2022 7th International Conference on Intelligent Computing and Signal Processing (ICSP) | 978-1-6654-7857-1/22/$31.00 ©2022 IEEE | DOI: 10.1109/ICSP54964.2022.9778323

Analysis on ordered weighted averaging operators in different types and applications for decision making

Ziwei Shu[1*]
[1] Faculty of Statistics, Complutense University of Madrid, Madrid 28040, Spain
*Corresponding author's e-mail: ziweishu@ucm.es

***Abstract*—Since Yager initially introduced the ordered weighted averaging (OWA) operator in 1988, it has been used to aggregate multiple decision-maker attitudes to yield an overall decision. This article uses science mapping analysis to provide an overview of the types and applications of OWA operators. The main goals are to discuss the OWA development path, its formulas and possible extensions in the literature, and the identification of the domains where the OWA operators have been most used. The results verify that, on the one hand, the OWA operators have been primarily used in the decision-making system; on the other hand, it can be extended by including various measurements into the OWA operator's formula. OWA and its applications have been expanding in a variety of fields, including computer science, business, economics, environment and natural resource, finance, geography, mathematics, and so on. The main use of the OWA operators in multiple application fields is to select the optimal strategy or plan by calculating the weight or importance of different criteria and taking into account the attitudinal character or degree of optimism of the decision-maker. In a word, this paper examines over 19,000 articles indexed by Web of Science, whose main contribution is to provide some future research directions on OWA.**

Keywords: ordered weighted averaging operator; science mapping analysis; word cloud; OWA development; decision-making

I. Introduction

The first introduction of ordered weighted averaging (OWA) operator was made in 1988 by Yager [1], during that time, the main researchers about the OWA operators were Yager [1], Fodor [2], Herrera [3], Mitchell [4] and Filev [5]. Based on the conceptualization of OWA and its expansions produced by various authors, the OWA operators have been widely used in several research fields, and they have grown increasingly popular in decision-making issues in the past few years. Thus, the objectives of this article are based on the following considerations:

- The OWA development trajectory should be analyzed systematically.
- The various types of OWA operators should be discussed to see the internal connection with each other.
- The areas where the OWA operators have been most applied should be analyzed.

In this study, Science Mapping Analysis (SMA) has been used to trace the evolution of OWA research over the last 34 years, to present an overview of OWA operators, and to synthesize OWA applications in a variety of scientific domains. The SMA is carried out by using the Biblioshiny software, which can interact with Web of Science (WoS) data. In addition, the keyword analysis is done implemented through using the Python programming.

The main contribution of this article is to review the development of OWA and its applications in different fields, which provides some future research directions for those who are interested in this topic.

II. Development of the OWA

Since the OWA operator was created in 1988, it has attracted a growing amount of attention. Through topic search on WoS with the query "ordered weighted averaging", 19,370 bibliographic records have been obtained about OWA operators from 1988 to 2021. The citations retrieved were counted up to January 2022.

Based on the documents downloaded, Science Mapping Analysis (SMA) has been used to describe the OWA development trajectory, to detect patterns in the literature, and to synthesize OWA applications in many scientific fields. The software Biblioshiny has been used to perform the SMA, with the support of the main functions of the bibliometrix package in the R language. Furthermore, as Biblioshiny has its limitations in cleaning words repetition, wordcloud package in the Python language has also been applied in the keyword analysis.

