# Summary on basic time series studies

tensor data analysis with different data types

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### 1 High-dimentional $\alpha$ -PCA method

#### 1.1 Overall Summary

This article considers the estimation and inference of the **low rank** components in high-dimentional matrixvariate models(tensor), and we propose an estimation method called  $\alpha$ -PCA and it has some benefits with the high dimensions data favorably compared with other methods(traditional PCA, etc) based on the performance in the simulation.

#### 1.2 Main model

The model is shown as the following:

$$\mathbf{Y}_t = \mathbf{R}\mathbf{F}_t\mathbf{C}^T + \mathbf{E}_t$$

 $\mathbf{Y_t}: \mathbf{Y_t} \in \mathbb{R}^{p \times q}, \ 1 \le t \le T, \text{ observations},$ 

 $\mathbf{F_t}: \mathbf{F_t} \in \mathbb{R}^{k \times r}$ , where  $k \ll p$  and  $r \ll q$  (low rank), latent matrix,

 $\mathbf{E_t} : \mathbf{E_t} \in \mathbb{R}^{p \times q}$ , noise matrix.

#### 1.3 Main Statistics

An estimation procedure, namely  $\alpha$ -PCA, aggregates the information in both first and second moments. Specifically, the two statistics are defined:

$$\widehat{\mathbf{M}}_{R} \stackrel{\Delta}{=} \frac{1}{pq} \left( (1+\alpha) \cdot \overline{\mathbf{Y}} \overline{\mathbf{Y}}^{T} + \frac{1}{T} \sum_{t=1}^{T} (\mathbf{Y}_{t} - \overline{\mathbf{Y}}) (\mathbf{Y}_{t} - \overline{\mathbf{Y}})^{T} \right)$$

$$\widehat{\mathbf{M}}_C \stackrel{\triangle}{=} \frac{1}{pq} \left( (1+\alpha) \cdot \overline{\mathbf{Y}}^T \overline{\mathbf{Y}} + \frac{1}{T} \sum_{t=1}^T (\mathbf{Y}_t - \overline{\mathbf{Y}})^T (\mathbf{Y}_t - \overline{\mathbf{Y}}) \right)$$

 $\alpha: \alpha \in [-1, +\infty)$ , a hyperparameter,

$$\overline{\mathbf{Y}} = \frac{1}{T} \sum_{i=1}^{T} \mathbf{Y}_{t}$$
, the sample mean.

Based on these two statistics, estimation of  $\mathbf{R}$  and  $\mathbf{C}$  can be obtained as  $\sqrt{p}$  times the top k eigenvectors of  $\widehat{\mathbf{M}}_{C}$  and  $\sqrt{q}$  times the top q eigenvectors of  $\widehat{\mathbf{M}}_{C}$  respectively, in descending order by corresponding eigenvalues.

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<sup>&</sup>lt;sup>1</sup>Generalized Factor Model