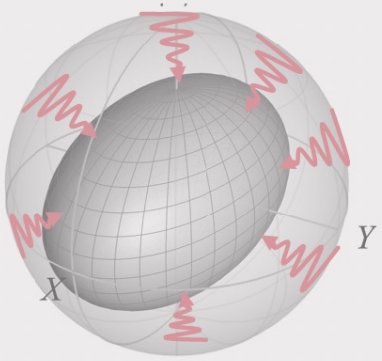
Projection noise

SPAM: Noisy meters



Incoherent noise

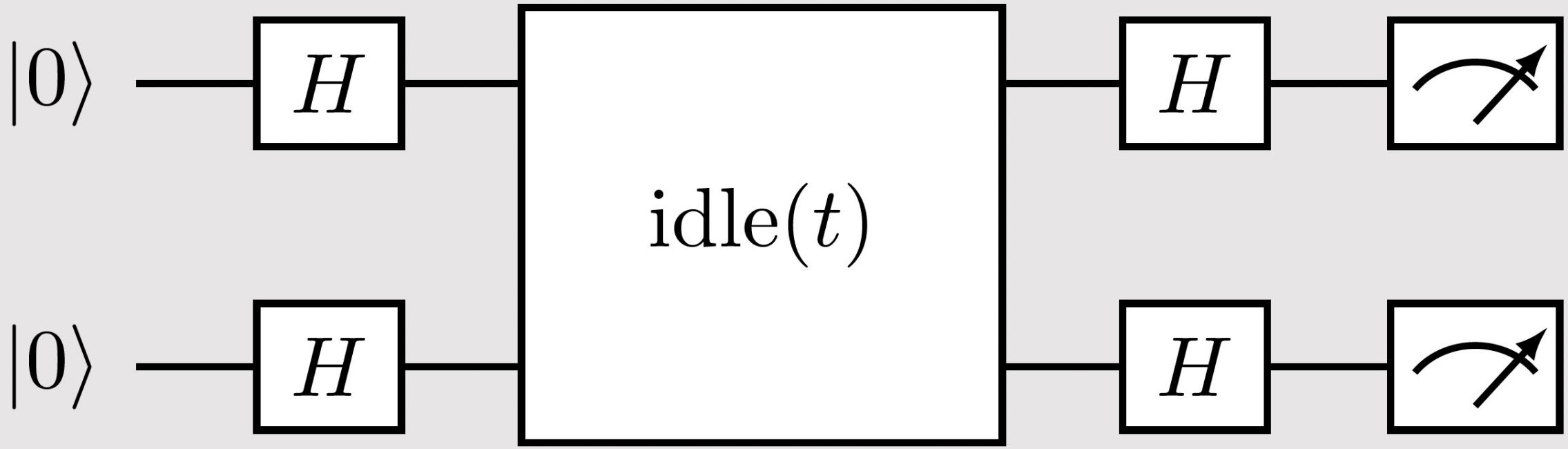




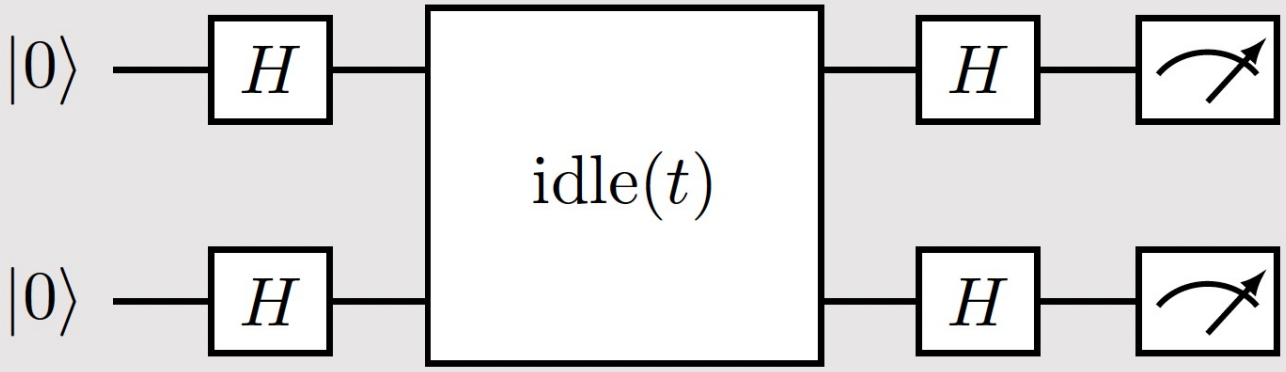
Bonus content

coin toss: flaticon; spam: make it move;
road based on: freepik

Bonus content: two-qubit coherent ZZ error



Bonus content: two-qubit ZZ error

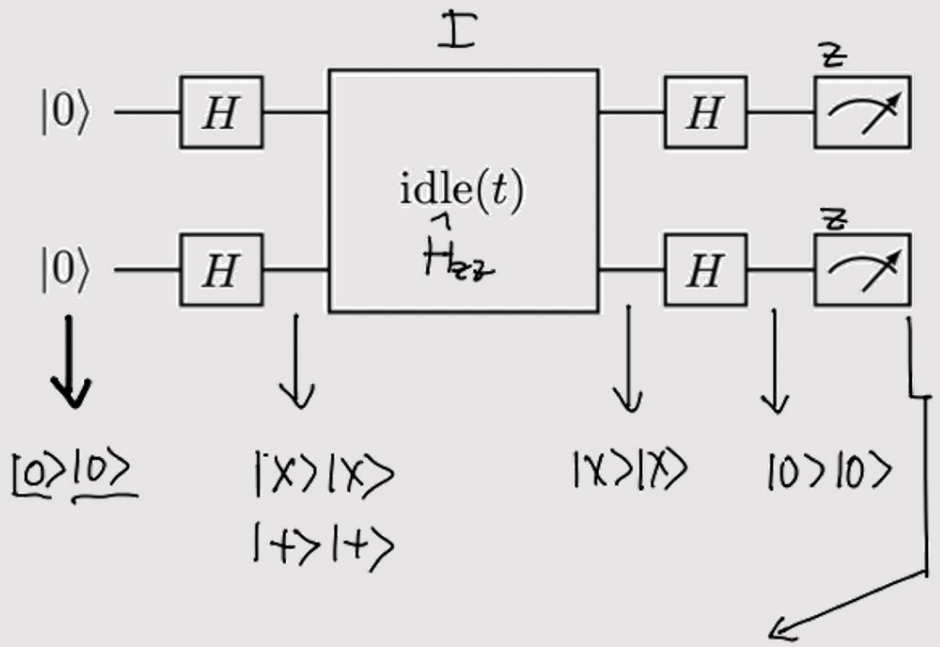


Introduction to quantum noise

Coherent errors

Qiskit Global Summer School on Quantum Machine Learning

Zlatko K. Minev



Hadamard gate

H

$$H = \begin{matrix} \\ |0\rangle \\ |1\rangle \end{matrix} \begin{matrix} \langle 0| & \langle 1| \\ \end{matrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{cases} H|0\rangle = |+x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ H|1\rangle = |-x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{cases}$$

$\langle ZI \rangle = +1$
$\langle IZ \rangle = +1$
$\langle ZZ \rangle = \langle 01| \langle 01| Z \, Z |01\rangle |01\rangle$
$= \langle 0|Z|0\rangle \langle 0|Z|0\rangle$
$= (+1)(+1)$
$= +1$

$X|+x\rangle = +1|+x\rangle$
$X|-x\rangle = -1|-x\rangle$
$|+x\rangle := |+\rangle$
$|-x\rangle := |-\rangle$