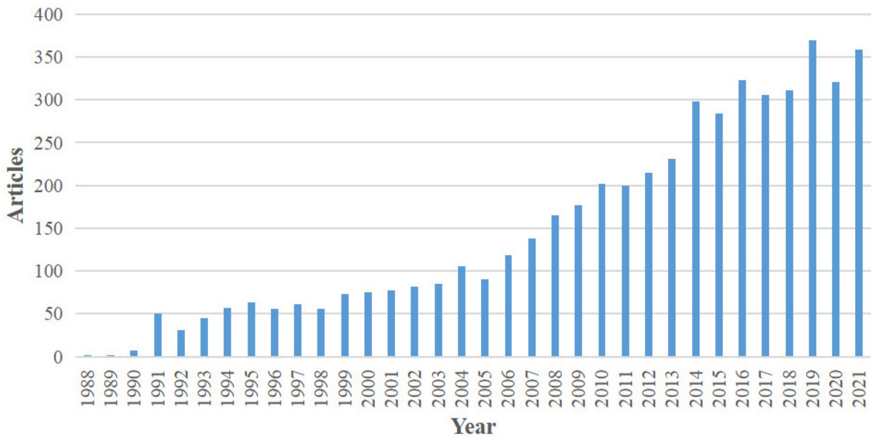

Fig.1 Construction and geometrical dimensions of specimens

Fig.1 shows the evolution of publishing volume with a subject associated with OWA operators from 1988 to 2021. In the first three years, the number of publications was very low. In 1991, it has an evidently grow to 50 articles. Even though a slight decrease appeared in the following two years, the number of publications maintained at the same level as in 1991 until 1998.

Since 1999, an increasing number of academics have explored this topic, whose total publication had over 100 in 2004, and over 150 in 2009. The publications associated with the OWA operators reached 200 in 2010. It has been steadily expanding since then, and has surpassed 350 by 2019. The number of articles reduced in 2020, however, it continuously remained above 300, and exceeded 350 again in 2021. It is obvious to see that this topic has received growing attention in the recent years.

Moreover, the keywords of the articles downloaded have been analyzed to investigate what terms that researchers used are relevant to the OWA operators. By doing this, the area where OWA has been applied commonly can be found. The keyword table was generated by Biblioshiny. After that, Python was used to conduct keyword cleaning for it so that Fig.2 can demonstrate a word cloud about the terms whose frequency is equivalent to or greater than 10 times. In addition to the two phrases related to the OWA operator ("ordered weighted averaging" and "aggregate operator"), the keywords with the similar largest font are: “group decision making”, “decision making”, “multiple attribute decision making” and “multi-criteria decision making”, respectively. This confirms the idea, proposed by Yager in his paper, which means that the OWA operators could be widely applied to decision-making [1].

Fig.2 Keyword analysis of OWA from 1988 to 2021. The word size represents the frequency of keyword occurrence. The larger is the word in the figure, the more frequently it appears.

In order to deeply study the relationship between keywords, Louvain Clustering has been used in Biblioshiny Co-occurrence Network to create the keyword network clustering map. Fig.3 presents the main three clusters. Each circular node represents a keyword, and the thickness of the connection between the nodes indicates the level of the co-occurrence frequency coefficient between them. It can be seen from Fig.3 that the level of the co-occurrence frequency coefficient between the keywords "owa operator" and "decision making" is clearly the highest in the cluster of red dots, followed by "decision making" and "aggregation operators". In the cluster composed of blue dots, it is clear to see that the level of the co-occurrence frequency coefficient between the keyword "group decision making" and "aggregation operator" is the highest. Furthermore, the green dot cluster has the highest amount of co-occurrence frequency coefficient between the terms "ordered weighted averaging" and "aggregation".

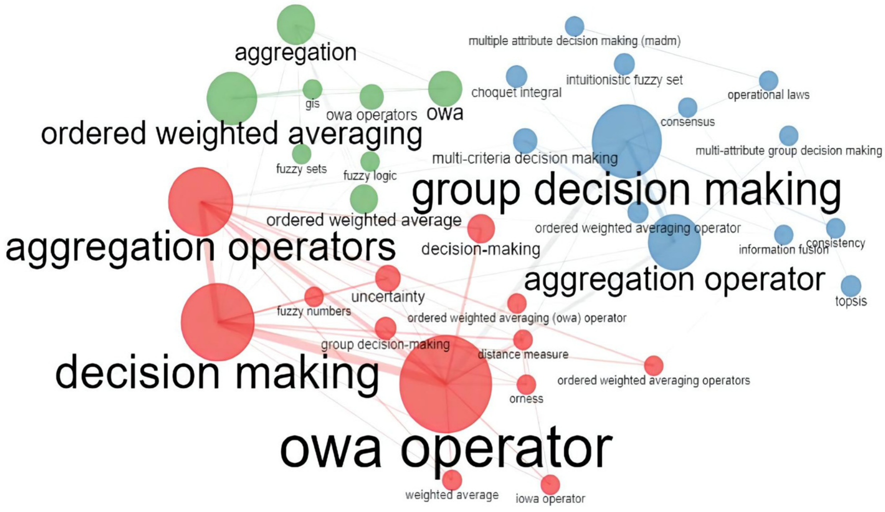

Fig.3 Networks clustering of keyword co-occurrence. The word size represents the frequency of keyword occurrence. The larger is the word in the figure, the more frequently it appears.

