A Stochastic Model of n-Day Precipitation

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ABSTRACT

General expressions are derived for the distribution functions of the total amount of precipitation and the largest daily precipitation occurring in an n-day period. Two special cases are considered: (i) the probability of occurrence of precipitation on any day in an n-day period is a constant (binomial counting process) and (ii) the probability of occurrence of precipitation on any day depends on whether the previous day was wet or dry (Markov chain counting process). The distribution function for daily precipitation was assumed to be exponential. Analytic expressions are derived for the distribution functions for total precipitation or precipitation greater than a threshold. For the numerical example chosen, the Markov chain-exponential model is slightly superior to the binomial-exponential model. This stochastic model seems to have several advantages over present approaches.

L: precipitation means the distribution function P(w) change from the uniform to \delta-like.

1. Introduction

This paper is concerned with a stochastic model for the description and analysis of certain aspects of the random structure of the precipitation phenomenon, utilizing daily precipitation records. Such a model is needed in the evaluation of excessive discharges or droughts, and in the development and management of water resource systems.

A rainfall model based on daily precipitation is attractive because relatively long and reliable records are readily available and such a model is frequently sufficient for many practical problems.

If the following, an attempt is made to determine the distribution function of (i) the total amount of precipitation in a given period of time, and (ii) the largest daily value of precipitation in the same period.

The model adopted here to study these problems does not provide the relation between the rainfall itself and various climatological factors such as temperature, wind and origin of air masses. Therefore, it cannot physically explain various features of the rainfall phenomenon.

2. Definitions

In the following we define the most important notation used in this paper. Consider a certain period of time that consists of n days. With each day of this n-day period, we associate a random variable η_j which may assume only two values: 0 if the day is dry and 1 if the day is wet. In other words, for the jth day of the n-day period, the associated random variable η_j is defined as

$$\eta_j=\begin{cases}1, & \text{if } j\text{th day is wet}\\ 0, & \text{if } j\text{th day is dry}\end{cases} \tag{1}$$

where $j=1, 2,\ldots, n$.

If N_n stands for the number of rainy days in the n-day period, then

$$N_n=\sum_{j=1}^{n}\eta_j. \tag{2}$$

It is apparent that the possible values of the random variable N_n are $0, 1,\ldots, n$.

We denote by ξ_ν the daily value of precipitation of the νth rainy day of the n-day period [notice that the νth rainy day may be any day after the $(\nu-1)$th day of the period]. According to these definitions, the total amount of precipitation $S(n)$ during this period is equal to

$$S(n)=\sum_{\nu=0}^{N_n}\xi_\nu,\quad \xi_0\equiv 0. \tag{3}$$

From this it immediately follows that

$$P\{S(n)=0\}=P\{\eta_1=0, \eta_2=0,\ldots, \eta_n=0\},$$

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and that

$$P\{S(n)>0\}=1-P\{S(n)=0\}.$$

Let $\chi(n)$ denote the largest daily value of precipitation in the n-day period, i.e.,

$$\chi(n)=\sup\xi_\nu,\ 0\leqslant\nu\leqslant N_n. \tag{4}$$

It is apparent that

$$P\{\chi(n)=0\}=P\{\eta_1=0,\eta_2=0,\ldots,\eta_n=0\},$$
$$P\{\chi(n)>0\}=1-P\{\chi(n)=0\}.$$

From these definitions [(3) and (4)] it follows that both $S(n)$ and $\chi(n)$ are discrete parameter stochastic processes such that $0\leqslant S(1)\leqslant S(2)\leqslant\ldots$ and $0\leqslant\chi(1)\leqslant\chi(2)\leqslant\ldots$. In the following, provided certain regularity assumptions hold, the one-dimensional distribution function and mathematical expectation of $S(n)$ and $\chi(n)$ are determined.

