

Efficient frontier with N stocks

Administrative Details

Topic 1: Foundations

Risksharing

▷ Risk and diversification

CAPM

“Time diversification”

Risk management

Classification

Financial markets and

instruments

Failures

Topic 2: Hedging in equity and fixed income markets

Topic 3: Endogenous risk and limits to arbitrage

Topic 4: Value at Risk

Topic 5: Credit risk

Topic 6: Credit derivatives and asset-backed securities

Topic 7: Regulation and the credit crisis

- Formally, the problem of finding the efficient frontier of n -stock portfolios is as follows:

$$\sigma_p = \min_{w_1, w_2, \dots, w_N} \sqrt{\sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_i \sigma_j \rho_{ij}}$$

subject to:

$$w_1 + \dots + w_N = 1,$$

$$w_1 \mu_1 + \dots + w_N \mu_N = \mu_p$$

- Investor achieves minimal risk for given expected return μ_p . Hence, the optimal portfolio is on the efficient frontier.
- This optimization problem can be solved explicitly. However, this is beyond our scope. You can use Excel solver to solve for optimal weights w_p and σ_p given μ_p .

Properties of efficient frontiers

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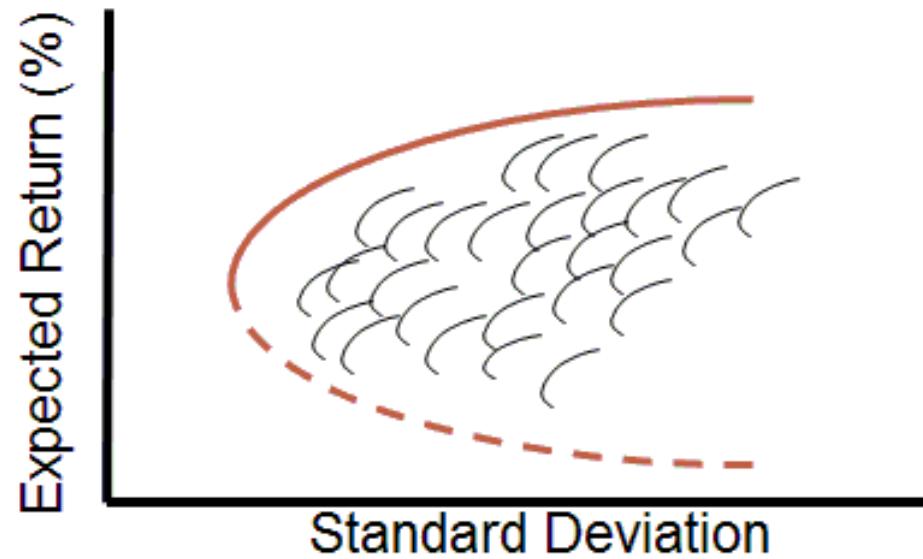
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- Among all possible portfolios with a given expected return the portfolios on the efficient frontier have the lowest variance.
- If portfolios A and B lie on the frontier, then all portfolios that invest $w\%$ in A and $(1 - w)\%$ in B also lie on the frontier.
- Portfolios on the N -stock efficient frontier dominate the portfolios on 2-, 3-, ..., $N-1$ stock frontiers:



Lending and borrowing allowed

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Lending or borrowing at the risk free rate (r_0) allows us to achieve the returns beyond the efficient frontier, i.e. higher returns for a given risk, or lower risk for a given return. E.g. you can lend by investing in Treasury bills, or borrow money which is equivalent to shorting Treasury bills.