It can be found in Fig.3 that the keywords such as "owa operator", "decision making", "group decision making" and "aggregation operator" have a relatively large dot, showing that various writers have given them greater attention. In general, based on Fig.2 and Fig.3, it reveals that the OWA operators have been primarily used in the decision-making system (DMS), which has begun to form an academic research network.

III. Different types of the OWA operators

OWA operator is a symmetric aggregation equation that assigns weights in line with the input value and summarizes the information of a set of data to generate an overall decision equation [1]. It can analyze the effect of risk-averse and risk-prone Decision Markers (DMs) on making decisions by taking into account their optimism and pessimism levels in a quantifiable way. Thus, it has been used to solve many Multi Attribute Decision Making (MADM) difficulties, generally following three steps:

- The input arguments are rearranged in descending order.
- The order weights are determined by using a proper method.
- These reordered arguments are multiplied by the OWA weights.

The method utilized to determine the order weights should be considered. One of the most common ways is to calculate the weights with linguistic quantifiers (all, most, many, half, some, few, quite a few, at least one, and so on), which improves the translation between natural language text and formal mathematical terms [6]. In addition, Yager proposed to apply Regular Increasing Monotone (RIM) quantifiers [7] to obtain a subset parameterized in the unit interval as presented in Formula 1:

$$Q(p) = p^{\gamma}, \quad \gamma > 0 \tag{1}$$

where Q is a linguistic quantifier that can be expressed as a fuzzy subset over the unit interval [0, 1]; for each p in the unit interval, Q (p) is the grade of membership that shows the compatibility of p with the concept signified by Q.

The ordered weights w_i ($w_i \in [0,1]$) are calculated using Formula 2:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right) = \left(\frac{i}{n}\right)^{\gamma} - \left(\frac{i-1}{n}\right)^{\gamma}, \gamma > 0 \tag{2}$$

where n is the quantity of decision makers, i=1,...,n, and γ is the value associated with each linguistic quantifier. The larger γ is, the more optimistic the decision maker's opinion is. γ=1 stands for a neutral stance.

The OWA can be calculated following its definition [1]: an OWA operator of dimension n is a mapping of OWA: $R^n \rightarrow R$ with a related weight vector W of dimension n such that $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$, based on the following Formula 3:

$$OWA\ (a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i \tag{3}$$

where b_i is the ith largest element of the collection a_i .

Although the input arguments are usually in descending order, in some cases, it could be in ascending order by using $w_i = w_{n+1-i}$, where w_i is the ith weight of the OWA and $w_i = w_{n+1-i}$ is the ith weight of the ascending OWA (AOWA) operator.

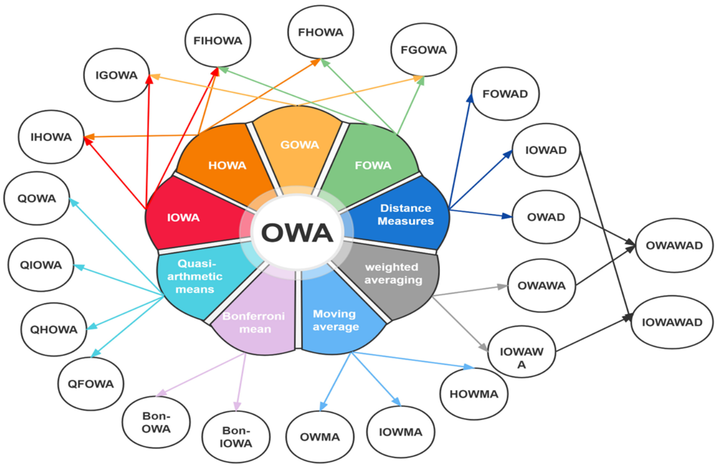

Fig.4 Different OWA extensions based on OWA formulas

Fig.4 illustrates the different applications of OWA derived from the concepts and formulations of OWA presented previously. It is apparent that, based on the concept of OWA and its formulas, a number of OWA extensions have been produced. One of its extensions can be seen in Fig.4 is the induced OWA operator (IOWA) [5]. Besides, the heavy ordered weighted averaging (HOWA) is useful when some part of the information is independent, whose values of the weighting vector is obtained by adjusting the weighted vector values in Formula 3 from 1 to ∞ [8].