3. On the process $\chi(n)$

By definition, the sequence of events $\{N_n=0\}$, $\{N_n=1\},\ldots,\{N_n=n\}$ represents a finite partition of the sample space. It means that

$$\{N_n=i\}\cap\{N_n=j\}=\theta \quad\text{and}\quad \sum_{\nu=0}^{n}P\{N_n=\nu\}=1 \tag{5}$$

for every $i\neq j$ where θ denotes the empty set. In the following we assume that:

1) $\xi_1,\xi_2,\ldots,\xi_n$ are independent, identically distributed random variables with $H(x)=P\{\xi_\nu\leqslant x\}$ where $k=N_n\leqslant n$.
2) $\xi_1,\xi_2,\ldots,\xi_n$ are independent of N_n.

On the basis of these two regularity conditions, we shall determine the distribution function and mathematical expectation of $\chi(n)$ in terms of the distribution function $H(x)$ and the probabilities $P\{N_n=k\}$ and write

$$\phi_n(x)=P\{\chi(n)\leqslant x\},\ x\geqslant 0.$$

Then, on the basis of (4) and (5) we have

$$\begin{aligned}\phi_n(x)&=P\{\chi(n)\leqslant x,\ \bigcup_{k=0}^{n}\{N_n=k\}\},\\&=\sum_{k=0}^{n}P\{\sup\xi_\nu\leqslant x,\ N_n=k\},\quad 0\leqslant\nu\leqslant N_n,\\&=\sum_{k=0}^{n}P\{\sup\xi_\nu\leqslant x\}P\{N_n=k\},\quad 0\leqslant\nu\leqslant k.\end{aligned}$$

Taking into account conditions 1) and 2) we obtain

$$\begin{aligned}\phi_n(x)&=P\{N_n=0\}+\sum_{k=1}^{n}P\{\sup\xi_\nu\leqslant x\}P\{N_n=k\},\\&\qquad\qquad 1\leqslant\nu\leqslant k,\\&=P\{N_n=0\}+\sum_{k=1}^{n}\{H(x)\}^kP\{N_n=k\},\end{aligned} \tag{6}$$

because

$$P\{\sup\xi_\nu\leqslant x\}=\prod_{\nu=1}^{k}P\{\xi_\nu\leqslant x\}=\{H(x)\}^k,\quad 1\leqslant\nu\leqslant k.$$

To compute the mathematical expectation of $\chi(n)$ we consider

$$\begin{aligned}E\{\chi(n)\}&=\int_0^\infty xd\phi_n(x),\\&=\int_0^\infty x\sum_{k=1}^{n}P\{N_n=k\}d\{H(x)\}^k,\\&=\sum_{k=1}^{n}kP\{N_n=k\}\left(\int_0^\infty x\{H(x)\}^{k-1}h(x)dx\right),\end{aligned}$$

where $h(x)=dH(x)/dx$. In the same way one can show that

$$E\{\chi(n)\}^\nu=\sum_{k=1}^{n}kP\{N_n=k\}\left(\int_0^\infty x^\nu\{H(x)\}^{k-1}h(x)dx\right).$$

4. On the process $S(n)$

We denote the distribution function of $S(n)$ by $F_n(x)=P\{S(n)\leqslant x\}$. To determine this distribution function in terms of probabilities of the ξ_ν's and N_n we shall proceed as follows: Bearing in mind relations (5), we have for every $x\geqslant 0$

$$\begin{aligned}P\{S(n)\leqslant x\}&=P\{\sum_{\nu=0}^{N_n}\xi_\nu\leqslant x,\ \bigcup_{k=0}^{n}\{N_n=k\}\},\\&=\sum_{k=0}^{n}P\{\sum_{\nu=0}^{k}\xi_\nu\leqslant x,\ N_n=k\}.\end{aligned}$$

