### Feature-Guided Black-Box Safety Testing of Deep Neural Networks

Matthew Wicker, Xiaowei Huang, and Marta Kwiatkowska TACAS 2018

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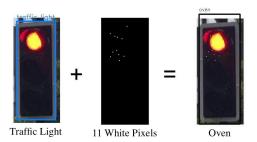
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#### Introduction

Deep neural networks (DNNs or networks, for simplicity) have been developed for a variety of tasks, including malware detection, abnormal network activity detection, and self-driving cars.

An **adversarial example** is an input which, though initially classified correctly, is misclassified after a minor, perhaps imperceptible, perturbation.



#### Introduction

They propose a *feature-guided* approach to test the resilience of image classifier networks against adversarial examples.

A key observation of this paper is that feature extraction methods enable us to identify elements of an image which are most vulnerable to a visual system such as a CNN.

They use the scale-invariant feature transform approach, or SIFT, to detect features.

Using the SIFT features, whose number is much smaller than the number of pixels, they represent the image as a two-dimensional Gaussian mixture model.

#### Introduction

We formulate the process of crafting adversarial examples as a two-player turn-based stochastic game, where *player I* selects features and *player II* then selects pixels within the selected features and a manipulation instruction.

After both players have made their choices, the image is modified according to the manipulation instruction, and the game continues.

While *player I* aims to minimise the distance to an adversarial example, *player II* can be cooperative, adversarial, or nature who samples the pixels according to the Gaussian mixture model.

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Let N be a network with a set C of classes.

Given an input  $\alpha$  and a class  $c \in C$ , we use  $N(\alpha, c)$  to denote the confidence of N believing that is in class c.

They write  $N(\alpha) = \arg \max_{c \in C} N(\alpha, c)$  for the class into which N classifies c.

The metrics used to compute the distance between images typically include  $L_0$ ,  $L_1$  (Manhattan distance),  $L_2$  (Euclidean distance), and  $L_{\infty}$ .

The input domain D is a vector space, which can be represented as  $\Re^{wxhxch}_{[0,255]}$  where w; h; ch are the width, height, and number of channels of an image, respectively

They write  $||\alpha_1 - \alpha_2||_k$  with  $k \ge 0$  for the distance between two images  $\alpha_1$  and  $\alpha_2$  with respect to the  $L_k$  measurement.

Given an image  $\alpha$ , a distance measure  $L_k$ , and a distance d, we define  $\eta(\alpha,k,d)=\{\alpha'|||\alpha'-\alpha||_k\leq d\}$  as the set of points whose distance to  $\alpha$  is no greater than d with respect to  $L_k$ .

**Definition 1.** (Constraints) Given an input  $x \in D$ , a distance measure  $L_k$  for some  $k \geq 0$ , and a distance d, an adversarial example  $\alpha'$  of class  $c \neq N(x)$  is such that  $\alpha' \in \eta(\alpha,k,d)$ ,  $N(\alpha) \neq N(\alpha')$ , and  $N(\alpha') = c$ . Moreover, we write  $adv_{N,k,d}(\alpha,c)$  for the set of adversarial examples of class c and let  $adv_{N,k,d}(\alpha) = \bigcup_{c \in C, c \neq N(\alpha)} adv_{N,k,d}(\alpha,c)$ . A targeted safety of class c is defined as  $adv_{N,k,d}(\alpha,c) = \emptyset$ , and a non-targeted safety is defined as  $adv_{N,k,d}(\alpha) = \emptyset$ .

#### Feature Extraction

The SIFT algorithm proceeds through the following steps:

- scale-space extrema detection (detecting relatively darker or lighter areas in the image)
- keypoint localization (determining the exact position of these areas)
- keypoint descriptor assignment (understanding the context of the image w.r.t its local area).

Let  $\Lambda(\alpha)$  be a set of features of the image  $\alpha$  such that each feature  $\lambda \in \Lambda(\alpha)$  is a tuple  $(\lambda_x, \lambda_y, \lambda_s, \lambda_r)$ , where  $(\lambda_x, \lambda_y)$  is the coordinate of the feature in the image,  $\lambda_s$  is the size of the feature, and  $\lambda_r$  is the response strength of the feature.

