Towards Human-Centric Aggregation via Ordered Weighted Aggregation Operators and Linguistic Data Summaries: A New Perspective on Zadeh's Inspirations

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***Abstract*—**This work presents a new perspective on how Zadeh's ideas related to fuzzy logic and computing with words have influenced the crucial issue of information aggregation and have led to what may be called a human-centric aggregation. We indicate a need to develop tools and techniques to reflect some fine shades of meaning regarding what can be considered the very purpose of human-centric aggregation, notably stated by various modalities in natural language specifications, in particular the usuality. We advocate the use of the ordered weighted average (OWA) operator, which is a formidable tool that can easily be tailored to a user's intention as to the purpose and method of aggregation, generalizing many simple and natural aggregation types, such as the arithmetic mean, maximum and minimum, and probability. We show some of the most representative extensions and generalizations, including the induced OWA, the generalized OWA, the probabilistic OWA, and the OWA distance. We show their use in the basic case of the aggregation of numerical values and in social choice (voting) results. Then, we claim that linguistic data summaries in Yager's sense can be considered an "ultimately human consistent" form of human-centric aggregation and show how the OWA operators can be used therein.

Digital Object Identifier 10.1109/MCI.2018.2881641
Date of publication: 10 January 2019

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I. Introduction

The purpose of this special paper is manifold. First, the paper is part of a memorial issue of the IEEE Computational Intelligence Magazine (IEEE CIM) dedicated to the late Professor Lotfi A. Zadeh, to commemorate his great conceptual and technical achievements, but—maybe above all—his vision and inspiration, which have had a tremendous impact on so many areas of science and technology, both related to fuzzy logic and to soft computing or computational intelligence, and well beyond, and which will certainly stay with us and be remembered in years to come. Second, the IEEE CIM is a very special scientific journal that is meant to present and advocate crucial developments in the broadly perceived area of computational intelligence that have already made an impact or are viewed to be promising for the future. This encompasses many different paper styles, notably reviews and state of the art articles, with a critical analysis of the history, and some discussion articles, which emphasize nonobvious connections or views on possible newer developments. Clearly, a combination of these two types of exposition can be interesting, and this paper is meant to be so, though mainly focusing on a novel view and the potential applications of some methods proposed in the literature.

To be more specific, we will look at the issue of the broadly perceived aggregation of some (numeric) pieces of evidence, less from the purely formal (mathematical) direction, see Mesiar and his collaborators [1], [2], and more from the point of view of how to reflect human-specific intentions, judgments and visions as to what and how to aggregate. The latter is often expressed in natural language, notably reflecting fine shades of meaning that are notably reflected by various modalities in natural language specifications, in particular the modality of usuality. This may be viewed as an attempt at what may be called a human-centric aggregation. We present a new perspective on how Zadeh's [3], [4] ideas related to fuzzy logic and computing with words can help attain such an aggregation.

The term human-centric (centered) is used here to denote that we attempt to develop solutions to problems that, as explicitly as possible, involve the human perspective in the sense of employing tools and techniques for the problem statement, solution, and interpretation of the results obtained that are consistent with the human cognitive abilities and communication skills. This is in line of, first of all, Dertouzos's [5] concept of a human-centric computing but has also an intrinsic relation to other concepts, notably the human-centered design (cf. [6]). In our context, we advocate the use of natural language, which is the only fully natural means of human articulation and communication, that is, as a quintessence of human centricity. A related perspective, based on the concept of granularity, is proposed by Pedrycz and Gomide [7]. More specifically, from this perspective, we view the OWA operators as the most human-centric operators due to their universality and their ability to reflect various human-specific aspects in judgments, specifications, or evaluations. Regarding the second topic considered, related to linguistic summaries, they may be the best example of the human-centricity of information summarization (closely related to aggregation, as well) because they explicitly employ natural language to represent the (aggregated) content of data.

For the human-centric aggregation, we first indicate a need to develop tools and techniques to reflect some fine shades of meaning, notably reflected by various modalities in natural language specifications, in particular the usuality, and we advocate the use of Yager's [8] ordered weighted average (OWA) operators, which can provide a wide array of aggregation modes that can easily be tailored to a user's intention, generalizing many simple and natural aggregation types, such as the arithmetic mean, maximum and minimum, and probability. We show some of the most representative extensions and generalizations, including the induced OWA, the generalized OWA, the probabilistic OWA, and the OWA distance. We show their use in the basic case of the aggregation of numerical values and in social choice (voting) results [9].

The first problem of interest in this paper is the aggregation process in which the information (data) available is expressed in a (simpler) numerical form. Needless to say that, with similarly as the decision making process, the aggregation process has many aspects, notably cognitive and psychological as well as formal (mathematical, equated mainly with the development of proper aggregation operators/functions). We will be mainly concerned here with a formal aspect, although more algorithmically oriented, focusing on how to involve some human-specific aspects in addition to formal technicalities.

> **… all these aspects are strongly related to the fact that in our intended setting the human being is a key element of the aggregation process, which is to be done by humans for humans in virtually all nontrivial tasks.**

An aspect that should be clarified is that aggregation is not the same as fusion, even though the two concepts are often interchangeably used. Basically, information fusion may be viewed as the process of merging several sources and types of information to use them for a purpose. These types can include data from sensors, databases or data repositories, expert testimonies and reports, and text and multimedia repositories data can include objective and subjective observations, commonsense knowledge, preferences and intentions, and more or less formal regulations, and can be in numeric, qualitative or linguistic forms, and can also include some uncertainty and belief qualifications. For a general exposition of data fusion, see [10], and for a perspective on the role of aggregation operators in data fusion, see [11]–[13].

Though the data fusion as outlined above is certainly more important in practice, it is more difficult, mostly due to the involvement of various types of information and its sources, different time scales and sampling periods. For our purposes, since in this paper we will mainly be concerned with some more basic issues, we limit our attention to aggregation that will basically serve the purpose of taking (without a loss of generality) two input values from [0, 1] and produce one output value as a result, also from [0, 1]. This will be valid for our aggregation related discussion, while for the linguistic summarization related discussion, the "value" of the output will be a (simple and possibly short) sentence in a (quasi) natural language.

For the aggregation mentioned before, the key problem is to properly define an aggregation function. Basically, it is defined as a function agg: $[0, 1] \times \ldots \times [0, 1] \rightarrow [0, 1]$ which is monotone and satisfies some boundary conditions, usually that $\text{agg}(0, \ldots, 0) = 0$ and $\text{agg}(1, \ldots, 1) = 1$. These simple conditions imply that the family of aggregation operators is very large and that many deep and interesting mathematical properties can be formulated and proved. For more information, see [11]–[14], among others.

The aggregation operators developed within the above formal direction have been the subject of deep analyses and strong results have been obtained, and applications have also found in a variety of fields [15]. However, there may be some doubts. First, it is not always clear whether a particular aggregation operator, or even a class thereof, reflects some real human intention or preference as to the very essence and aim of aggregation, for instance:

- How many items should be taken into account during the aggregation process: all, some (crisp) percentage, some imprecise specified (fuzzy) part exemplified by "most", "almost all", "much more than a half;"
- Which items are more important and to what extent are they more important than the other ones, is the importance structure flat or hierarchical; and
- Which items should be possibly neglected in the aggregation because their inclusion can interfere with the inclusion of other items?