$\begin{cases} Z|0\rangle = +1|0\rangle \\ Z|1\rangle = -1|1\rangle \end{cases}$

Noisy ZZ Interaction

$$\hat{H} = \frac{1}{2}\hbar\omega\,\hat{Z}\hat{Z}$$

$$\hat{U}(t) = \exp\left(-i\hbar^{-1}\hat{H}t\right)$$

$$= \exp\left(-i\frac{\omega t}{2}\hat{Z}\hat{Z}\right)$$

$$= \cos\left(\frac{\omega t}{2}\right)\hat{I} - i\sin\left(\frac{\omega t}{2}\right)\hat{Z}\hat{Z}$$

$$= \hat{R}_{ZZ}(\theta = \omega t)$$

$$\hat{R}_X(\theta) = \exp\left(-i\frac{\theta}{2}\hat{X}\right) \qquad X^2 = I$$

$$= \cos\left(\frac{\theta}{2}\right)I - i\sin\left(\frac{\theta}{2}\right)X$$

$$(\hat{Z}\hat{Z})^2 = Z^2Z^2 = I_2 \otimes I_2 = I_4$$

$|0\rangle|0\rangle \xrightarrow{HH} |+\rangle|+\rangle \xrightarrow{\text{idle}} R_{zz}(\theta)|+\rangle|+\rangle = \cos\frac{\theta}{2}|+\rangle|+\rangle - i\sin\frac{\theta}{2}|-\rangle|-\rangle \xrightarrow{HH} \cos\theta/2\,|0\rangle|0\rangle - i\sin\theta/2\,|1\rangle|1\rangle$

$$R_{zz}(\theta)|+\rangle|+\rangle = \cos\frac{\theta}{2}|+\rangle|+\rangle - i\sin\frac{\theta}{2}(Z|+\rangle)_1 \otimes (Z|+\rangle)_2$$