#### Feature Extraction

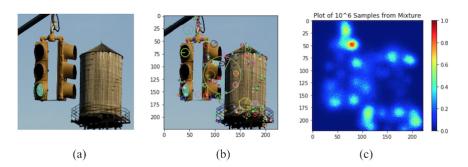


Figure: Illustration of the transformation of an image into a saliency distribution. (a) The original image, provided by ImageNet. (b) The image marked with relevant keypoints  $\Lambda(\alpha)$ . (c) The heatmap of the Gaussian mixture model  $G(\Lambda(\alpha))$ .

#### Distance w.r.t. Human Perception

Given an image  $\alpha$  and its set  $\Lambda(\alpha)$  of keypoints, we define for  $\Lambda_i$   $in\Lambda(\alpha)$  a two-dimensional Gaussian distribution  $G_i$  such that, for pixel  $(p_x; p_y)$ , we have

$$G_{i,x} = \frac{1}{\sqrt{2\pi\lambda_{i,s}^2}} exp(\frac{-(p_x - \lambda_{i,x})^2}{2\lambda_{i,s}^2}) \quad G_{i,y} = \frac{1}{\sqrt{2\pi\lambda_{i,s}^2}} exp(\frac{-(p_y - \lambda_{i,y})^2}{2\lambda_{i,s}^2}) \quad (1)$$

They define a set of weights  $\Phi = \{\phi_i\}_{i \in \{1,2,...,k\}}$  such that  $k = |\Lambda(\alpha)|$  and  $\phi_i = \lambda_{i,r} / \sum_{j=0}^k \lambda_{j,r}$ .

Then, they can construct a Gaussian mixture model  $G_x = \prod_{i=1}^k \phi_i * G_{i,x}$  and  $G_y = \prod_{i=1}^k \phi_i * G_{i,y}$ 

#### Pixel Manipulation

They write (x; y; z) for the value of the z – channel of the pixel positioned at (x; y) on the image.

They define pixel manipulations  $\delta_{X;i}:D\to D$  for  $X\subseteq P_0$  a subset of input dimensions

$$\delta_{X,i}(\alpha)(x,y,z) = \begin{cases} \alpha(x,y,z) + \tau, \text{ if } (x,y) \in X \text{ and } i = +\\ \alpha(x,y,z) - \tau, \text{ if } (x,y) \in X \text{ and } i = -\\ \alpha(x,y,z) & \text{otherwise} \end{cases}$$

for all pixels (x; y) and channels  $z \in \{1, 2, 3\}$ .

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Every image represents a point in the input vector space D.

Most existing investigations of the safety (or robustness) of DNNs focus on optimising the movement of a point along the gradient direction of some function obtained from the network.

Therefore, these approaches rely on the knowledge about the DNN.

Furthermore, the current understanding about the transferability of adversarial examples are all based on empirical experiments

A reasonable interpretation is that transferable adversarial examples, if they exist, do not rely on the gradient direction suggested by a network but instead may be specific to the input.

They propose a *feature-guided* approach which, instead of using the gradient direction as the guide for optimisation, it relies on targeting and manipulating image features as recognised by human perception capability.

Their method needs neither the knowledge about the network nor the necessity to massively sample the network for data to train a new network, and is therefore a *black-box* approach

## Safety Against Manipulations based on Human Perception Objective to Optimise

The sets  $adv_{N;k;d}(\alpha;c)$  and  $adv_{N;k;d}(\alpha)$  can be infinite. Therefore, we may be interested in finding a finite subset:

**Definition 2.** (Objective) Among all the adversarial examples in the set  $adv_{N,k,d}(\alpha,c)$  (or  $adv_{N,k,d}(\alpha)$ ), the one with minimum distance to the original image  $\alpha$  is defined as follows:

$$\arg\min_{\alpha'} \{ sev_{\alpha}(\alpha') \mid \alpha' \in adv_{N,k,d}(\alpha,c) (or \, adv_{N,k,d}(\alpha)) \}$$
 (2)

where  $sev_{\alpha}(\alpha') = ||\alpha - \alpha'||_k + e*||\alpha - \alpha'||_H$  is the severity of the adversarial example  $\alpha'$  against the original image  $\alpha$  and e is an adjustable constant.

