STOR 556: Time Series **Spring 2026**

Homework 2

Instructor: Hang Zhou *Due date: 11:00 am, Feb 3*

Homework is a crucial step in your learning journey for this course. I strongly suggest you spend time on it and complete it independently.

Question 1: Let X and Y be two random variables with $\mathbb{E}(Y) = \mu$ and $\mathbb{E}(Y^2) < \infty$.

(a) Show that the constant c that minimizes $\mathbb{E}(Y - c)^2$ is $c = \mu$.

(b) Deduce that the random variable $f(X)$ that minimizes $\mathbb{E}[(Y - f(X))^2 \mid X]$ is $f(X) = \mathbb{E}[Y \mid X]$.

(c) Deduce that the random variable $f(X)$ that minimizes $\mathbb{E}(Y - f(X))^2$ is also $f(X) = \mathbb{E}[Y \mid X]$.

Question 2: Suppose that $X_1, X_2, \ldots$ is a sequence of random variables with $\mathbb{E}(X_t^2) < \infty$ and $\mathbb{E}(X_t) = \mu$.

(a) Show that the random variable $f(X_1, \ldots, X_n)$ that minimizes the conditional mean squared error

$$\mathbb{E}\big[(X_{n+1} - f(X_1, \ldots, X_n))^2 \mid X_1, \ldots, X_n\big]$$

is

$$f(X_1, \ldots, X_n) = \mathbb{E}[X_{n+1} \mid X_1, \ldots, X_n].$$

(b) Deduce that the random variable $f(X_1, \ldots, X_n)$ that minimizes the unconditional mean squared error

$$\mathbb{E}\big[(X_{n+1} - f(X_1, \ldots, X_n))^2\big]$$

is also

$$f(X_1, \ldots, X_n) = \mathbb{E}[X_{n+1} \mid X_1, \ldots, X_n].$$

(c) If $X_1, X_2, \ldots$ are i.i.d. with $\mathbb{E}(X_i^2) < \infty$ and $\mathbb{E}X_i = \mu$, where μ is known, what is the minimum mean squared error predictor of X_{n+1} in terms of $X_1, \ldots, X_n$?

Question 3: Let $\{Z_t\}$ be a sequence of independent normal random variables, each with mean 0 and variance σ^2, and let a, b, and c be constants. Which, if any, of the following processes are stationary? For each stationary process specify the mean and autocovariance function.

(a) $X_t = a + bZ_t + cZ_{t-2}$

(b) $X_t = Z_1 \cos(ct) + Z_2 \sin(ct)$

(c) $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$

(d) $X_t = a + bZ_0$

(e) $X_t = Z_0 \cos(ct)$

(f) $X_t = Z_t Z_{t-1}$

Question 4: This problem requires using the ITSM package.

If you are using Windows PC, having installed ITSM package and with the tutorial (Appendix E) in hand, do the following. Produce time plots of the time series `airpass.tsm`. Print it out, for example, by copying it to clipboard and then pasting it to an MSWord document. Next to each plot write which types of variations (trend, cyclic variations or irregular fluctuations) the time series exhibit. Which types of variations, in your opinion, might be of interest when analyzing this time series? Short description of the time series can be found in the textbook (Examples 9.5.2.)

For R users: Once you start R, load `itsmr` package. The series `airpass.tsm` can be plotted with `plotc(airpass)`. You can save the plot as `.jpeg` file.