$$Z|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix} = |-\rangle$$

$$Z|+\rangle = |-\rangle$$

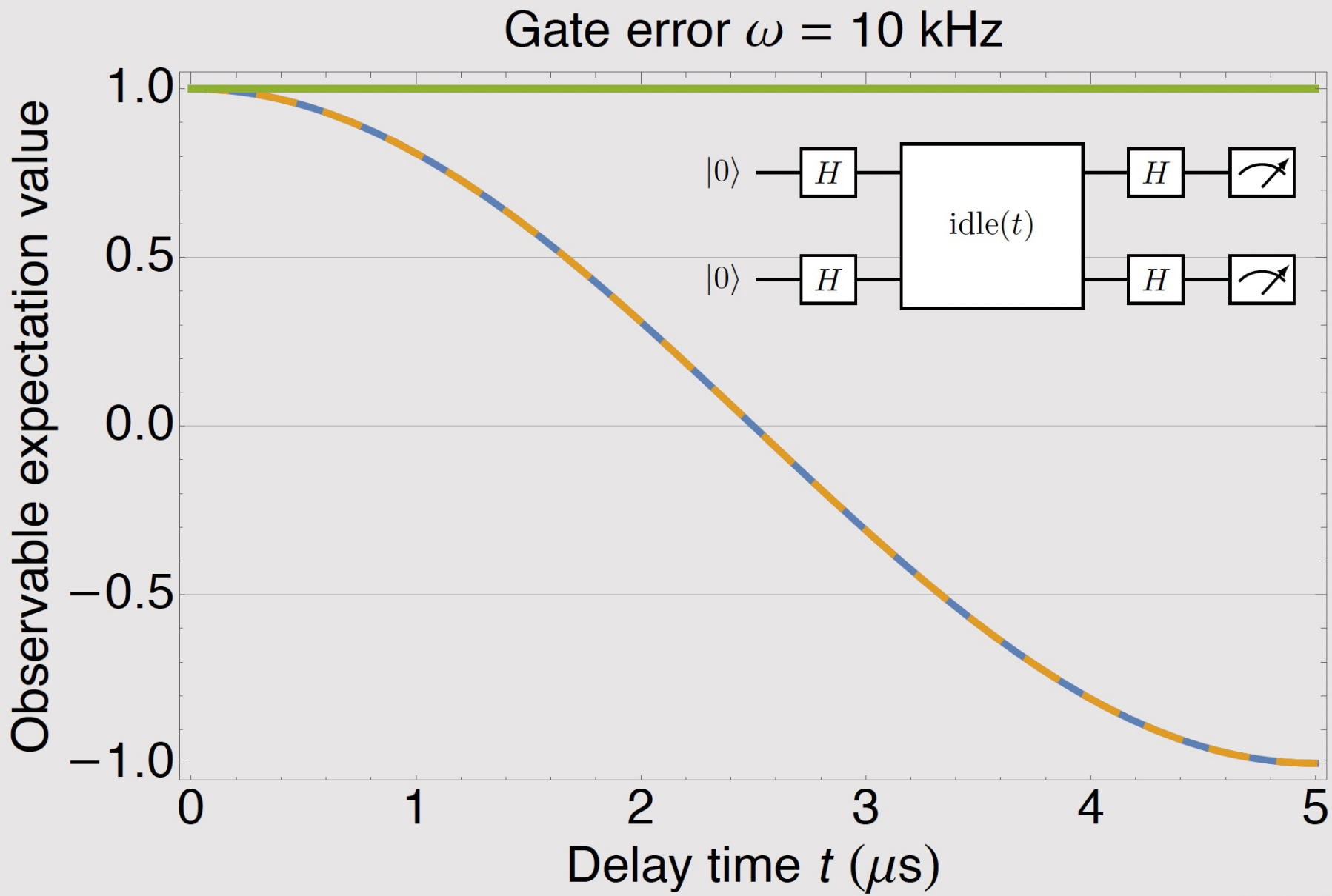
$$Z|-\rangle = |+\rangle$$

$$\langle ZI\rangle = \cos\omega t$$

$$\langle IZ\rangle = \cos\omega t$$

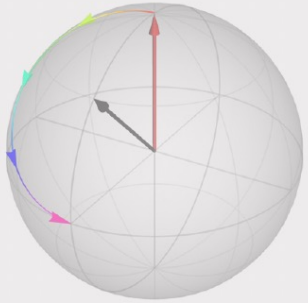
$$\langle ZZ\rangle = 1$$

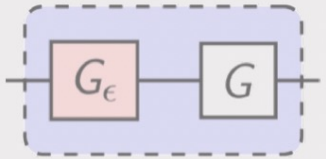
Two-qubit ZZ error plotted



Bonus content

Coherent noise





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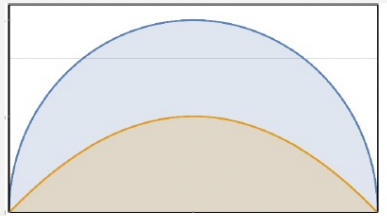
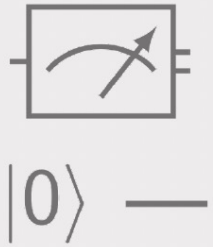
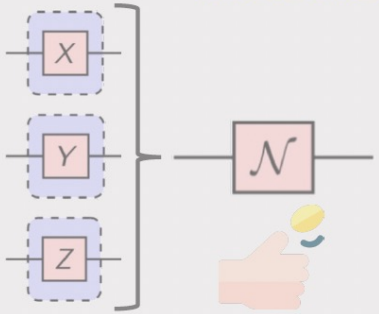
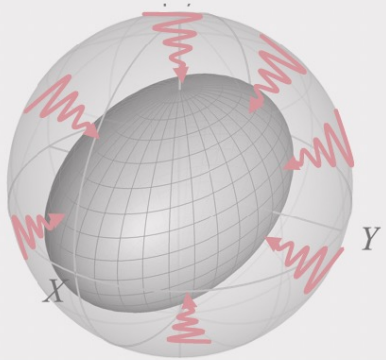