Taking into account condition 2), it follows that

$$P\{S(n)\leqslant x\}=\sum_{k=0}^{n}P\{X_k\leqslant x\}P\{N_n=k\},$$

where

$$X_0=0;\ X_k=\xi_1+\xi_2\cdots+\xi_k,\ k=0,1,\ldots,n,$$

and the distribution function of $S(n)$ is

$$F_n(x)=P\{N_n=0\}+\sum_{k=1}^{n}P\{X_k\leqslant x\}P\{N_n=k\}. \tag{7}$$

It can be readily shown that the mean and variance of $S(n)$ are

$$\left.\begin{aligned}E\{S(n)\}&=\alpha E\{N_n\}\\ \mathrm{Var}\{S(n)\}&=\beta E\{N_n\}+\alpha^2\,\mathrm{Var}\{N_n\}\end{aligned}\right\}, \tag{8}$$

where $\alpha=E\{\xi_\nu\}$ and $\beta=\mathrm{Var}\{\xi_\nu\}<\infty$.

5. Computation of probabilities $P\{X_k \leqslant x\}$ and $P\{N_n = k\}$

To determine the distribution functions $F_n(x)$ and $\phi_n(x)$ it is necessary to compute the probabilities

$$P\{X_k \leqslant x\} \text{ and } P\{N_n = k\},\ k=0, 1\dots, n.$$

To determine the probability $P\{X_k \leqslant x\}$ we shall assume that the daily amount of precipitation is an exponentially distributed random variable, i.e.,

$$H(x) = 1 - e^{-\lambda x}. \tag{9}$$

From this and the supposition that $\xi_1, \dots, \xi_k$ are independent, it follows that

$$P\{X_k \leqslant x\} = \frac{\lambda^k}{\Gamma(k)} \int_0^x u^{k-1} e^{-\lambda u} du,$$

where $\Gamma(k) = (k-1)!$

To compute the probability $P\{N_n = k\}$ we shall consider the following two cases:

(a) $\eta_1, \eta_2, \dots, \eta_n$ is a sequence of independent random variables with

$$P\{\eta_\nu = 1\} = p,\ 0 < p < 1;\ \nu = 1, 2, \dots, n.$$

Under these assumptions it follows immediately (bearing in mind relation (2)] that

$$P\{N_n = k\} = \binom{n}{k} p^k (1-p)^{n-k}.$$

(b) $\eta_1, \eta_2 \dots, \eta_n$ represents a Markov chain with

$$\left.\begin{aligned} P\{\eta_k = 1 \mid \eta_{k-1} = 0\} = q_0 \\ P\{\eta_k = 1 \mid \eta_{k-1} = 1\} = q_1 \end{aligned}\right\} \tag{10}$$

for all $k = 1, 2, \dots, n$.

The following is a physical interpretation of these assumptions: Case (a) means that the probability of occurrence of precipitation on any day of an n-day period is a constant and does not depend on the occurrence of precipitation on any other day within the period. Case (b) means that the probability of rainfall on any day of the n-day period depends only on whether the previous day was wet or dry. Here η_0, which appears in (10) for $k=1$, denotes the random variable which refers to the day preceding the first day of the n-day period, assuming only two values: 1 if the day was wet and 0 if the day was dry.

Now to compute $P\{N_n = k\}$ we make the definition

$$\psi_1(k,n) = P\{N_n = k \mid \eta_0 = 1\}. \tag{11}$$

Then as was shown by Gabriel (1959) (see also Gabriel and Neuman, 1962) it follows that

$$\Psi_1(k,n) = q_1^k (1-q_0)^{n-k} + \sum_{c=1}^{c_1} \binom{k}{a} \binom{n-k-1}{b-1} \left(\frac{1-q_1}{1-q_0}\right)^b \left(\frac{q_0}{q_1}\right)^a, \tag{12}$$

where

$$c_1 = \begin{cases} n + \frac{1}{2} - |2k - n + \frac{1}{2}|, & \text{if } k < n \\ 0, & \text{if } k = n \end{cases} \tag{13}$$