Crafting Adversarial Examples as a Two-Player Turn-Based Game

Assume two players I and II.

Let  $M(\alpha, p, d) = (S \cup (Sx\Lambda(\alpha)), s_0, \{T_\alpha\}_{\alpha \in \{I, H\}}, L)$  be a **game model**, where S is a set of game states belonging to player I such that each state represents an image in  $\eta(\alpha, k, d)$ , and  $Sx\Lambda(\alpha)$  is a set of game states belonging to player II.

They write  $\alpha(s)$  for the image associated to the state  $s \in S$ .  $s_0 \in S$  is the initial image state such that  $\alpha(s_0)$  is the original image  $\alpha$ .

The transition relation  $T_I: Sx\Lambda(\alpha) \to Sx\Lambda(\alpha)$  is defined as,  $T_I(s,\lambda) = (s,\lambda)$ 

Transition relation  $T_{II}: (Sx\Lambda(\alpha))xP(P_0)xI \to S$  is defined as,  $T_{II}((s,\lambda),X,i) = \delta_{X,i}(\alpha(s))$ 

Crafting Adversarial Examples as a Two-Player Turn-Based Game

Intuitively, on every game state  $s \in S$  player I will choose a keypoint  $\lambda$ , and, in response to this, player II will choose a pair (X, i), where X is a set of input dimensions and i is a manipulation instruction.

The **labelling function**  $L: S \cup (Sx\Lambda(\alpha)) \to CxG$  assigns to each state s or  $(s; \lambda)$  a class  $N(\alpha(s))$  and a two-dimensional Gaussian mixture model  $G(\Lambda(\alpha(s)))$ .

A **path** (or game play) of the game model is a sequence  $s_1u_1s_2u_2...$  of game states such that, for all  $k \ge 1$ , we have  $u_k = T_I(s_k, \lambda_k)$  for some feature  $\lambda_k$  and  $s_{k=1} = T_{II}((s_k, \lambda_k), X_k, i_k)$  for some  $(X_k, i_k)$ .

Crafting Adversarial Examples as a Two-Player Turn-Based Game

Let  $last(\rho)$  be the last state of a finite path  $\rho$  and  $Path_a^F$  be the set of finite paths such that  $last(\rho)$  belongs to player  $a \in \{I, II\}$ .

A **stochastic strategy**  $\sigma_I: Path_I^F \to D(\Lambda(\alpha))$  of player I maps each finite paths to a distribution over the next actions, and similarly for  $\sigma_{II}: Path_{II}^F \to D(P(P_0)xI)$  for player II.

They call  $\sigma = (\sigma_I, \sigma_{II})$  a strategy profile.

Crafting Adversarial Examples as a Two-Player Turn-Based Game

They define a **reward**  $R(\sigma, \rho)$  for a given strategy profile  $\sigma = (\sigma_I, \sigma_{II})$  and a finite path  $\rho \in \bigcup_{a \in \{I,II\}} Path_a^F$ .

Let  $\alpha'_{\rho} = \alpha(\textit{last}(\rho))$  be the image associated with the last state of the path  $\rho$ .

They write  $t(\rho)$  for the expression  $N(\alpha'_{\rho}) = cV||\alpha'_{\rho} - \alpha||_k > d$  representing that the path has reached a state whose associated image either is in the target class c or lies outside the region  $\eta(\alpha, k, d)$ .

The path  $\rho$  can be terminated whenever  $t(\rho)$  is satisfiable.

Crafting Adversarial Examples as a Two-Player Turn-Based Game

Then they define the reward function  $R(\sigma, \rho)$ :

$$\begin{cases} 1/sev_{\alpha}(\alpha'_{\rho}) & \text{if } t(\rho) \text{ and } \rho \in Path_{\mathtt{I}}^F \\ \sum_{\lambda \in A(\alpha)} \sigma_{\mathtt{I}}(\rho)(\lambda) \cdot R(\sigma, \rho T_{\mathtt{I}}(last(\rho), \lambda)) & \text{if } \neg t(\rho) \text{ and } \rho \in Path_{\mathtt{I}}^F \\ \sum_{(X,i) \in \mathcal{P}(P_0) \times I} \sigma_{\mathtt{II}}(\rho)(X,i) \cdot R(\sigma, \rho T_{\mathtt{II}}(last(\rho), X, i)) & \text{if } \rho \in Path_{\mathtt{II}}^F \end{cases}$$

where  $\sigma_I(\rho)(\lambda)$  is the probability of selecting  $\lambda$  on  $\rho$  by player I and  $\sigma_{II}(\rho)(X,i)$  is the probability of selecting (X,i) based on  $\rho$  by player II.