The generalized ordered weighted averaging (GOWA) operator, another extension developed by Yager, can generalize a wide variety of aggregation operations, including the OWA operator and its special cases [9]. Yager also introduced a mixing of the t-norms with the OWA operator (TOWA) [10], and gave the central scores with the most weight in the OWA operator, forming the Centered OWA (COWA) [11].

Other authors extended OWA by using uncertain situations where the information can be assessed with fuzzy numbers [3-4, 12] and linguistic variables [13], such as the fuzzy ordered weighted averaging (FOWA).

Additional extensions have been developed by integrating into one formulation the extensions of OWA operators previously introduced. For example, the HOWA operator has been extended by using order-inducing variables (IHOWA) [14] and fuzzy numbers (FHOWA) [15]. In the same way, the GOWA operator has its variation such as the fuzzy generalized ordered weighted averaging (FGOWA), the induced generalized ordered weighted averaging (IGOWA) [14] and so on. Furthermore, Merigo & Casanovas introduced the fuzzy induced heavy OWA (FIHOWA) operator by combining the characteristics of the FOWA, IOWA, and HOWA operators into a single operator, which takes into account the different results that may come out in a given situation using fuzzy information provided in the form of interval numbers with an induced order [15].

The OWA operator can be further extended by using quasi-arithmetic means, which is called Quasi-OWA operator [2]. Other quasi-arithmetic means extensions of the OWA operator can be found in [15-17]. Additionally, the weighted averages can be used to create more expansions of the OWA operator. The ordered weighted averaging–weighted averaging (OWAWA or POWA) operator includes two different vectors: the first is the conventional weighting vector of OWA operator, while the second is a probabilistic operator based on the likelihood that the occasions occurred [18]. Similarly, the induced ordered weighted averaging – weighted averaging (IOWAWA or IPOWA) operator can be obtained by unifying the weighted averaging and the IOWA operator [19]. Other generalizations of the OWA operator by using weighted averages can be found in [20-22].

Some other studies unify the OWA with the power average [23-25], moving averages [26-28], distances measures [14-15, 19, 29-33], bonferroni means [34-35], and a set of sums [36-37]. Obviously, it can create other new extensions of the OWA by including some or all of the different measures in the same formulation. Note that the bonferroni probabilistic ordered weighted averaging (Bon-POWA or Bon-OWAWA) can be obtained by combining vectors of weights considered by the degree of importance for decision-makers, and the probability in the same formulation [35,37].

In general, the literature has a wide range of OWA operator extensions, most of which are based on Yager and Merigó's concepts. As a result, by including other measurements into the OWA operator's formula, more and more extensions can be added.

IV. Application of the OWA in different fields

The OWA operator is a useful tool for decision-making [1], which has become widely used in a variety of fields. By analyzing the papers published in recent 7 years, Fig.5 demonstrates top 10 areas in terms of the number of articles published on the web of science collection, with a total of 2154 publications.

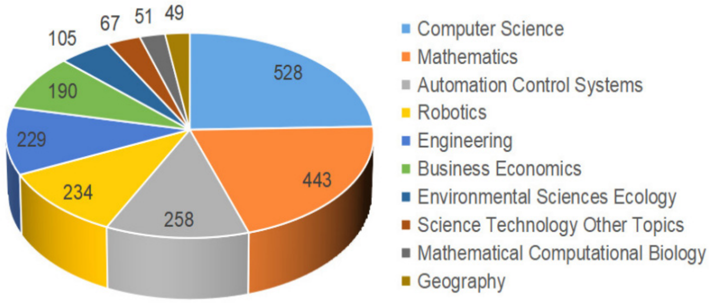

Fig.5 Distribution of the research areas (2014-2021)

In Fig.5, it is obvious that the number of publications published about the OWA's application in computer science is roughly a quarter of the total, followed closely by mathematics (20.67%). However, in other fields, such as automation control systems (11.98%), robotics (10.86%), engineering (10.63%), business economics (8.82%) and so on, have substantially fewer publications about application of OWA.

OWA operators and their extensions have been applied in recent years. In the business area, it can be used to determine the weight of the recency, frequency, and monetary value in the RFM model to segment customers by its RFM score [38]. The study of Teresa and Evangelos showed the possibility to apply OWA in the SERVQUAL method to assess customer satisfaction with logistics services attributes [39]. Combining OWA with the induced variables and picture fuzzy, the picture fuzzy induced OWA (PFIOWA) operators can be obtained to evaluate the enterprise marketing capability with picture fuzzy information [40].