If we define

$$\Psi_0(k,n) = P\{N_n = k \mid \eta_0 = 0\}, \tag{14}$$

then

$$\Psi_0(k,n) = q_1^k (1-q_0)^{n-k} + \sum_{c=1}^{c_0} \binom{k-1}{b-1} \binom{n-k}{a} \left(\frac{1-q_1}{1-q_0}\right)^a \left(\frac{q_0}{q_1}\right)^b, \tag{15}$$

where

$$c_0 = \begin{cases} n + \frac{1}{2} - |2k - n + \frac{1}{2}|, & \text{if } k > 0 \\ 0, & \text{if } k = 0 \end{cases} \tag{16}$$

The constants a and b are defined as

$$\left.\begin{aligned} a = \inf\{\nu; \nu \geqslant \tfrac{1}{2}(c-1)\} \\ b = \inf\{\nu; \nu > \tfrac{1}{2}c\} \end{aligned}\right\}. \tag{17}$$

On the basis of relations (11) and (14) it follows that the probability of k wet days in an n-day period is equal to

$$P\{N_n = k\} = R\psi_1(k,n) + (1-R)\psi_0(k,n), \tag{18}$$

where

$$R = P\{\eta_0 = 1\}.$$

The expected number of wet days in n days is

$$E\{N_n\} = nQ + (R-Q)\frac{(1-d^n)}{1-d} d,$$

where

$$d = q_1 - q_0;\quad Q = \frac{q_0}{1-d}.$$

6. Some special cases

On the basis of the previous results we can compute the distribution functions $\phi_n(x)$ and $F_n(x)$ in some special cases for every $n = 1, 2, \dots$, including all previously considered cases.

To obtain the distribution function for the extreme daily rainfall, $\phi_n(x)$, for case (a), we utilize Eqs. (6) and (9) to obtain

$$\begin{aligned} \phi_n(x) &= (1-p)^n + \sum_{k=1}^{n} \binom{n}{k} [p(1-e^{-\lambda x})]^k (1-p)^{n-k}, \\ &= (1-pe^{-\lambda x})^n. \end{aligned} \tag{19}$$

For case (b) relations (9), (10) and (18) hold, and using (6), the distribution function $\phi_n(x)$ becomes

$$\phi_n(x) = (1-q_0-Rd)(1-q_0)^{n-1} + \sum_{k=1}^{n} [R\psi_1(k,n) + (1-R)\psi_0(k,n)](1-e^{-\lambda x})^k. \tag{20}$$

It can be easily verified that (7), the distribution function for n-day precipitation, for case (a) under the assumption that (9) holds, becomes

$$F_n(x)=(1-p)^n\left[1+\int_0^x e^{-\lambda u}\frac{\alpha_n(p,u)}{u}du\right], \qquad (21)$$

where

$$\alpha_n(p,u)=\sum_{\nu=1}^{n}\left(\frac{\lambda p u}{1-p}\right)^{\nu}\frac{\binom{n}{\nu}}{\Gamma(\nu)}.$$

We shall call (21) the Binomial-Exponential (BE) precipitation model.

For case (b), provided that Eqs. (9) and (10) hold, one obtains

$$F_n(x)=(1-q_0-Rd)(1-q_0)^{n-1}+\sum_{k=1}^{n}\left[R\psi_1(n,k)+(1-R)\psi_0(n,k)\right]\times\frac{\lambda^k}{k}\int_0^x u^{k-1}e^{-\lambda u}du. \qquad (22)$$

We shall refer to (22) as the Markov Chain-Exponential (MCE) model.

