

B27

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Stats 110 - HW 4

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1.

```
# load the data
pulse = read.csv("../data/Pulse.csv")
```

T-test for the slope:

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

```
# t-test
fit_lm = lm("Rest ~ Hgt", data=pulse)
summary(fit_lm)

##
## Call:
## lm(formula = "Rest ~ Hgt", data = pulse)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -26.153  -5.982  -0.571   5.565  33.618
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 112.4141    11.6346   9.662  < 2e-16 ***
## Hgt         -0.6457     0.1702  -3.793  0.00019 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.673 on 230 degrees of freedom
## Multiple R-squared:  0.05887,    Adjusted R-squared:  0.05478
## F-statistic: 14.39 on 1 and 230 DF,  p-value: 0.0001902
```

p-value is low, therefore reject H_0 in favor of H_1 .

ANOVA test for significance:

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

```
anova(fit_lm)

## Analysis of Variance Table
##
## Response: Rest
##              Df Sum Sq Mean Sq F value    Pr(>F)
## Hgt             1  1346.2  1346.18   14.387 0.0001902 ***
## Residuals    230 21520.5    93.57
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

p-value is low, therefore reject H_0 in favor of H_1 .

T-test for Correlation: $H_0 : \rho = 0$

$H_1 : \rho \neq 0$

```
cor.test(pulse$Rest, pulse$Hgt, method="pearson")
```

```
##
## Pearson's product-moment correlation
##
## data: pulse$Rest and pulse$Hgt
## t = -3.7931, df = 230, p-value = 0.0001902
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -0.3601758 -0.1175064
## sample estimates:
## cor
## -0.2426329
```

p-value is low, therefore reject H_0 in favor of H_1 .

2. Population model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma^2)$$

3. Fit regression model for X_1 , X_2 , and X_3

```
fit_multi = lm("Rest ~ Hgt + Wgt + Smoke", data=pulse)
summary(fit_multi)
```

```
##
## Call:
## lm(formula = "Rest ~ Hgt + Wgt + Smoke", data = pulse)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -25.684  -5.938  -1.085    5.852   34.291
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  111.02244    14.25754   7.787 2.39e-13 ***
## Hgt          -0.60484     0.25644  -2.359 0.01919 *
## Wgt          -0.01295     0.03039  -0.426 0.67031
## Smoke         5.80259     2.01400   2.881 0.00434 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.543 on 228 degrees of freedom
## Multiple R-squared:  0.09193,    Adjusted R-squared:  0.07998
## F-statistic: 7.694 on 3 and 228 DF,  p-value: 6.414e-05
```

3-(a). $\hat{R}_{est} = 111.022 - 0.606(Hgt) - 0.013(Wgt) + 5.803(Smoke)$

3-(b). There is a positive linear trend with coefficient 5.803 if the data case belongs to the smokers group, else 0.

3-(c).

```
rest_hat = 111.022 - 0.606*65 - 0.013*150 + 5.803*1
rest_hat
```

```
## [1] 75.485
```

3-(d).

$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

$H_1 : \text{At least one } \beta_i \neq 0 \text{ (for } i = 1, 2, 3)$

4.

```
anova(fit_multi)
```

```
## Analysis of Variance Table
##
## Response: Rest
##          Df Sum Sq Mean Sq F value    Pr(>F)
## Hgt        1  1346.2   1346.18   14.7814 0.0001566 ***
## Wgt         1    0.0     0.03   0.0003 0.9857373
## Smoke       1   756.0    755.99   8.3009 0.0043405 **
## Residuals 228 20764.5    91.07
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4-(a).

$SSTotal = 1346.2 + 0.0 + 756.0 + 20764.5 = 22866.7$

$SSE = 20764.5$

$SSModel = 2102.2$

4-(b).

$SSTotal = 22866.7$

$SSModel = 1346.2$

$SSE = 22866.7 - 1346.2 = 21520.5$

5.

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

```
summary(fit_multi)
```

```
##
## Call:
## lm(formula = "Rest ~ Hgt + Wgt + Smoke", data = pulse)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -25.684  -5.938  -1.085   5.852  34.291
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 111.02244   14.25754   7.787 2.39e-13 ***
## Hgt         -0.60484    0.25644  -2.359 0.01919 *
## Wgt         -0.01295    0.03039  -0.426 0.67031
## Smoke        5.80259    2.01400   2.881 0.00434 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.543 on 228 degrees of freedom
## Multiple R-squared:  0.09193,    Adjusted R-squared:  0.07998
## F-statistic: 7.694 on 3 and 228 DF,  p-value: 6.414e-05
```

5-(a).

p-value is low, therefore reject H_0 in favor of H_1 . Hence, there is sufficient evidence to suggest that height is a significant predictor of resting pulse rate.

5-(b).

The significance of height decreased after adding Wgt and Smoke. Hence adding more terms may be overfitting the model.

6.

β_0 = intercept of Rest

β_1 = slope of Rest against Wgt

β_2 = slope of Rest against Smoke

$Rest = \beta_0 + \beta_1 Wgt + \beta_2 Smoke + \epsilon$, $\epsilon \sim \mathcal{N}(0, \sigma^2)$

7.

```
fit_wgt_smoke = lm("Rest ~ Wgt + Smoke", data=pulse)
summary(fit_wgt_smoke)
```

```
##
## Call:
## lm(formula = "Rest ~ Wgt + Smoke", data = pulse)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -25.872  -6.207  -0.719   5.794  37.128
```

```
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) 78.24692    3.22061  24.296 < 2e-16 ***
## Wgt         -0.06697    0.02017  -3.319  0.00105 **
## Smoke        6.04288    2.03136   2.975  0.00325 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.638 on 229 degrees of freedom
## Multiple R-squared:  0.06978,    Adjusted R-squared:  0.06165
## F-statistic: 8.589 on 2 and 229 DF,  p-value: 0.0002531
```

$$H_0 : \beta_2 = 0$$

$$H_1 : \beta_2 \neq 0$$

Low p-value for Smoke, hence there is significant evidence that the intercepts are different.

8.

β_0 = intercept of Rest

β_1 = slope of Rest against Wgt

β_2 = slope of Rest against Smoke

β_3 = slope of Rest against the interaction of Wgt and Smoke

$$Rest = \beta_0 + \beta_1 Wgt + \beta_2 Smoke + \beta_3 Wgt * Smoke + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma^2)$$

9.

$$H_0 : \beta_3 = 0$$

$$H_1 : \beta_3 \neq 0$$