In computer science, it can be applied in recommender systems. One study has already demonstrated its applicability in the book recommendation technique by integrating the positional aggregation based scoring (PAS) technique with OWA to assign weights to book's scores and find the top books [41]. In another article about the mobile phone recommender system, the weights of fuzzy ordered weighted averaging (OWA) have been linked with gray relational analysis to calculate the recommended F1 indices [42], which has increased accuracy of recommendation results. In some other cases, it can also be used for video face recognition [43] or emotion-driven polarity detection [44].

In economics, the moving averages with heavy aggregation (HOWMA) operators, one of the OWA extensions, can be applied in the econometric forecasting to unify the historical data with the information available and the knowledge of the decision-maker [28]. Another application of OWA is in the calculation of the Human Development Index (HDI), which uses the prioritized induced ordered weighted geometric average (PIOWGA) operator to consider the three components of the HDI [45]. Similar to its use in determining the HDI, another extension of OWA, the prioritized IOWAWA (PIOWAWA) operator can be employed to consider the level of importance, reordering and weight factors assigned to the information in the same formulation by the decision maker [46].

In environment and natural resources, OWA and its extensions can be applied in resource allocation problems [47-50]. In the engineering field, Zhou applied OWA operator to evaluate the e-marketing performance of different enterprises, compared its evaluation value and generated the corresponding strategy [51]. In finance, OWA can be applied in the stock selection. Amin and Hajjami used it to involve optimistic and pessimistic scenarios in data envelopment analysis (DEA) methods [52], so that to generate interval OWA scores for all stocks and choose the best stock. In another study, Amin and Hajjami applied it in the linear programming model to select superior stocks to invest without requiring any re-ordering process [53].

In geography, Kiavarz & Jelokhani-Niaraki applied OWA in the geographical information system (GIS) method to generate geothermal prospectivity maps considering the concept of risk [54]. Xiao et al. employed OWA to modify the application of fuzzy AHP in GIS method [55]. Firozjaei et al. used OWA in GIS analysis to discover the optimal areas for solar power plants installation [56]. In another study, it combined GIS method with a modified version of the analytic hierarchy process based on the OWA technique to assess citrus cropland location [57].

In mathematics and statistics, OWA and its extensions, including IOWA, POWA, HOWA, IPOWA, IHOWA are used to create new volatility formulas [45]. In the case of classification, it has been applied to rewrite the hinge loss function [58], or to cope with a cost-sensitive expansion of the conventional Support Vector Machine [59].

To sum up, it is far feasible to use OWA operators in different fields which include business, computer science, economics, environment and natural resource, engineering, finance, geography, mathematics, statistics, etc. The main application of OWA is to select the optimal strategy or plan by calculating the weight or importance of different criteria and considering the attitudinal character or degree of optimism of the decision-maker. Many OWA extension models have been proposed since 1988, and the conventional OWA model remains to be used directly by many researchers in their works.

V. Conclusion

Since the OWA operator was launched in 1990s, it has been applied by several researchers in various research fields. This study investigates the development of OWA operators from its introduction until 2021. Based on the science mapping analysis on 19,370 bibliographic records acquired from the WoS, it reveals that the OWA operators have gained increasing attention over the last few years, with the main use in the decision-making system.

Besides, a brief overview of the OWA formulas and its various types has been made by analyzing some representative articles in the literature. Even though many authors extended the OWA operator by using various methods such as order-inducing variables, fuzzy numbers, quasi-arithmetic means, weighted averages, moving averages, distance measures, bonferroni means, and so on, their methods were developed primarily based on the concepts introduced by Yager and Merigó. This has proved the possibility that OWA can be flexibly combined with various mathematical concepts to create more extensions.

In order to comprehensively understand the application of OWA operators in different fields, the publications in the last 7 years have been involved for further analysis. Ten areas have been summarized in terms of the number of papers published on the web of science collection. The field of computer science has the most publications, followed closely by mathematics. The result of analyzing the applications of OWA and its expansions in the literatures indicates that the main use of OWA operators is to select the optimal strategy or plan by calculating the weight or importance of different criteria and taking into account the attitudinal character or degree of optimism of the decision-maker.