Note that a path only terminates on player I states.

Crafting Adversarial Examples as a Two-Player Turn-Based Game

**Definition 3.** The goal of the game is for player I to choose a strategy  $\sigma_I$  to maximise the reward  $R((\sigma_I, \sigma_{II}), s_0)$  of the initial state  $s_0$ , based on the strategy  $\sigma_{II}$  of the player II, i.e.,

$$\arg\max_{\sigma_{\text{I}}} \operatorname{opt}_{\sigma_{\text{II}}} R((\sigma_{\text{I}}, \sigma_{\text{II}}), s_0). \tag{3}$$

where option  $\operatorname{opt}_{\sigma_{11}}$  can be  $\max_{\sigma_{11}}$ ,  $\min_{\sigma_{11}}$ , or  $\operatorname{nat}_{\sigma_{11}}$ , according to which player II acts as a cooperator, an adversary, or nature who samples the distribution  $\mathcal{G}(\Lambda(\alpha))$  for pixels and randomly chooses the manipulation instruction.

A strategy  $\sigma$  is deterministic if  $\sigma(\rho)$  is a Dirac distribution, and is memoryless if  $\sigma(\rho) = \sigma(last(\rho))$  for all finite paths  $\rho$ . We have the following result:

**Theorem 1.** Deterministic and memoryless strategies suffice for player I, when  $opt_{\sigma_{II}} \in \{ \max_{\sigma_{II}}, \min_{\sigma_{II}}, nat_{\sigma_{II}} \}$ .

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They present an approach based on Monte Carlo tree search (MCTS) to find an optimal strategy asymptotically.

Consider the case of  $opt_{\sigma_{II}} = max_{\sigma_{II}}$ .

An MCTS algorithm, gradually expands a partial game tree by sampling the strategy space of the model  $M(\alpha, p, d)$ .

MCTS has a theoretical guarantee that it converges to optimal solution when the game tree is fully explored. (Exploration-Exploitation tradeoff)

They use two termination conditions  $tc_1$  and  $tc_2$  to control the pace of the algorithm. The terminating conditions can be, e.g., bounds on the or the number of iterations, etc.

 $tc_1$  controls whether the entire procedure should be terminated, and  $tc_2$  controls when a move should be made.

On the partial tree, every node maintains a pair (r, n), which represents the accumulated reward and the number of visits, respectively.

#### Algorithm 1 Monte-Carlo Tree Search for $opt_{\sigma_{II}} = max_{\sigma_{II}}$

```
1: Input: A game model M(\alpha, p, d), two termination conditions tc_1 and tc_2, a target class c
2: Output: An adversarial example \alpha'
3: procedure MCTS(M(\alpha, p, d), tc_1, tc_2, c)
4:
        root \leftarrow s_0
5:
        While (\neg tc_1):
           While (\neg tc_2):
6:
7:
              leaf \leftarrow selection(root)
8:
              newnodes \leftarrow expansion(M(\alpha, p, d), lea f)
9:
              for node in newnodes:
                 v \leftarrow Simulation(M(\alpha, p, d), node, c)
10:
11:
                 backPropogation(node, v)
12:
           root \leftarrow bestChild(root)
13:
        return root.
```

The **selection** travels from the root to a leaf according to an exploration-exploitation balance.

After **expanding** the children of the leaf node, we call **Simulation** to run simulation on every child node.

Every simulation terminates when reaching a terminated node  $\alpha'$ , on which a reward  $1/sev(\alpha')$  can be computed.

This reward is then **backpropagated** from the new child node through its ancestors until reaching the root.