Notice that all these aspects are strongly related to the fact that in our intended setting the human being is a key element of the aggregation process, which is to be done by humans for humans in virtually all nontrivial tasks. Aggregation is therefore, from our perspective, an explicitly human-centric task, and therefore, we will advocate a new concept of a human-centric aggregation in this paper.

The above remarks clearly suggest that to perform aggregation in an adequate, human consistent way—which should help attain easier human acceptability of results—one should try to use, in the problem formulation, selection of algorithms and interpretation of results, some human-centric tools and techniques. Fuzzy logic and computing with words seem to provide, in this respect, ultimately human-centric tools and techniques, mainly because imprecision (fuzziness) in values, relations or more general procedures can be well represented and then used in computation.

II. Fuzzy Logic and Computing with Words (CWW) as an Effective and Efficient Set of Tools and Techniques for a Human-Centric Aggregation and Linguistic Summarization

In this section, we will briefly present the idea of fuzzy logic, which is widely known, and our perspective on Zadeh's computing with words (CWW) paradigm, which—in our context—is the ultimate method for the implementation of our human-centric view on aggregation and linguistic summarization.

In 1965, Zadeh, already a world-famous control and systems theorist, proposed a theory of fuzzy sets [16], a calculus of imprecision associated with classes of non-crisp boundaries, i.e., consisting of elements belonging to the class to some degree between 0, which stands for the full non-membership, to 1, which stands for the full membership, through all intermediate values. Therefore, he was in a position to define classes of "tall people," "large numbers," etc., as well as fuzzy relations such as "slightly higher than." The theory of fuzzy sets has since developed rapidly and found applications in a vast array of areas in science and technology exemplified by mathematics, optimization, control, economics, humanities and social sciences—in general, in all systems in which the human being is a key element and when, as for all human beings, the natural language is the only fully natural means of articulation and communication so that an inherent imprecision (fuzziness) plays a crucial role.

One should bear in mind, however, that the shock occurred earlier, namely, in 1962, when Zadeh published, in the prestigious Proceedings of IRE (Institute of Radio Engineers, which later became the IEEE), in the midst of his accomplished career, working in areas in which mathematical rigor and precision were sine qua non conditions, a "strange" paper [17] in which he stated: "… *In fact, there is a fairly wide gap between what might be regarded as "animate" system theorists and "inanimate" system theorists at the present time, and it is not at all certain that this gap will be narrowed, much less closed, in the near future. There are some who feel that this gap reflects the fundamental inadequacy of the conventional mathematics—the mathematics of precisely defined points, functions, sets or probability measures—for coping with the analysis of biological systems and that to deal effectively with such systems, which are generally orders of magnitude more complex than man-made systems …*" Notice that this was a real manifesto for all what Zadeh attempted later, that is, trying to bridge that gap, in various contexts.

Computing with words (and perceptions) appeared in the early to mid-1990s with the main paper of Zadeh [4], followed by many papers, and large edited volumes, e.g., [18]–[20]. Zadeh himself had considered it to be the main conceptual and algorithmic development of fuzzy logic. As opposed to traditional computing, which involves mainly numbers and symbols, computing with words takes as its inputs values, relations or courses of actions specified in an inherently imprecise (quasi) natural language, and yields results in (quasi) natural language too. It constitutes, therefore, a fusion of natural language and computations on fuzzy variables and relations. Obviously, they are strongly related to information granulation, linguistic variables and linguistic modeling. Needless to say, these aspects may have a considerable impact on the broadly perceived aggregation.

For our purposes, it may be expedient to look at the very essence of computing with words in terms of some "generations" [19]. Arguably, the following generations can be distinguished:

Generation -2
This concerns the first, pre-fuzzy period of Zadeh's activities, when he began advocating the necessity to rethink the very essence of formal tools and techniques for the representation and analysis of complex systems and processes. This has best been reflected in his famous, though somehow forgotten paper, Zadeh [17], which we have already cited and commented upon. His remarks on the inadequacy of conventional mathematics to deal with complex, heavily human-centric problems and systems are in fact a manifesto and the first announcement of fuzzy logic.

Generation -1
This generation comprises fuzzy logic in its basic version [16]. Notice that, although in the original form Zadeh's fuzzy sets theory and fuzzy logic (in the broad sense) are not explicitly related to what we can consider as computing with words, he has been correctly using linguistic terms to describe the very essence of a fuzzy set, fuzzy number and fuzzy relation exemplified by "tall men," "around 5," "is much higher than," etc. In fact, it is difficult to imagine any constructive approach to fuzzy sets theory or fuzzy logic, and applications thereof, without any linguistic denotation. That is why it makes sense to term fuzzy sets theory or fuzzy logic some generation of computing with words.

Generation 0
This generation, which is practically explicitly related to computing with words, though this term has not been used, has been triggered by Zadeh's famous papers on linguistic modeling. First, in [20], in which he has defined how to use linguistic values, rules and algorithms to model processes and systems in which the human being, with its perception and cognitive limitations, may preclude the use of precise mathematical and algorithmic tools and techniques. His concepts of a linguistic variable, fuzzy conditional statement, compositional rule of inference, and, finally, a fuzzy algorithm, have set the stage for many areas, notably, fuzzy (logic) control [21], linguistic modeling in economics and management [18], to name a few.

These powerful concepts, which have explicitly focused on the use of (quasi) natural language, with its inherent imprecision to be represented and handled by fuzzy sets, have then been extended and further elaborated in Zadeh's [22] papers on a deeper analysis of a linguistic variable and its relevance for the broadly perceived approximate reasoning process. A culmination of this early involvement in issues related to natural languages, with their inherent imprecision, is Zadeh's [23] concept of PRUF, a meaning representation language for natural language. These ideas are clearly explicitly and strongly related to the later computing with words. As brilliant and powerful as they have been, they have not found, unfortunately, much resonance outside of the fuzzy community, most likely because of an inadequate relation to what might be called natural language technology, exemplified by computational linguistics, NLU (national language understanding), NLP (natural language processing) and NLG (natural language generation). We will comment upon this later.

Generation 1
This is the first "real" use of computing with words, that is, when the term "computing with words" (or, very often "computing with words and perceptions") is used. This started in 1996 with Zadeh's [4] famous paper in which he has stated, even in the title, that fuzzy logic equals computing with words. This has been a strong and far-reaching statement that, as everything, has a very positive but also a less positive impact. On the one hand, it emphasized that computing with words, as a set of tools and techniques for dealing with some linguistic descriptions and relations, is based on fuzzy logic, which is presumably the easiest way to constructively reflect the imprecision of meaning. On the other hand, however, it suggested a weak (lack of) relation to both computing as such and, more importantly, to the abovementioned "natural language technology". The latter has had a detrimental effect and has made it difficult for computing with words to become a widely employed paradigm beyond the fuzzy sets community.