References

[1] Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision-making. IEEE Transactions on Systems, Man, and Cybernetics, 18(1):183-190.

[2] Fodor, J., Marichal, J.-L., & Roubens, M. (1995). Characterization of the ordered weighted averaging operators. IEEE Transactions on Fuzzy Systems, 3(2): 236-240.

[3] Herrera, F., & Herrera-Viedma, E. (1997). Aggregation Operators for linguistic weighted information. Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, 27: 646-656.

[4] Mitchell, H. B., & Estrakh, D. D. (1998). An OWA operator with fuzzy ranks. International Journal of Intelligent Systems, 13(1): 69-81.

[5] Yager, R. R., & Filev, D. P. (1999). Induced ordered weighted averaging operators. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 29(2): 141-150.

[6] Zadeh, L. A. (1983). A computational approach to fuzzy quantifiers in natural languages. Computers & Mathematics with Applications, 9(1): 149-184.

[7] Yager. (1996). Quantifier guided aggregation using OWA operators. International Journal of Intelligent Systems, 11(1): 49-73.

[8] Yager, R. R. (2003). Monitored heavy fuzzy measures and their role in decision making under uncertainty. Fuzzy Sets and Systems, 139(3): 491-513.

[9] Yager, R. R. (2004). Generalized OWA Aggregation Operators. Fuzzy Optimization and Decision Making, 3(1): 93-107.

[10] Yager, R. (2005). Extending multicriteria decision making by mixing t‐norms and OWA operators. International Journal of Intelligent Systems, 20(4): 453–474

[11] Yager, R. R. (2007). Centered OWA Operators. Soft Computing - A Fusion of Foundations, Methodologies and Applications, 11(7): 631-639.

[12] Xu, Z. S., & Da, Q. L. (2002). The uncertain OWA operator. International Journal of Intelligent Systems, 17(6): 569 - 575

[13] Merigó, J. M., & Gil-Lafuente, A. M. (2009). The induced 2-tuple linguistic generalized OWA operator and its application in linguistic decision making (Working Papers in Economics N.º 232). Universitat de Barcelona. Espai de Recerca en Economia.

[14] Merigó, J. M., & Casanovas, M. (2010a). Induced and heavy aggregation operators with distance measures. Journal of Systems Engineering and Electronics, 21(3): 431-439.

[15] Merigo, J. M., & Casanovas, M. (2010b). Fuzzy Generalized Hybrid Aggregation Operators and its Application in Fuzzy Decision Making. International Journal of Fuzzy Systems, 12(1): 15-24.

[16] Merigo, J. M., & Casanovas, M. (2010c). The Fuzzy Generalized OWA Operator and Its Application in Strategic Decision Making. Cybernetics and Systems, 41: 359-370.

[17] Merigo, J. M., & Casanovas, M. (2010d). Induced and uncertain heavy OWA operators. https://www.studocu.com/es-mx/document/universidad-tecmilenio/finanzas-internacionales/caie-2011-iuhowa/5817073

[18] Merigo, J. M. (2009). On the Use of the OWA Operator in the Weighted Average and its Application in Decision Making. In: World Congress on Engineering, London. pp.82-87

[19] Merigó, J. M., & Gil-Lafuente, A. M. (2010). DECISION MAKING WITH DISTANCE MEASURES, WEIGHTED AVERAGES AND INDUCED OWA OPERATORS. In: XV Spanish Congress on Fuzzy Logic and Technologies (ESTYLF), Huelva. pp. 291-296.

[20] Merigó, J. M., & Casanovas, M. (2010e). USING HEAVY AGGREGATIONS IN A UNIFIED MODEL BETWEEN THE WEIGHTED AVERAGE AND THE OWA OPERATOR. In: XV Spanish Congress on Fuzzy Logic and Technologies (ESTYLF), Huelva. pp. 285-290.

[21] Merigó, J. M., & Wei, G. (2011). Probabilistic aggregation operators and their application in uncertain multi-person decision-making. Technological and Economic Development of Economy, 17(2): 335-351.

[22] Zeng, S., Merigó, J. M., & Su, W. (2013). The uncertain probabilistic OWA distance operator and its application in group decision making. Applied Mathematical Modelling, 37(9): 6266-6275.