Every time a new reward v is backpropogated through a node, we update its associated pair to (r + v, n + 1). The bestChild(root) returns the child of root who has the highest value of the expression r/n.

Severity Interval from the Game

There exists a severity interval  $SI(\alpha, p, d)$  with respect to the role of player II:

$$[sev(M(\alpha, p, d), max_{\sigma_{II}}), sev(M(\alpha, p, d), min_{\sigma_{II}})]$$

We have that  $sev(M(\alpha, p, d), nat_{\sigma_{II}})$ 

Safety Guarantee via Optimal Strategy

An image  $\alpha' \in \eta(\alpha, k, d)$  is a  $\tau$ -grid image if for all dimensions  $p \in P_0$  we have  $|\alpha'(p) - \alpha(p)| = n * \tau$  for some  $n \ge 0$ .

Let  $\tau(\alpha, k, d)$  be the set of  $\tau$ -grid images in  $\eta(\alpha, k, d)$ . We have the following conclusion for the case when player II is cooperative:

**Theorem 2.** Let  $\alpha' \in \eta(\alpha, k, d)$  be any  $\tau$ -grid image such that  $\alpha' \in adv_{N,k,d}(\alpha, c)$ . Then we have that  $sev_{\alpha}(\alpha') \geq sev(M(\alpha, p, d), max_{\sigma_{\Pi}})$ .

The idea of the proof is to show that every  $\tau$ -grid image can be reached by some game play.

Safety Guarantee via Optimal Strategy

**Theorem 3.** If all  $\tau$ -grid images are misclassification aggregators with respect to  $\tau/2$ , and  $sev(M(\alpha, p, d), max_{\sigma_{11}}) > d$ , then  $adv_{N,k,d}(\alpha, c) = \emptyset$ .

Proof. (Sketch) First, we can show that  $\eta(\alpha,k,d)\subseteq\bigcup_{\alpha_1\in\tau(\alpha,k,d)}\eta(\alpha_1,1,\tau/2)$ . Now assume that  $adv_{N,k,d}(\alpha,c)\neq\emptyset$ . Then there must exist an image  $\alpha'$  such that  $\alpha'\in adv_{N,k,d}(\alpha,c)$ . Because all  $\tau$ -grid images are misclassification aggregators with respect to  $\tau/2$ , there must exist a  $\tau$ -grid image  $\alpha''$  such that  $\alpha''\in adv_{N,k,d}(\alpha,c)$ . By Theorem 2 we have  $sev_{\alpha}(\alpha'')\geq sev(M(\alpha,p,d,\tau),max_{\sigma_\Pi})$ . By the hypothesis that  $sev(M(\alpha,p,d),max_{\sigma_\Pi})>d$ , we have  $sev_{\alpha}(\alpha'')>d$ , which is impossible because  $\alpha''\in adv_{N,k,d}(\alpha,c)\subset\eta(\alpha,k,d)$ .

Safety Guarantee via Optimal Strategy

**Definition 4.** Network N is a Lipschitz network with respect to the distance measure  $L_k$  and a constant  $\hbar > 0$  if, for all  $\alpha, \alpha' \in D$ , we have  $|N(\alpha', N(\alpha)) - N(\alpha, N(\alpha))| < \hbar \cdot ||\alpha' - \alpha||_k$ .

Note that all networks whose inputs are bounded, including all image classification networks we studied, are Lipschitz networks.

$$I = \min\{|\textit{N}(\alpha',\textit{N}(\alpha)) - \textit{N}(\alpha,\textit{N}(\alpha))||\alpha,\textit{a}\alpha' \in \textit{D},\textit{N}(\alpha' \neq \textit{N}(\alpha))\}$$

Safety Guarantee via Optimal Strategy

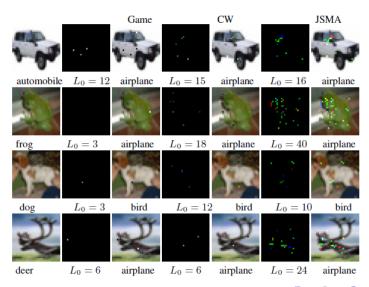
**Theorem 4.** Let N be a Lipschitz network with respect to  $L_1$  and a constant  $\hbar$ . Then when  $\tau \leq \frac{2\ell}{\hbar}$  and  $sev(M(\alpha,p,d),max_{\sigma_{\text{II}}}) > d$ , we have that  $adv_{N,k,d}(\alpha,c) = \emptyset$ .