Generation 2
This generation is basically related to Generation 1 in that computing with words is still considered in the original Zadeh's perspective but now many more elements are added

that can help represent many fine shades of meaning. Notably, this concerns the use of linguistic quantifiers, which started with Zadeh's [3] paper in which he has presented an algorithmic approach for dealing with linguistically quantified statements. This has provided powerful tools and techniques for the very aggregation as proposed, for example, Yager's [24] ingenious concept of a linguistic data summary, Kacprzyk's [25], [26] concept of a fuzzy majority for collective decisions and consensus reaching, Kacprzyk and Ziółkowski's [27], and Kacprzyk, Zadrożny and Ziółkowski's [28] database querying systems in which the aggregation of partial cores from the satisfaction of conditions on attribute values has been driven by a linguistic quantifier.

In this generation, Zadeh [29] also more explicitly mentioned the role of modalities in natural language, notably that of usuality, and has proposed the use of a calculus of so-called dispositions. This problem of modalities will be discussed later in this paper as it is relevant for our discussion.

Generation 3

This generation of computing with words boils down to a synergistic and explicit integration with the "natural language technology" (computational linguistics, NLU/NLP/NLG, maybe also Haliday's SFL (systemic functional linguistics)). This has been proposed by a series of papers by Kacprzyk and Zadrożny [30], in which—what is directly related to the topic of this paper—the derivation of Yager's [24] linguistic data summaries has been cast as a problem of natural language generation (NLG). There are obvious advantages of this new concept, namely, that the NLP is a very well developed area, with many well-known people involved, widely available commercial and academic software, and well established commercial companies that provide software systems for the commercial derivation of linguistic summaries. This is clearly a sine qua non-condition of making computing with words a general, widely used paradigm, which it greatly deserves [31].

Of course, these generations of computing with words well characterize the particular stages of the development of this paradigm, with their increasing extent of human centricity, and can also be related to what we consider important for the development of both general aggregation (fusion) and linguistic summaries, as well as the exposition of the role of Zadeh for the development of these areas. This constitutes the focus point of this paper.

One can also imagine that some new developments in the broadly perceived direction of human-centric systems and computing, humanistic intelligence, human/society in the loop paradigms, cognitive informatics, to just name a few, will make it possible in the future to further develop computing with words, and more specifically the aggregation (fusion) and linguistic summarization.

Now, to discuss the importance of computing with words for aggregation and linguistic summaries, we will briefly present the idea of computing with words. Basically, for our purposes, the very essence of computing with words is that the meaning of a proposition, p, in a natural language may be represented as a generalized constraint, as follows:

$$X \text{ isr } R$$

where: X is a constrained variable taking on some fuzzy value, r is the so called indexing variable that refers to a modality, and R is a constraining relation. For instance, if we are concerned with inflation in the next year (X), the above statement can be "it is possible (r) that inflation (X) in the next year will be much less that 2 percent (R)".

As proposed by Zadeh [4], the principal types of constraints are as follows:

- Equality: X is $= R$ $(X = R)$
- Possibilistic constraint: X is (poss) R (R is a possibilistic distribution)
- Probabilistic constraint: X is (prob) R (R is a probability distribution)
- Usuality constraints: X is u R [usually (X is R)]
- Veristic, rough set type, etc.

They clearly correspond to some modalities in natural language, though not to some more sophisticated ones. Luckily enough, the usuality has been dealt with, and it is of paramount importance for the aggregation and, in particular, linguistic summarization because it boils down to what usually happens or is valid, which is the very essence of all data analysis, data mining and knowledge discovery. We can only touch upon the huge problem of modalities in natural languages, and we limit our discussion to just a few of the more relevant and intuitively appealing modalities, as follows [32]:

- Usuality–how frequently something occurs,
- Probability, possibility or certainty–the likelihood of something happening or being the case,
- Obligation or necessity–how necessary it is for things to be done or to be a certain way,
- Ability–the ability of someone or something to do something,
- Inclination–the inclination or willingness of someone to do something.

It is easy to see that computing with words can handle probability, possibility or certainty, and of course, usuality well! However, there is presumably some problem in the representation and processing of ability and inclination.

More generally, we can have the following:

- Alethic modalities, or modalities of truth, like: possibility ("It is possible that"), necessity ("It is necessary that") or impossibility ("It is impossible that");
- Temporal modalities, like: "It was the case that," "It has always been that," "It will be that," "It will always be that," or "It is usually that;"
- Deontic modalities, such as "It is obligatory that," "It is permissible that;"
- Epistemic modalities, or modalities of knowledge, such as "It is known that;" and
- Doxastic modalities, or modalities of belief, such as "It is believed that."

Notice that it would yield a new quality for aggregation and linguistic summarization if these, and maybe also other modalities, were explicitly included in models. It is, however, not so easy.

An interesting connection related to the above can be given between words used in various research communities that are related to the linguistic descriptions of probability and usuality, which is so important for us, as shown in Table 1.

As we have already mentioned, usuality is of a particular importance because it expresses what we consider to be true or to happen in practice. It is clearly strongly related to the linguistic quantifier employed, which, in our context, is the linguistic quantifier that drives the aggregation and linguistic summarization process. This can be presented simply as in Table 2.

Now, we will briefly present our proposed framework in which elements of computing with words, notably some modalities, can be employed for aggregation (fusion), followed by the linguistic summarization. We will start with a very brief exposition of what is meant by aggregation, notably the aggregation operators, in the formal direction. Then, we will outline a possible approach to the use of ideas related to fuzzy logic and computing with words in the very process of aggregation, followed by linguistic summarization. The purpose is to present some tools and techniques, more algorithmic than formal, to reflect some fine shades of meaning regarding what can be considered the very purpose of aggregation, notably stated by various modalities in natural language specifications, in particular, the usuality. We will advocate the use of Yager's [8] OWA operator, which can provide a wide array of aggregation modes that can easily be tailored to a user's intentions and preferences, generalizing many simple and natural aggregation types, such as the arithmetic mean, maximum and minimum, or probability. We will show some of the most representative extensions and generalizations, including the induced OWA, the generalized OWA, the probabilistic OWA and the OWA distance and will show their use in the basic case of aggregation of numerical values and in social choice (voting) results, as well indicate how some modality can be reflected. The approach proposed can be termed a human-centric aggregation.

Then, we will briefly present a linguistic summarization, which can be viewed as an ultimate form of human-centric aggregation. We will again indicate Zadeh's inspirations in this respect and present how the concept has developed towards a comprehensive and implementable verbalization technology.

III. The OWA Operator—a General Purpose Human-Centric Aggregation Operator

In general, an aggregation operator (function) is usually defined as a function agg: $[0, 1] \times \ldots \times [0, 1] \rightarrow [0, 1]$ that is monotone and satisfies some boundary conditions, usually that $\text{agg}(0, \ldots, 0) = 0$ and $\text{agg}(1, \ldots, 1) = 1$. This clearly implies a huge family of aggregation operators, usually very interesting from a mathematical point of view, but not always intuitively appealing and implementable [11]–[15].