[23] Yager, R. R. (2001). The power average operator. IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans, 31(6): 724–731

[24] Song, Y., & Deng, Y. (2019). A new soft likelihood equation based on power ordered weighted average operator. International Journal of Intelligent Systems, 34(11): 2988-2999.

[25] Chen, Z., Yang, L., Rodríguez, R. M., Xiong, S., Chin, K., & Martínez, L. (2021). Power‐average‐operator‐based hybrid multiattribute online product recommendation model for consumer decision‐making. International Journal of Intelligent Systems, 36(6): 2572-2617.

[26] Yager, R. (2008). Time Series Smoothing and OWA Aggregation. Fuzzy Systems, IEEE Transactions on, 16: 994-1007.

[27] Merigo, J. M., & Yager, R. (2013). Generalized moving averages, distance measures and OWA operators. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 21 (4): 533−559.

[28] León-Castro, E., Avilés-Ochoa, E., Merigó, J. M., & Gil-Lafuente, A. M. (2018). Heavy Moving Averages and Their Application in Econometric Forecasting. Cybernetics and Systems, 49(1): 26-43.

[29] Merigo, J. M., & Casanovas, M. (2010f). Decision Making with Distance Measures and Linguistic Aggregation Operators. International Journal of Fuzzy Systems, 12: 190-198.

[30] Merigó, J. M., & Gil-Lafuente, A. M. (2008a). Using the OWA Operator in the Minkowski Distance. International Journal of Economics and Management Engineering, 2(9): 1032-1040.

[31] Merigo, J. M., & Gil-Lafuente, A. M. (2008b). On the Use of the OWA Operator in the Euclidean Distance. International Journal of Electrical and Systems Engineering, 2: 170-176.

[32] Merigo, J. M., Casanovas, M., & Zeng, S. (2014). Distance measures with heavy aggregation operators. Applied Mathematical Modelling, 38: 3142-3153.

[33] Yusoff, B., Merigó, J. M., & Hornero, D. C. (2018). Analysis on Extensions of Multi-expert Decision Making Model with Respect to OWA-Based Aggregation Processes. En A. M. Gil-Lafuente, J. M. Merigó, B. K. Dass, & R. Verma (Eds.), Applied Mathematics and Computational Intelligence, 730:179-196.

[34] Yager, R. R. (2009). On generalized Bonferroni mean operators for multi-criteria aggregation. International Journal of Approximate Reasoning, 50(8): 1279-1286.

[35] Blanco-Mesa, F., León-Castro, E., Merigó, J. M., & Xu, Z. (2019).

Bonferroni means with induced ordered weighted average operators. International Journal of Intelligent Systems, 34(1): 3-23.

[36] Merigo, J. M., Alrajeh, N., & Peris-Ortiz, M. (2016). Induced aggregation operators in the ordered weighted average sum. 2016 IEEE Symposium Series on Computational Intelligence (SSCI), 1-6.

[37] Espinoza-Audelo, L. F., Olazabal-Lugo, M., Blanco-Mesa, F., León-Castro, E., & Alfaro-Garcia, V. (2020). Bonferroni Probabilistic Ordered Weighted Averaging Operators Applied to Agricultural Commodities' Price Analysis. Mathematics, 8(8): 1350.

[38] Cao, J., Yu, X., & Zhang, Z. (2015). Integrating OWA and data mining for analyzing customers churn in E-commerce. Journal of Systems Science and Complexity, 28(2): 381-392.

[39] Teresa, G., & Evangelos, G. (2015). Importance of logistics services attributes influencing customer satisfaction. In: 2015 4th International Conference on Advanced Logistics and Transport (ICALT). Valenciennes. pp.53-58.

[40] Li, D.-X., Dong, H., & Jin, X. (2017). Model for evaluating the enterprise marketing capability with picture fuzzy information. Journal of Intelligent & Fuzzy Systems, 33(6): 3255-3263.

[41] Sohail, S. S., Siddiqui, J., & Ali, R. (2015). OWA based Book Recommendation Technique. Procedia Computer Science, 62: 126-133.

[42] Wang, S.-T., & Li, M.-H. (2018). Mobile Phone Recommender System Using Information Retrieval Technology by Integrating Fuzzy OWA and Gray Relational Analysis. Information, 9: 326.