SPAM: Noisy meters



Incoherent noise

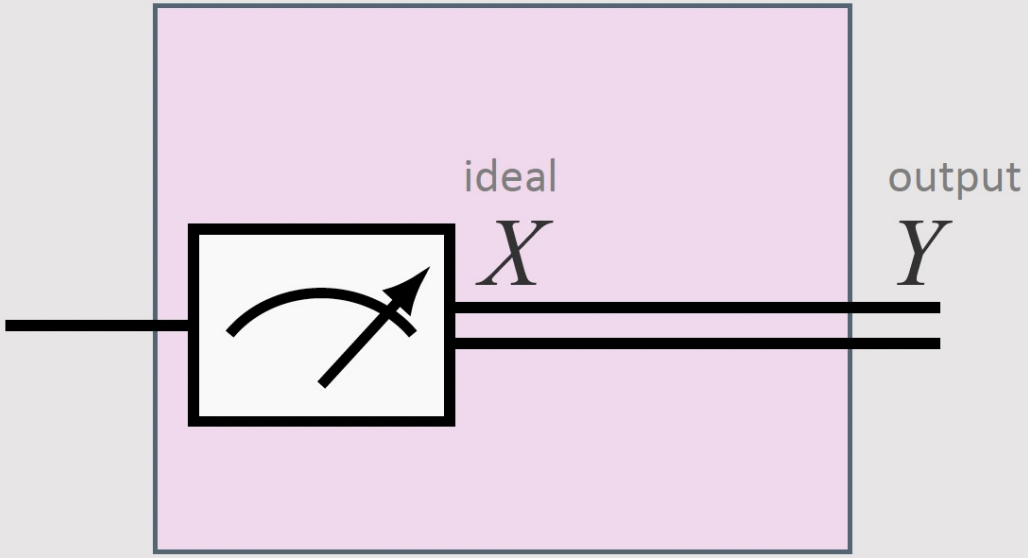




Bonus content

coin toss: flaticon; spam: make it move;
road based on: freepik

Recall the *A* matrix



$$\mathbf{A}_{\text{ideal}} = \begin{matrix} \\ Y=0 \\ Y=1 \end{matrix} \begin{matrix} \begin{matrix} X=0 & X=1 \end{matrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

$$\mathbf{p}_{\text{noisy}} = \mathbf{A}\mathbf{p}_{\text{ideal}}$$

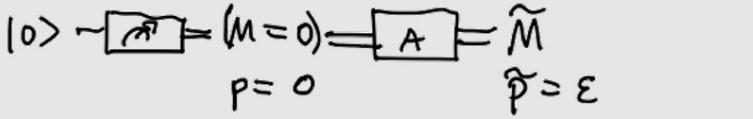
$$\mathbf{A} = \begin{matrix} \\ Y=0 \\ Y=1 \end{matrix} \begin{matrix} \begin{matrix} X=0 & X=1 \end{matrix} \\ \begin{pmatrix} 1-\epsilon & \nu \\ \epsilon & 1-\nu \end{pmatrix} \end{matrix}$$

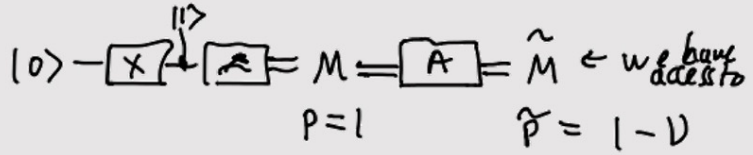
* Monster image by jcomp on Freepik

Deeper dive on the readout *A* matrix & Shannon entropy

Bonus section content:

Reconstruct A matrix





Noise mitigation

we know A

measure $\tilde{p}$, $\tilde{P}_M$ noisy

find p, P_M ideal

dim $A = 2^n \times 2^n$ n = #qubits

$$\tilde{P}_M = A P_M$$

$$P_M = A^{-1} \tilde{P}_M$$

$$p =$$

Assignment Fidelity

$$F_0 = 1 - \tfrac{1}{2}\left[p(\tilde{M}=1 \mid M=0) + p(\tilde{M}=0 \mid M=1)\right]$$