A powerful and widely used operator is Yager's [8] OWA operator, which yields aggregation results between the minimum and the maximum. The OWA operator has been extended and generalized in different ways [33], [34]. A common approach to provide a general formulation is by using the generalized OWA (GOWA) operator [35] and the quasi-arithmetic OWA (Quasi-OWA) operator [36]. Some particular cases of these approaches are the ordered weighted geometric average (OWGA), the ordered weighted harmonic average (OWHA) and the ordered weighted quadratic average (OWQA). Some other generalizations of the OWA are the induced OWA (IOWA) operator [37], which reorders the information by using order inducing variables, the probabilistic OWA (POWA) [38], [39], the probabilistic weighted OWA [40] and related extensions [41], the OWA distance (OWAD) [42], [43] and the OWA norm (OWAN) [44]. All these extensions can help reflect fine shades of meaning, including modalities, of the real human judgments, intentions and preferences of what and how is to be aggregated. This is exactly what is needed for our purposes.

A. What is the OWA Operator

The OWA operator was introduced by Yager [8] and it constitutes a parameterized family of aggregation operators between the maximum and the minimum. It is widely used in information analysis and decision making under uncertainty and in many other related disciplines [9], [34], [45]–[48].

Definition 1

An OWA operator of dimension n is a mapping $OWA: R^n \rightarrow R$ that has an associated weighting vector $W = (w_1, \ldots, w_n)$ of dimension n with $\Sigma_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (1)$$

where b_j is the jth largest element of the a_i's.

TABLE 1 Various terms related to the assessment of probability and usuality.

PROBABILITY	USUALITY	VERBS
DEFINITELY	ALWAYS	IT MUST BE
CERTAINLY	USUALLY	IT WILL BE
PROBABLY	SOMETIMES	IT MAY BE
MAYBE	OCCASIONALLY	IT MIGHT BE
UNLIKELY	RARELY	IT COULD BE
DEFINITELY NOT	NEVER	IT WON'T BE

TABLE 2 Terms related to usuality and their possible corresponding linguistic quantifiers.

USUALITY	LINGUISTIC QUANTIFIERS
USUALLY	FOR MOST, ALMOST ALL
SOMETIMES	SOME "IN-BETWEEN" QUANTIFIERS
OCCASIONALLY	FOR SOME
RARELY	FOR AT LEAST ONE (A FEW)
NEVER	FOR NONE

The OWA operator is a mean or averaging operator [49]. This is a reflection of the fact that the operator is commutative, monotonic, bounded and idempotent. Different families of the OWA operators can be obtained by taking various weight vectors [8], [50]–[52], for example: the maximum, the minimum, the arithmetic mean, the Hurwicz criteria, the median, the Step-OWA, the Window-OWA, the Olympic-OWA, the Centered-OWA, the S-OWA.

Note that for some types of aggregation behavior, which can be termed a linguistic quantifier driven aggregation (e.g., to aggregate most pieces of information), Yager [50], [53] suggested a simple method to derive the OWA weights from Zadeh's [3] linguistic quantifier.

An important element in the OWA-based aggregation is the weighting vector [54] because it represents the degree of orness/andness used in the analysis, which basically expresses the attitude of the decision maker towards the underestimation or overestimation of information. Therefore, several works [55] have characterized the weighting vector by using a wide range of techniques, including the degree of orness [8], the entropy of dispersion [8] based on Shannon entropy [56], the balance operator [57] and the divergence [58]. Many methods for generating the weighting vector based on the previous measures are known [59]. A very popular one is the maximum entropy OWA (MEOWA), which was introduced by O'Hagan [60]. This method calculates the weighting vector that attains the maximum entropy subject to a specific degree of orness. Following the MEOWA approach, several other studies have also developed some related extensions by using the Renyi entropy [61], the minimax disparity approach that minimizes the maximum difference between two adjacent weights [62], equidifferent weights [63] and the least-squared method [64]. Zadrożny and Kacprzyk [65] analyze this problem from the point of view of fuzzy querying.

B. Generalized OWA Operators (GOWA)

The GOWA operator [35] provides a more complete framework of the OWA operator by using generalized means. Thus, apart from the arithmetic mean, it can consider many other types of aggregations, including geometric, harmonic, quadratic and cubic means. The GOWA operator is formulated as follows:

$$GOWA(a_1, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda}, \tag{2}$$

where b_j is the jth largest element of the a_i's and λ is a parameter such that $\lambda \in \{-\infty, \infty\} - \{0\}$.

The GOWA operator has similar properties and more particular cases than the OWA operator. However, it is worth noting that some differences may arise [66]. The key feature of the GOWA operator is the parameter λ, which can take different values between $-\infty$ and ∞, generating a different type of OWA aggregation. For example:

- If $\lambda = 1$, the GOWA becomes the classic OWA operator.
- If $\lambda = 2$, the GOWA becomes the ordered weighted quadratic average (OWQA).
- If $\lambda = 3$, the ordered weighted cubic average (OWCA).
- If $\lambda = -1$, the ordered weighted harmonic average (OWHA).
- If $\lambda \to 0$, the ordered weighted geometric average (OWGA) [67], [68].
- If $\lambda = \infty$, the maximum aggregation.
- If $\lambda = -\infty$, the minimum aggregation.

This can be viewed as a way to handle some modalities as well.

A further generalization of the OWA operator can be developed by using quasi-arithmetic means, forming the quasi-arithmetic OWA (Quasi-OWA) operator [36]. It replaces the parameter λ by a strictly continuous monotonic function. Therefore, this approach provides a more general representation that includes the GOWA and all its particular cases.

C. The Induced OWA Operator (IOWA)

The IOWA operator [37] is an extension of the OWA operator that uses a more general reordering process of the information based on order inducing variables. The main advantage of this approach is that it can consider complex attitudes of the decision maker, where the information may not necessarily be ordered numerically in a decreasing or increasing way. For our context, this can also be viewed as an attempt to represent some modalities, notably their combinations. The IOWA is formulated as follows:

$$IOWA(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is the a_i value of the IOWA pair $\langle u_i, a_i \rangle$ having the jth largest u_i, u_i is the order-inducing variable, and a_i is the argument variable.

The IOWA operator includes a wide range of particular cases, including the OWA operator and the weighted average. It also accomplishes similar properties to the OWA operator and has equivalent measures for characterizing the weighting vector [37], [69], [70]. However, note that the degree of orness [8] may have different interpretations here. First, we can reorder the weights in (3) according to the numerical values of their associated arguments to see numerically the orness or andness of the aggregation [71]. Second, we may simply use the weights in (3) according to the initial ordering $w_1, \ldots, w_n$. In this case, this measure would represent an induced orness or andness where we want to focus on this perspective instead of the classical numerical one.

The IOWA operator can be extended by using generalized and quasi-arithmetic means yielding the induced generalized OWA (IGOWA) and the quasi-arithmetic IOWA (Quasi-IOWA) operator [69]. Following (3), the Quasi-IOWA operator is formulated as follows:

$$Quasi\text{-}IOWA(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = g^{-1}\left(\sum_{j=1}^{n} w_j g(b_j) \right), \tag{4}$$

where g is a strictly continuous monotonic function.