[43] Rivero-Hernández, J., Morales-González, A., Denis, L. G., & Méndez-Vázquez, H. (2021). Ordered Weighted Aggregation Networks for Video Face Recognition. Pattern Recognition Letters, 146: 237-243.

[44] Serrano-Guerrero, J., Romero, F. P., & Olivas, J. A. (2021). Ordered Weighted Averaging for Emotion-Driven Polarity Detection. Cognitive Computation, 13(1): 1-18.

[45] Leon-Castro, E., Blanco-Mesa, F., Romero-Serrano, A. M., & Velázquez-Cazares, M. (2021). The Ordered Weighted Average Human Development Index. Axioms, 10(2): 87.

[46] Perez-Arellano, L. A., Leon-Castro, E., Blanco-Mesa, F., & Fonseca-Cifuentes, G. (2021). The ordered weighted government transparency average: Colombia case. Journal of Intelligent & Fuzzy Systems, 40(2): 1837-1849.

[47] Mianabadi, H., Sheikhmohammady, M., Mostert, E., & Van de Giesen, N. (2014). Application of the Ordered Weighted Averaging (OWA) method to the Caspian Sea conflict. Stochastic Environmental Research and Risk Assessment, 28(6): 1359-1372.

[48] Llopis-Albert, C., & Palacios-Marques, D. (2017). Applications of ordered weighted averaging (OWA) operators in environmental problems. Multidisciplinary Journal for Education, Social and Technological Sciences, 4(1): 52-63.

[49] Llopis-Albert, C., Merigó, J. M., Liao, H., Xu, Y., Grima-Olmedo, J., & Grima-Olmedo, C. (2018). Water Policies and Conflict Resolution of Public Participation Decision-Making Processes Using Prioritized Ordered Weighted Averaging (OWA) Operators. Water Resources Management, 32(2): 497-510.

[50] Mijani, N., Alavipanah, S. K., Hamzeh, S., Firozjaei, M. K., & Arsanjani, J. J. (2019). Modeling thermal comfort in different condition of mind using satellite images: An Ordered Weighted Averaging approach and a case study. Ecological Indicators, 104: 1-12.

[51] Zhou, F. (2015). Enterprise E-marketing Performance Evaluation Based on Interval-valued Intuitionistic Fuzzy Sets OWA Operator. Chemical Engineering Transactions, 46: 637-642.

[52] Amin, G. R., & Hajjami, M. (2016). Application of Optimistic and Pessimistic OWA and DEA Methods in Stock Selection: OPTIMISTIC AND PESSIMISTIC OWA AND DEA METHODS FOR STOCK. International Journal of Intelligent Systems, 31: 1220-1233.

[53] Hajjami, M., & Amin, G. R. (2018). Modelling stock selection using ordered weighted averaging operator. International Journal of Intelligent Systems, 33(11): 2283-2292.

[54] Kiavarz, M., & Jelokhani-Niaraki, M. (2017). Geothermal prospectivity mapping using GIS-based Ordered Weighted Averaging approach: A case study in Japan's Akita and Iwate provinces. Geothermics, 70: 295-304.

[55] Xiao, Y., Yi, S., & Tang, Z. (2017). Integrated flood hazard assessment based on spatial ordered weighted averaging method considering spatial heterogeneity of risk preference. Science of The Total Environment, 599-600: 1034-1046.

[56] Firozjaei, M. K., Nematollahi, O., Mijani, N., Shorabeh, S. N., Firozjaei, H. K., & Toomanian, A. (2019). An integrated GIS-based Ordered Weighted Averaging analysis for solar energy evaluation in Iran: Current conditions and future planning. Renewable Energy, 136: 1130-1146.

[57] Zabihi, H., Alizadeh, M., Kibet Langat, P., Karami, M., Shahabi, H., Ahmad, A., Nor Said, M., & Lee, S. (2019). GIS Multi-Criteria Analysis by Ordered Weighted Averaging (OWA): Toward an Integrated Citrus Management Strategy. Sustainability, 11(4): 1009.

[58] Maldonado, S., Merigó, J., & Miranda, J. (2018). Redefining support vector machines with the ordered weighted average. Knowledge-Based Systems, 148: 41-46.

[59] Marín, A., Martínez-Merino, L. I., Puerto, J., & Rodríguez-Chía, A. M. (2021). The soft-margin Support Vector Machine with ordered weighted average. Knowledge-Based Systems, 107705.