*Proof.* We need to show that  $\tau \leq \frac{2\ell}{\hbar}$  implies that all  $\tau$ -grid images are misclassification aggregators with respect to  $\tau/2$ . First of all, by the definition of Lipschitz network, we have  $|N(\alpha_2,N(\alpha_2))-N(\alpha_1,N(\alpha_2))|<\hbar\cdot||\alpha_2-\alpha_1||_1$ . Then by the definition of  $\ell$ , we have  $||\alpha_2-\alpha_1||_1>\ell/h$  when  $N(\alpha_2)\neq N(\alpha_1)$ . Second, we notice that, the statement that all  $\tau$ -grid images are misclassification aggregators with respect to  $\tau/2$  is equivalent to say that for any  $\tau$ -grid image  $\alpha_1$  such that  $N(\alpha_1)=N(\alpha)$ , we have that for any  $\alpha_2$ ,  $N(\alpha_2)\neq N(\alpha_1)$  implies that  $||\alpha_2-\alpha_1||_1>\tau/2$ . Finally, we notice that  $||\alpha_2-\alpha_1||_1>\tau/2$  holds when  $||\alpha_2-\alpha_1||_1>\ell/h$  and  $\tau\leq \frac{2\ell}{\hbar}$ .

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#### Comparison with Existing Approaches



Comparison with Existing Approaches

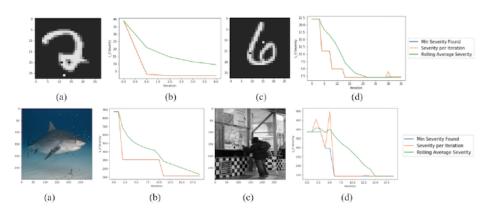
| $L_0$   | CW ( $L_0$ algorithm) | Game (timeout = 1m) | JSMA-F | JSMA-Z |
|---------|-----------------------|---------------------|--------|--------|
| MNIST   | 8.5                   | 14.1                | 17     | 20     |
| CIFAR10 | 5.8                   | 9                   | 25     | 20     |

Table 1. CW vs. Game vs. JSMA

Experiments are conducted by setting the termination conditions  $tc_1 = 20s$  and  $tc_2 = 60s$  for every image.

Note that JSMA needs several minutes to handle an image, and CW is 10 times slower than JSMA.

#### Convergence in Limited Runs

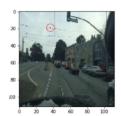


#### **Evaluating Safety-Critical Networks**

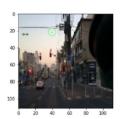
They apply the method to a network used for classifying traffic light images collected from dashboard cameras. The Nexar traffic light challenge made over eighteen thousand dashboard camera images publicly available.

Each image is labeled either green, or red or null if there is no traffic light appearing in the image

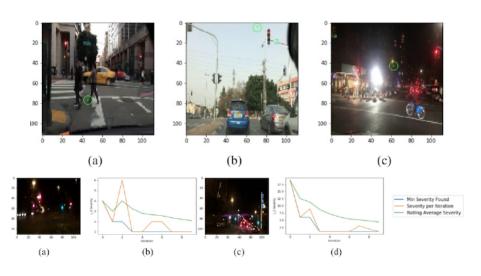
They test the winner of the challenge which scored an accuracy above 90%.







#### **Evaluating Safety-Critical Networks**



Counter-claim to Statements in [25]

A recent paper [25] argued that, under specific circumstances, there is no need to worry about adversarial examples because they are not invariant to changes in scale or angle in the physical domain.

This approach, which is inherently scale and rotationally invariant, can easily counter-claim such statements.

In [25] it is suggested that realistic camera movements – those which change the angle and distance of the viewer – reduce the phenomenon of adversarial examples to a curiosity rather than a safety concern.

This work shows that their adversarial examples, which are predicated on scale and rotationally invariant methods, defeat these claims.

Counter-claim to Statements in [25]



