Optimization
Homework 1

Hector G. Flores Rodriguez (Id. 25721714)

October 11, 2017

1. Problem 1

1-(b) The Fibonnacci method was used to optimized the functions:

$$f_1(x) = x^2 + 2x$$
$$f_2(x) = x^3 - 3x^2 + 36$$

Prior to optmizing the provided functions, an initial interval for candidate starting points was determined by visually inspecting the graphs of the functions (shown in Figure 1).

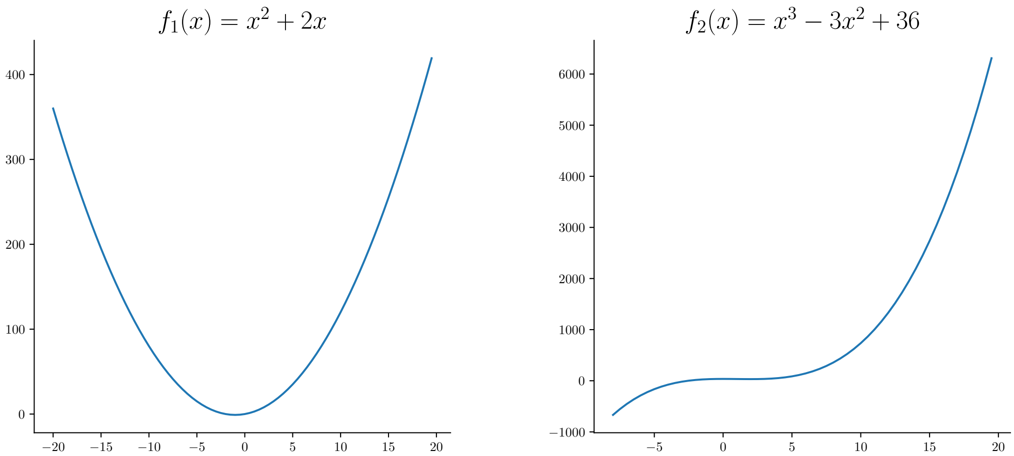

Figure 1: Quadratic and non-quadratic functions for optmization.

The estimated value for x_1^* lies near -1, whereas the estimated value for x_2^* lies near 8. Hence, we can sample from a normal distribution for both functions:

$$X_1 \sim \mathcal{N}(\mu_1 = -1, \, \sigma_1 = 1)$$
$$X_2 \sim \mathcal{N}(\mu_2 = 8, \, \sigma_2 = 1)$$

Furthermore, we will derive the number of iterations (stopping conditions) needed for the Fibonacci method to provide an accuracy of within 5%. We will define I_0 to be the initial interval, I_n to be the final interval, and F_n to be the n^{th} Fibonnaci term.

Hence,

$$\frac{I_n}{2I_0} \leq \frac{1}{100}$$
$$\frac{I_n}{I_0} \leq \frac{1}{50}$$
$$\frac{1}{F_n} \leq \frac{1}{50}$$

Therefore,

$$F_n \geq 50$$

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The value $n = 10$ was computed using Binet's formula for the Fibonacci terms. Fifteen random samples were then generated for both functions using the stopping criterion of $n = 10$ iterations. The table of results for both functions are shown in both Table 1 and Table 2.

Table 1: Results for $f_1(x) = x^2 + 2x$

	x^*	$f_1(x^*)$	function_calls	clock_time
	-1.003858	-0.999985	13.0	0.000258
	-0.999863	-1.000000	12.0	0.000121
	-0.458484	-0.706761	12.0	0.000142
	-0.133349	-0.248917	12.0	0.000113
	-0.995906	-0.999983	12.0	0.000107
	-1.794326	-0.369046	12.0	0.000126
	-0.998712	-0.999998	12.0	0.000121
	-0.675915	-0.894969	12.0	0.000112
	-0.849852	-0.977456	12.0	0.000097
	-0.998391	-0.999997	12.0	0.000108
	-1.006449	-0.999958	12.0	0.000099
	-1.009986	-0.999900	12.0	0.000097
	-0.998913	-0.999999	12.0	0.000097
	-1.293503	-0.913856	12.0	0.000130
	-0.989732	-0.999895	12.0	0.000165
	-0.493534	-0.743493	12.0	0.000159
	-1.455641	-0.792392	12.0	0.000164
	-0.779196	-0.951246	12.0	0.000162
	-1.010637	-0.999887	12.0	0.000120
	-0.995907	-0.999983	12.0	0.000100
Avg:	-0.947108	-0.879886	12.050	0.000158
Std:	0.351413	0.216419	0.224	0.000056

Our final optimum for $f_1(x)$ is:

$$x_1^* = -0.947 \pm 0.351$$
$$f_1(x_1^*) = -0.880 \pm 0.216$$

Relative distance is:

$$\begin{aligned} x_{true} - x_1^* &= -1 - (-0.947) \\ &= -0.053 \\ f_{true} - f_1(x_1^*) &= -1 - -0.880 \\ &= -0.12 \end{aligned}$$

Table 2: Results for $f_2(x) = x^3 - 3x^2 + 36$

	x^*	$f_2(x^*)$	function_calls	clock_time
	7.699171	314.553840	13.0	0.000316
	7.630426	305.599131	12.0	0.000154
	7.940157	347.457533	12.0	0.000132
	7.030768	235.247719	12.0	0.000120
	7.778227	325.086647	12.0	0.000136
	7.676198	311.540361	12.0	0.000131
	7.471320	285.591838	12.0	0.000118
	9.357814	592.745461	12.0	0.000137
	7.949493	348.780494	12.0	0.000145
	7.497876	288.862239	12.0	0.000126
	7.477308	286.326859	12.0	0.000160
	6.089015	150.528652	12.0	0.000166
	8.458776	426.580208	12.0	0.000126
	7.625984	305.026966	12.0	0.000138
	8.655849	459.757240	12.0	0.000249
	7.073400	239.804382	12.0	0.000280
	7.573978	298.386878	12.0	0.000229
	7.160760	249.349189	12.0	0.000226
	7.293491	264.392311	12.0	0.000221
	7.571402	298.060772	12.0	0.000213
Avg:	7.650571	316.683936	12.050	0.000176
Std:	0.664623	92.297258	0.224	0.000059

Our final optimum for $f_2(x)$ is:

$$\begin{aligned} x_2^* &= 7.651 \pm 0.665 \\ f_2(x_2^*) &= 316.684 \pm 92.297 \end{aligned}$$

Relative distance is:

$$\begin{aligned} x_{true} - x_2^* &= 2 - 7.650 \\ &= -5.65 \\ f_{true} - f_2(x_2^*) &= 32 - 316.684 \\ &= -284.684 \end{aligned}$$