Note that the Quasi-IOWA operator includes a wide range of particular cases by using different expressions in the strictly continuous monotonic function $g(b)$. If $g(b) = b$, we get the IOWA operator. If $g(b) = b^2$, the induced ordered weighted quadratic average (IOWQA) and if $g(b) \to 0$, the induced ordered weighted geometric average (IOWGA) [72], [73]. Here, again, for our context, this can be viewed as a way to deal with some modalities.

... it is also possible to develop more general frameworks by using probabilities, weighted averages and OWA operators in the same formulation.

D. The probabilistic OWA Operator (POWA)

The POWA operator [38] is an aggregation that uses probabilities and OWA operators in the same formulation. The main advantage of this approach is the possibility of under or over estimating the results of the expected value according to the attitude of the decision maker. Following (1), the POWA operator is defined as a mapping $POWA\colon R^n \to R$ of dimension n, if it has an associated weighting vector W, with $\Sigma_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$ and a probabilistic vector P, with $\Sigma_{j=1}^{n} w_j = 1$ and $p_i \in [0,1]$, such that:

$$POWA(a_1, \ldots, a_n) = \beta \sum_{j=1}^{n} w_j b_j + (1-\beta) \sum_{i=1}^{n} p_i a_i, \tag{5}$$

where b_j is the jth largest of the a_i's and $\beta \in [0,1]$.

Note that, in the literature, there are other frameworks for integrating the OWA operator with the expected value, such as the immediate probabilities [39], [74], [75]. The main advantage of the POWA is that it unifies both concepts by using a convex combination that considers the degree of importance that each piece of information has in the aggregation. Again, in our context, it can provide a means for the representation of some modalities, notably the combination of the usuality and some alethic modalities.

Following the OWA literature [34], [50], we can analyze similar properties, and notably particular cases [38] of the POWA operators, including the expected value, the OWA operator, the probabilistic maximum, the probabilistic minimum, the arithmetic OWA, and the arithmetic expected value.

Another interesting issue is to characterize the weights of the POWA operator. Following the OWA operator, we could develop the degree of orness of the POWA, the entropy and other related measures [38]. Note that the entropy of the POWA can be formulated as follows:

$$H(\hat{P}) = -\left(\beta \sum_{j=1}^{n} w_j \ln(w_j) + (1-\beta) \sum_{i=1}^{n} p_i \ln(p_i)\right) \tag{6}$$

and if $\beta = 0$, it is the Shannon entropy [56] and if $\beta = 1$, it is the OWA entropy [8].

Observe that similar developments could be used when dealing with the weighted average forming the weighted OWA (WOWA) [76]. However, note that in the literature, there are different methods for integrating the OWA operator with the weighted average, including the classic WOWA operator [77], the importance OWA [78], the hybrid average [73], the OWA weighted average (OWAWA) [76] and the immediate weights [79], [80].

Additionally, it is also possible to develop more general frameworks by using probabilities, weighted averages and OWA operators in the same formulation. Thus, we may form the probabilistic weighted OWA (PWOWA) operator (probabilistic OWAWA (POWAWA) operator [40], [41]). The POWAWA operator includes a wide spectrum of its particular cases, notably, the expected value, the weighted average, the OWA operator, the POWA operator, the WOWA operator, the probabilistic weighted average (PWA) [81], the maximum PWA, the minimum PWA, the arithmetic PWA, the arithmetic POWA and the arithmetic WOWA.

Furthermore, note that the POWAWA operator can be extended into a more general framework where the aggregation considers many more weighting vectors that are integrated into a collective one. The main advantage of this approach is that we can consider a wide range of sources of information that affect the aggregation and consider the degree of importance that each of them has in the problem. For simplicity, let us call this approach the unified weighted average (UWA) [82], which is defined as follows:

$$UWA(a_1, \ldots, a_n) = \sum_{h=1}^{m} \sum_{i=1}^{n} C_h w_i^h a_i, \tag{7}$$

where C_h is the degree of importance that each concept has in the aggregation with $C_h \in [0,1]$ and $\Sigma_{h=1}^{m} C_h = 1$, w_i^h is the ith weight of the hth weighting vector W with $w_i^h \in [0,1]$ and $\Sigma_{i=1}^{n} w_i^h = 1$.

Additionally, note that in the UWA operator, we only consider one level of aggregation, although it is possible to consider more levels forming a hierarchical aggregation. This is very practical in problems that deal with many variables, such as in a multicountry multicriteria multiexpert POWAWA operator [41].

Therefore, the richness of extensions of the concept of an OWA operator can be a decisive factor for an adequate and human consistent representation, and then processing, of many human specific judgments, intentions and preferences, which are the very essence of our approach.

E. The OWA Distance (OWAD)

The OWAD (or Hamming OWAD) operator [42], [43] is an extension of the normalized Hamming distance [83] by using the OWA operators. The main difference between the classic normalized Hamming distance and the OWAD operator is that in the OWAD, the reordering of the arguments of the individual distances is developed according to their values in a decreasing or increasing way. Therefore, it is possible to calculate the distance under or over the results obtained from the minimum to the maximum distance. The OWAD operator can be extended with generalized means forming the Minkowski OWAD (MOWAD) [43], [84]. It can be defined, in [0, 1], for two sets $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$, as a mapping MOWAD: $[0,1]^n \times [0,1]^n \to [0,1]$ that has an associated weighting vector W, with $\Sigma_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$ such that:

$$MOWAD(\langle x_1, y_1 \rangle, \dots, \langle x_n, y_n \rangle) = \left(\sum_{j=1}^{n} w_j D_j^{\lambda} \right)^{1/\lambda}, \tag{8}$$

where D_j is the jth largest individual distance of the $\left| x_i - y_i \right|$ and λ is a parameter such that $\lambda \in \{-\infty, \infty\} - \{0\}$.

Observe that this definition can be generalized for all the real numbers R by using MOWAD: $R^n \times R^n \to R$. Note that the difference of reordering in a decreasing or increasing way is connected by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the descending MOWAD (DMOWAD) operator and w^*_{n-j+1}, the jth weight of the ascending MOWAD (AMOWAD) operator. Moreover, if $\lambda = 1$, the MOWAD operator becomes the OWAD and if $\lambda = 2$, it becomes the Euclidean OWAD operator [84].

Note that by using the techniques mentioned in the previous sections, we can form various OWA distances, as follows:

- For the induced aggregation operators [84]: The induced OWA (IOWAD) distance.
- For the quasi-arithmetic means (also generalized) [84], [85]: The quasi-arithmetic OWAD (Quasi-OWAD) and the induced quasi-arithmetic OWAD (Quasi-IOWAD).
- For the probabilities [86]: The probabilistic OWAD (POWAD), the induced POWAD (IPOWAD), the quasi-arithmetic POWAD (Quasi-POWAD) and the induced quasi-arithmetic POWAD (Quasi-IPOWAD).
- For the weighted averages [87], [88]: The weighted OWAD (WOWAD), the induced WOWAD (IWOWAD), the quasi-arithmetic WOWAD (Quasi-WOWAD) and the induced quasi-arithmetic WOWAD (Quasi-IWOWAD).
- For the probabilities and weighted averages [89]: The weighted probabilistic OWAD (PWOWAD), the induced PWOWAD (IPWOWAD), the quasi-arithmetic PWOWAD (Quasi-PWOWAD) and the induced quasi-arithmetic PWOWAD (Quasi-IPWOWAD).