$$= \tfrac{1}{d} \mathrm{Tr}(A)$$

$$= 1 - \tfrac{1}{2}(\nu + \nu)$$

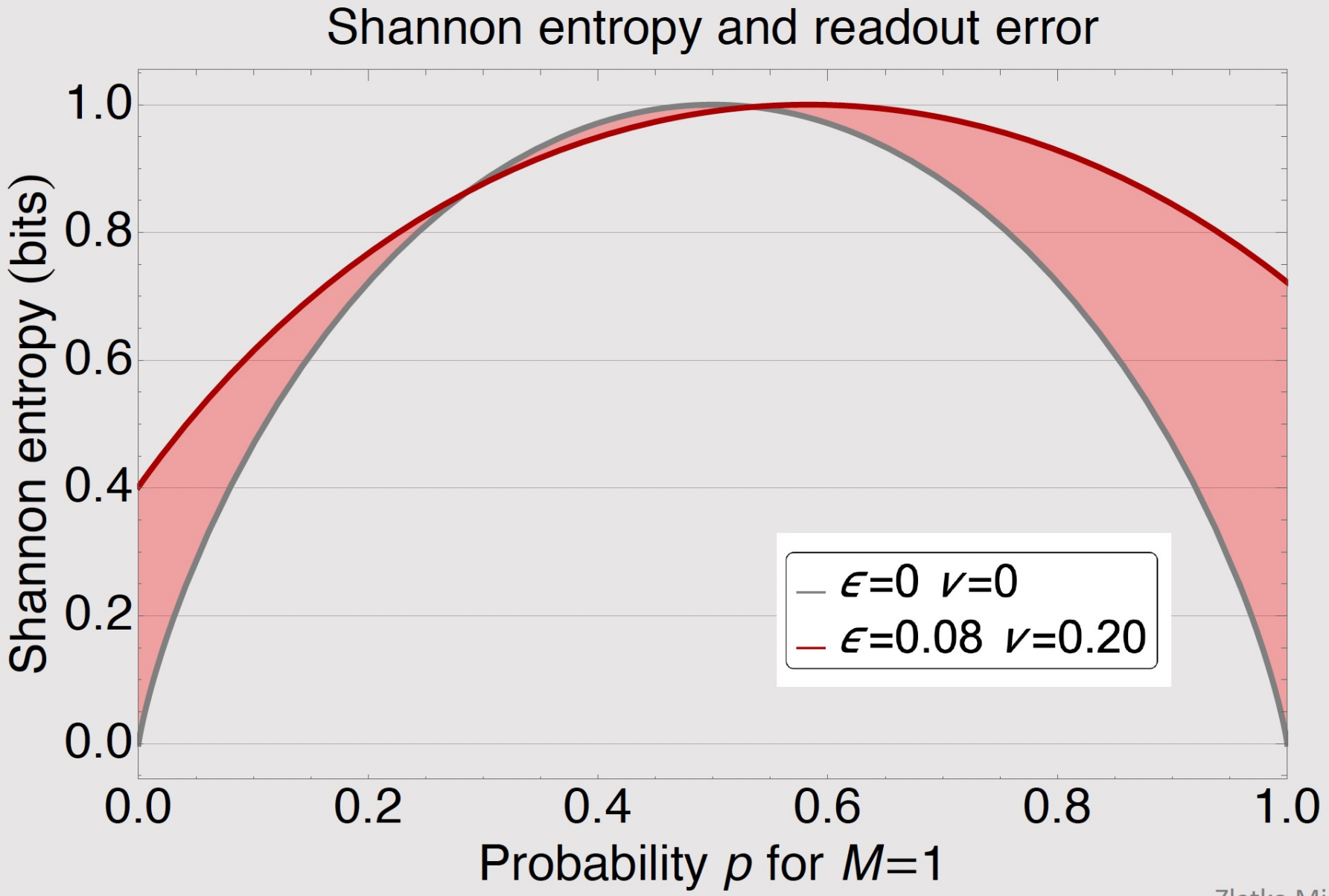
$d = 2^n$, n = #qubits

Shannon Entropy

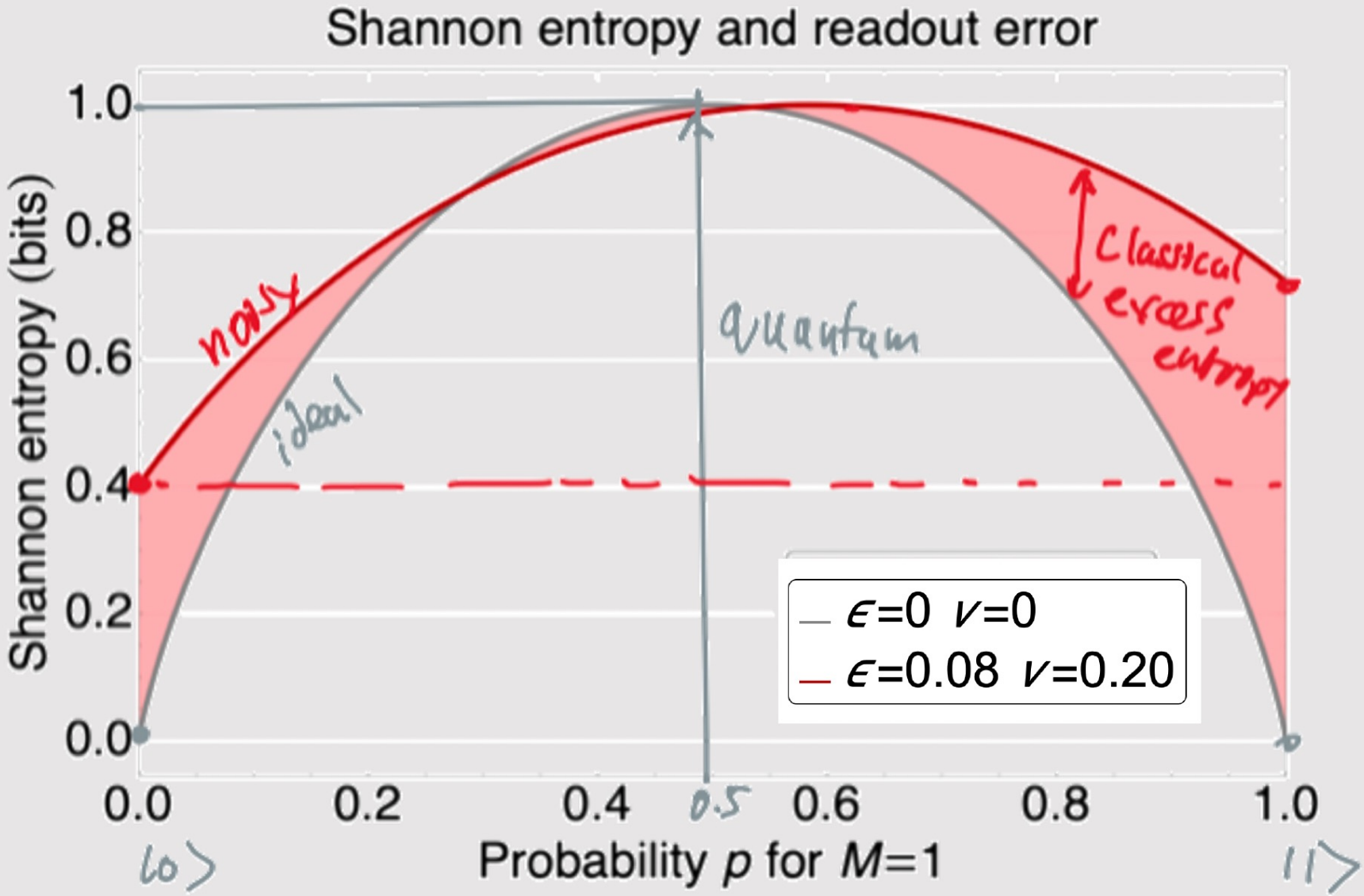
$$H(M) = H(P_M) = -\sum_m P_m \log_2 P_m = -(1-p)\log_2(1-p) - p \log_2 p$$

Binary entropy

Entropy


Shannon entropy and readout error
Shannon entropy (bits)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Probability p for M=1
ϵ=0 ν=0
ϵ=0.08 ν=0.20

Entropy


Shannon entropy and readout error
Shannon entropy (bits)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.5
0.6
0.8
1.0
Probability p for M=1
|0>
|1>
noisy
ideal
quantum
Classical excess entropy
ϵ=0 ν=0
ϵ=0.08 ν=0.20

The End!