F. The OWA Norm (OWAN)

The OWAN operator [44] is an aggregation operator that aggregates a set of norms into a single representative result of the norms by using the OWA operator. Thus, the OWAN can consider any result ranging from the minimum to the maximum. Compared to the OWAD operator, the OWAN can be seen as a generalization, where, instead of distances, the aggregation may consider any type of operation, including distances, sums, divisions and variances. Therefore, this approach provides a more general framework for the aggregation of two or more sets. The OWAN is defined as follows:

$$OWAN(|a_1|, \dots, |a_n|) = \sum_{j=1}^{n} w_j N_j, \tag{9}$$

where N_j is the jth largest of the individual norm $|a_i|$. Recall that a norm is a function $f: R^n \to [0, \infty)$ that has the following properties: (1) $f(x_1, x_2, \dots, x_n) = 0$ if and only if all $x_i = 0$; (2) $f(aX) = |a| f(X)$ and (3) $f(X) + f(Y) \geq f(X+Y)$, that is, the triangle inequality.

Some examples of operations that can be used in the OWAN are as follows [90]:

- For the sums: The OWA sum.
- For the subtractions: The OWA subtraction.
- For the multiplications: The OWA multiplication.
- For the divisions: The OWA division.
- For the variances [91]: The OWA variance.
- For the covariances [92]: The OWA covariance.
- For the Pearson coefficient [92]: The OWA Pearson coefficient.
- For the areas: OWA triangle, OWA polygon and OWA circle.
- For the volumes: OWA cube, OWA cylinder and OWA sphere.
- For the physical measures: OWA velocity, OWA temperature and OWA age.

Many other operations could be used in the aggregation as long as they accomplish the properties of a norm [44]. However, it is also possible to consider more complex operations in the aggregation that may accomplish some special properties.

Following the previous sections, the OWAN operator and its particular cases could be extended with induced aggregation operators, quasi-arithmetic means, probabilities and weighted averages. Note that there are other studies in the literature that use t-norms and t-conorms in the aggregation of the OWA operator for other related purposes [93], including the T-OWA and the ST-OWA [94].

G. Some Other Extensions of the OWA Operator

In the literature, there are many other extensions of the OWA operator that improve and extend some specific characteristics of the OWA aggregation [33], [34], [45].

First, to allow the aggregation to use a weighting vector where the sum of the weights is higher than 1 and up to n, Yager [58] introduced the heavy OWA operator. Further extensions have considered that the sum of the weights may move between $-\infty$ and ∞ and use induced and generalized aggregation operators [95], [96] and distance measures [97].

Another interesting extension of the OWA operator is the one that does not accomplish the monotonicity property and therefore forms the nonmonotonic OWA operator [98]. In this framework, some of the weights may be negative, producing a repulsion effect in the aggregation. This approach can also be studied with probabilities [38] and distance measures [42].

Further interesting extensions of the OWA operator are those that deal with uncertain information that can be represented by interval numbers, fuzzy numbers and linguistic variables. By using interval numbers, the OWA operator becomes

… different types of intervals and fuzzy numbers can be developed by using different extensions of the OWA operator, including … the subjective fuzzy number, the objective fuzzy number and the moving fuzzy number.

the uncertain OWA operator [99]; with fuzzy numbers, the OWA becomes the fuzzy OWA operator [100], [101]; and with linguistic information, we get the linguistic OWA operator [102], [103]. Note that there are other extensions [104] currently appearing in the literature that use other types of linguistic and fuzzy information, including intuitionistic fuzzy sets [105], picture fuzzy information [106], type 1 and 2 fuzzy sets [107], hesitant fuzzy sets [108], [109], uncertain linguistic information [110], [111], 2-tuple linguistic variables [112]–[114] and unbalanced information [115].

Some other works have developed extensions by using moving averages [116], [117], Bonferroni means [118], [119], prioritized aggregations [120], [121], logarithmic aggregations [55], [122], Einstein t-norm operations [123], Shapley weights [124], power averages [125], Choquet integrals [126], continuous aggregations [127], [128], Losonczi means [129], Heronian means [130], modular aggregations [131], lattices [132], density averages [133], Archimedean t-norms [134] and dependent information [50].

Note that, in the literature, there are many other developments of the OWA operator that use a wide range of tools and techniques [33], [34], [45]. Additionally, the expectation for the future is that many other extensions and generalizations will appear based on the use of other methods and techniques.

H. Applications of the OWA Operator

The OWA operator has been applied in a wide range of fields. Currently, it is very popular to apply the OWA operator in decision making due to the flexibility provided by the OWA that allows the decision maker to make decisions under or overestimating the information according to his degree of optimism or pessimism. From this point of view, the OWA operator unifies the classic methods for decision making under uncertainty in the same formulation [8], including the pessimistic and optimistic criteria, the Laplace criteria and the Hurwicz criteria. Some other extensions of the OWA operator provide a broader unification, such as the POWA operator [38], which unifies decision-making problems under risk and uncertainty in the same formulation.

Moreover, the OWA operator is widely used in many other decision making situations, including group decision making [135], multiperson aggregation systems [38], multipurpose decision-making [136], consensus processes [137], voting systems [138], representation and handling of imprecise (fuzzy) majorities [9], Dempster-Shafer belief structure [71], [139], minimization of regret [140], the analytic hierarchy process (AHP) [141] and utility theory [142].

Another important field where the OWA operator can be applied is statistics. Recall that the OWA operator and its generalizations include the arithmetic mean, the weighted average and the expected value as particular cases. Therefore, all the previous studies that use one of these tools can be revised and extended with the OWA framework providing a more general approach that can consider different complex scenarios and be reduced to the classic approach. Some representative examples are the OWA variance (OWAV) [91], [92], the OWA covariance (OWAC) [38], [92] and the OWA Pearson coefficient (OWAPC) [38], [92]. Additionally, many other developments can be implemented in regression analysis [38], [143], probability theory and descriptive statistics. Furthermore, statistics is widely used in many other sciences where the OWA operator could be implemented, including physical statistics, biostatistics and econometrics.

A key issue of the OWA operator is that it provides a parameterized family of aggregation operators between the minimum and the maximum. Therefore, by using these two extremes and some central values generated by an OWA aggregation, it is possible to construct interval and fuzzy numbers [38]. Therefore, all the studies that use interval or fuzzy numbers can also be improved by using the OWA operators. Note that different types of intervals and fuzzy numbers can be developed by using different extensions of the OWA operator, including subjective intervals based on the WOWA operator, objective intervals based on the POWA operator and moving intervals based on the OWA moving average (OWMA) [116]. With fuzzy numbers, we obtain the subjective fuzzy number, the objective fuzzy number and the moving fuzzy number.

Focusing on computational intelligence, the OWA operator has also been studied in many other problems apart from those mentioned above regarding fuzzy and probabilistic reasoning. The use of OWA operators in neural networks was already proposed by Yager [144]. Several authors have developed some extensions with neural networks [145], [146]. Some other works have also used the OWA operator in evolutionary computation [147]. However, the use of the OWA operator in neural networks and evolutionary computation is still in the initial stages. Therefore, many new contributions are expected in this field in the future.

The OWA operator has also been used in many different problems in computer science and engineering, for example, in data mining [148], support vector machines [149], clustering [150], image processing [151], linguistic summaries [48], web server analyses and design [48], geographical information systems [179], water resources management [152], continuous location [153], optimization [57], [154], [155], and data envelopment analysis [156].

Another important field where the OWA operator is being used significantly is in business and economics. Apart from decision-making problems regarding financial, economic, commercial and strategic issues [157], the OWA operator has been used to design new economic variables based on the analysis of economic averages. Some representative examples in this direction are the average price [41], the average sales [158], the average demand [159] and the average growth [160]. Moreover, there are many other applications of the OWA operator in business and economics, including portfolio selection [161], insurance management [162], [163], sales forecasting [164], human resource selection [165], social choice and voting [9], [166] and econometrics [97].

Finally, note that since the OWA operator generalizes basic concepts such as the arithmetic mean, the weighted average and the expected value, the expectation for the future is that many other new applications will appear in the literature in a wide range of fields. Therefore, it is clear that more research on the OWA operator is needed to continue the growth of this area.

To summarize this important section devoted to a comprehensive presentation of the many types of OWA operators, one can notice that by a proper choice of their parameters and structures, we can always find a form that will well reflect some fine shade of meaning, notably the desired modality, which is what we need to develop a human-centric aggregation.

IV. Linguistic Summaries

In the preceding sections, we have presented an approach to what might be termed a human-centric aggregation of data, or pieces of evidence in general. Basically, we showed a possible solution to a crucial problem of how to include and reflect in the aggregation process many fine shades of meaning of user intentions, judgments and preferences, specified in natural language, notably in the form of some modalities to be handled by tools of computing with words. To be specific, we have presented a comprehensive account of various forms and extensions of Yager's [8] OWA operators, which are general yet effective and efficient aggregation operators. However, one can still consider the OWA operators as not sufficiently (or ultimately) human-centric because, by applying them to numerical data, we also obtain numerical data as the aggregation result.

From this point of view, more human-centric, possibly "ultimately human-centric," are certainly Yager's [24] linguistic data summaries, which from a set (even very large) of numeric data yield, as a result of aggregation, a short natural language summary, e.g., for a personnel data set, "most younger collaborators are well qualified and well paid." We will consider these linguistic summaries as they are the most powerful and intuitively appealing, in the form introduced by Yager [24], then advanced by Kacprzyk and Yager [167], and Kacprzyk, Yager and Zadrożny [168], and then implemented in Kacprzyk and Zadrożny [169], [170]. We will not deal here, due to lack of space, with other approaches to linguistic data summarization exemplified by the use of type 2 fuzzy sets, linguistic summarization in the sense of mining IF-THEN rules, linguistic summaries via gradual rules, linguistic summarization by mining association rules or relations of linguistic summarization with NLG. Moreover, due to space limitations, we will only outline our approach proposed and refer the interested readers to the literature [171]–[174].

In this short review, we address the linguistic summaries in Yager's [24] sense, which are essentially in the form of a linguistically quantified statement. First, they are directly inspired by Zadeh's [3] concept of a linguistically quantified proposition, the truth of which can easily be determined by using Zadeh's classic approach based on fuzzy logic with linguistic quantifiers or OWA operators. We will concentrate, for practical reasons, on the use of type 1 fuzzy set approaches. Second, they are definitely the most conceptually powerful and widely used summaries, with many applications and implementations for solving real-life problems [170]. We will not deal here, however, with important aspects of linguistic summarization exemplified by their interpretability [175].

In the basic Yager's [8] approach, in its constructive form by Kacprzyk and Yager [167], and Kacprzyk, Yager and Zadrożny [168], and implemented in Kacprzyk and Zadrożny [169], [170], we have: (1) V, a quality (attribute) of interest, e.g., salary in a database of workers, (2) a set of objects (records) y_i that manifest quality V, e.g., the set of workers. Hence, $V(y_i)$ are values of quality V for objects y_i, and (3) $Y = \{V(y_1), \ldots, V(y_m)\}$ is a set of m pieces of data (the "database" in question). A linguistic summary of a data set consists of the following:

- ❏ a *summarizer* S (e.g. young, extendable to young and well paid),
- ❏ a *quantity in agreement* Q given as a fuzzy linguistic quantifier (e.g., most),
- ❏ *truth degree* T - e.g., 0.7,

and can be written as "*most of the employees are young*," which can be clearly valid (true) to some degree, written as, e.g., "T(*most of the employees are young*) = 0.7".

The calculation of the truth degree is equivalent to the calculation of the truth value (from [0, 1]) of a linguistically quantified statement, which may be done by using Zadeh's [3] calculus of linguistically quantified propositions [18], which can directly be represented via Yager's [8] OWA operators [34].

Such linguistic summaries, equated with linguistically quantified propositions, can be written as follows:

$$\underset{y\in Y}{Q}\, S(y) \ \text{(e.g., "Most elements of } Y \text{ possess property } S\text{")} \tag{10}$$

$$\underset{y\in Y}{QKS}(y) \ \text{(e.g., "Most elements of } Y \text{ with property } K \text{ possess also property } S\text{")} \tag{11}$$

and their truth values are calculated via the well-known Zadeh's [3] calculus as follows:

$$\begin{aligned} \mathrm{truth}\,(\mathrm{Q}P(X)) &= \mu_{\mathrm{Q}}\left(\sum \mathrm{Count}\,(P) \Big/ \sum \mathrm{Count}\,(X)\right) \\ &= \mu_{\mathrm{Q}}\left(\sum_{i=1}^{N} \mu_P(x_i)/m\right) \end{aligned} \tag{12}$$

$$\begin{aligned} \mathrm{truth}\,(\mathrm{Q}BP(X)) &= \mu_{\mathrm{Q}}\left(\sum \mathrm{Count}\,(P\cap B) \Big/ \sum \mathrm{Count}\,(B)\right) \\ &= \mu_{\mathrm{Q}}\left(\sum_{i=1}^{m} (\mu_P(x_i) \wedge \mu_B(x_i)) \Big/ \sum_{i=1}^{m} \mu_B(x_i)\right) \end{aligned} \tag{13}$$

where: $m = \text{card}(Y)$, $\Sigma\text{Count}(A) = \Sigma_{y_i \in Y} \mu_A(y_i)$, $\Sigma_{i=1}^{m} \mu_K(y_i) \neq 0$, $\wedge$ is a t-norm.

Simple and natural equivalents of the above formulas using the OWA operators have been proposed, e.g., by Yager [8], [53].

The basic validity criterion, i.e., the truth degree T, is certainly the most important but does not grasp all aspects of a linguistic summary. As to some other quality (validity) criteria, e.g., the informativeness, see Yager [8], and then five additional measures have been proposed by Kacprzyk and Yager [167], and Kacprzyk, Yager and Zadrożny [168], which are truth, degrees of imprecision, covering and appropriateness, and the length of a summary. For more measures, see Kacprzyk, Wilbik and Zadrożny [176], [177].

It is easy to see that the above linguistic summaries consider a static case, and a new approach to the linguistic summarization of time series has been proposed by Kacprzyk, Wilbik and Zadrożny [176], [177].

First, as always with time series analyses, we perform a segmentation of a time series to obtain segments of values which exhibit a uniform behavior, e.g., a slow increase, moderate variability. There are many methods for performing the segmentation, both bottom-up and top-down. Then, the linguistic summarization proceeds over such extracted trends (segments).

Next, we assume a piecewise linear representation of time series data, and we extract segments, i.e., the constituent straight lines that represent a uniform behavior of the data, using one of many popular methods.

Basically, we consider the following basic features of trends in the time series: dynamics of change (speed of change of the consecutive values described by the slope of a line representing a trend), duration (the length of a single trend), and variability (how "spread out" a group of data within a segment is). We use a small set of granulated linguistic labels, e.g., quickly increasing, increasing, slowly increasing, constant, slowly decreasing, decreasing, quickly decreasing, represented by fuzzy sets.

Many protoforms (abstract prototypes, or templates, of linguistically quantified propositions) of linguistic time series summaries are proposed by Kacprzyk, Wilbik and Zadrożny [176], [177], for instance:

Among all segments, Q are P

e.g: "Among all segments, most are slowly increasing".

Among all R segments, Q are P

e.g: "Among all short segments, most are slowly increasing".

We can also add a temporal dimension, E_T, such as "recently", "initially", "in the very beginning", and "in the early Spring of 2010", and obtain temporal protoforms such as the following:

E_T among all segments, Q are P

e.g.: "Recently, among all segments, most are slowly increasing".

E_T among all R segments, Q are P

e.g: "Initially, among all short segments, most are slowly increasing".

The truth values of such linguistic time series summaries can be calculated by using various calculi, notably Zadeh's [3] calculus of linguistically quantified propositions, Yager's [8] OWA operators, and the Choquet integral [126].

The linguistic summaries of time series have proven to be very useful in applications exemplified by the analyses of Web server logs [48], healthcare data [178], and some other approaches [179].

Therefore, the linguistic data(base) summaries in Yager's [24] are a very intuitively appealing and powerful tool for obtaining very easy to use, even for novice users, the results of data analyses and mining. Obviously, their generation is not trivial and various methods can be employed. First, Kacprzyk and Zadrożny's [176], [177] proposal to use for this fuzzy querying with linguistic quantifiers augmented with a protoform analysis is quite intuitive, easy to use and efficient. Basically, since fuzzy queries directly correspond to linguistic summaries, the derivation of a linguistic summary may proceed as follows: (1) the user formulates a set of linguistic summaries of interest (relevance) using the fuzzy querying, (2) the system retrieves records from the database and calculates the validity of each summary adopted, and (3) a most appropriate linguistic summary is chosen. One can also employ the mining of association rules and a genetic algorithm.

A novel, promising approach, which has been proposed by Kacprzyk and Zadrożny [180], boils down to the generation of linguistic summaries by using the tools and techniques, and software (both free and commercial), of NLG following the data to text summarization. The use of linguistic summaries in our sense makes it possible to account for an inherent imprecision (fuzziness) of natural language. This approach seems to have great potential and has been followed by some other researchers [31].

This concludes our brief exposition of linguistic data (database) summaries, which are very important for our purposes because, first of all, they can be viewed to constitute an "ultimately human-centric" way of aggregation of some pieces of data (evidence) since they operate on natural language, which is the only fully natural means of articulation and communication for humans. Second, formally they are based on the use of some aggregation operators that reflect the usuality modality in human judgments and intentions (expressing what usually, or in most cases, happens), and—as we have shown in Section III—the use of the more basic OWA operators can yield a new quality by an effective and efficient aggregation behavior but also an ingenious ability (notably in the case of more sophisticated extensions) of representing human-specific aspects, notably some other modalities and their combinations.

V. Conclusions

In this paper, we wished to pay a tribute to Professor Lotfi A. Zadeh. He has inspired a multitude of areas and researchers, including the authors and our line of research. To be more specific, we have considered here Zadeh's inspiration for a very important area of broadly perceived aggregation (fusion) of pieces of data, information or evidence. We have looked at aggregation not only from a technical point of view but more generally, as a process that is in general performed by humans for humans, i.e., essentially human-centric. Since for human being, natural language is the only fully natural means of articulation and communication, we have concentrated on the aggregation process, and tools and techniques, as perceived by humans. We have therefore considered a general setting in which the point of departure is the human perception, judgment, intention and vision expressed in natural language, with its inherent imprecision. We have used elements of Zadeh's computing with words to represent and then process them and have developed a new human-centric vision and perspective on the aggregation and then on linguistic summarization, which—in our context—is the ultimately human-centric aggregation. In particular, to express fine shades of meaning in human judgments, intentions and preferences, we have widely used in our models some natural language modalities, notably usuality, which is crucial for our purposes. We have also proposed models that can make it possible to employ other modalities. This yields a new concept of a human-centric aggregation, the idea of which has been proposed here.

For the new human-centric aggregation to be operational and then implementable, we have proposed to widely use the ingenious concept of the OWA operator as the main building block, notably its more sophisticated extensions, exemplified by the GOWA operator, the IOWA operator, the POWA operator, the OWAD and the OWAN operator. We have shown their main properties and applications but also commented upon their possible role as tools for a human-centric aggregation, notably their ability to represent various modalities in human specifications of what and how to aggregate. We have also indicated that some advanced OWA operators can implement combinations of modalities, even some sort of hybrid modalities. In particular, we have outlined the crucial role that can be played by the OWA operator driven aggregation in the context of linguistic data summaries.

We hope that our new idea of a human-centric aggregation will be further developed, as it can provide powerful tools for all types of human-centric systems in which the human being is the key element. Such systems dominate in all types of nontrivial decision making and problem solving.

Acknowledgments

We would like to thank the guest editors and the anonymous reviewers for valuable comments that have improved significantly the quality of the paper.

A partial support from Project 691249, RUC-APS: Enhancing and implementing Knowledge based ICT solutions within high Risk and Uncertain Conditions for Agriculture Production Systems (www.ruc-aps.eu), funded by the EU under H2020-MSCA-RISE-2015 (J. Kacprzyk), and a support from the Chilean Government through the Conicyt—Fondecyt Regular Program (project number 1160286) (J.M. Merigó), are acknowledged.

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