The mean is

$$E\{S(n)\}=\alpha\left\{nQ+(R-Q)\frac{(1-d^n)}{1-d}d\right\}.$$

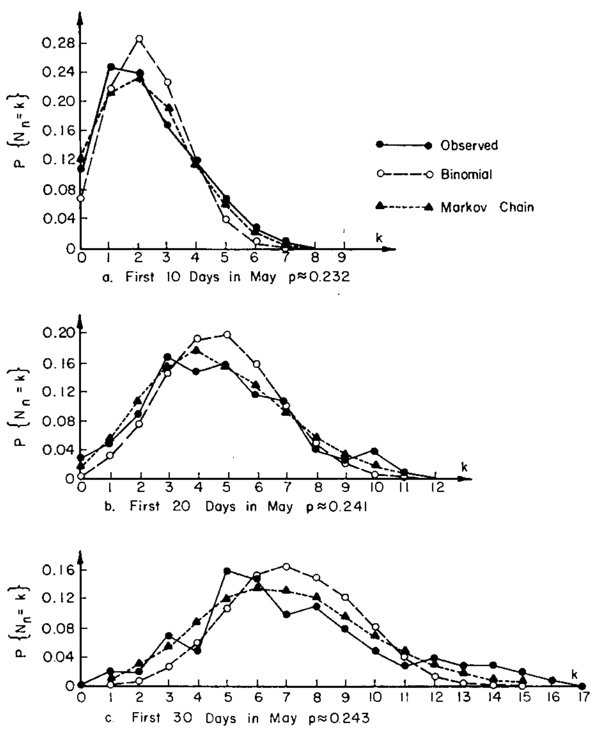

FIG. 1. Observed and theoretical distributions for the number of rainy days in May (Austin, Tex., 1861–1967).

For a more extensive coverage of this particular topic see Todorovic and Woolhiser (1974).

7. The effect of a threshold

Suppose that we only consider precipitation equal to or greater than some threshold T. As a practical matter only precipitation of 0.005 inch or greater is included in rainfall tabulations; smaller amounts are recorded as a trace. As an example, we may assume that precipitation smaller than a given amount is ineffective for plant growth so we are interested in the distribution of n-day precipitation subject to the condition that $\xi_\nu \geqslant T$.

If only precipitation greater than some threshold T is included, the distribution function for daily precipitation is the truncated exponential

$$H(x)=1-e^{-\lambda(x-T)},\quad T\leqslant x\leqslant\infty,$$

and the distribution function of the sum of k daily rainfall amounts is

$$P\{X_k\leqslant x\}=\frac{\lambda^k}{\Gamma(k)}\int_0^{x-kT}u^{k-1}e^{-\lambda u}du,$$

where

$$kT\leqslant x.$$

The above expressions may be substituted into (21) and (22) to obtain distribution functions for the extreme or for the sum of n-day precipitation. The parameter p in the BE model and the parameters R, q_0 and q_1 in the MCE model will also change.

For the BE model let p be as defined previously and let $p_T=P\{\xi_\nu\geqslant T\}p$, where the subscript T denotes the parameter with threshold T; thus,

$$p_T=pe^{-\lambda T}.$$

For the MCE model

$$R_T=Re^{-\lambda T}.$$

For the Markov chain parameters, the transition matrix for $T=0$ is

$$\begin{vmatrix}1-q_0 & q_0\\ 1-q_1 & q_1\end{vmatrix}$$

and the steady-state probabilities are

$$\left.\begin{aligned}\Pi_0=P\{\eta_\nu=0\}&=\frac{1-q_1}{1-q_1+q_0}\\ \Pi_1=P\{\eta_\nu=1\}&=\frac{q_0}{1-q_1+q_0}\end{aligned}\right\}.$$

If we reconstitute the process into a three-state Markov chain with states

$$\left.\begin{aligned}&\mu_\nu=0, \text{ if } \eta_\nu=0\\ &\mu_\nu=1, \text{ if } \eta_\nu=1 \text{ and } \xi_\nu<T\\ &\mu_\nu=2, \text{ if } \eta_\nu=1 \text{ and } \xi_\nu\geqslant T\end{aligned}\right\},$$

TABLE 1. Sample values of Markov chain parameters.

Period	$\hat{q}_0$	$\hat{q}_1$
1st 10 days	0.1837	0.3744
2nd 10 days	0.1921	0.3916
3rd 10 days	0.1727	0.4407
1st 30 days	0.1828	0.4025
$R=0.24$		

we obtain the transition matrix

$$\begin{vmatrix} 1-q_0 & q_0(1-e^{-\lambda T}) & q_0e^{-\lambda T} \\ 1-q_1 & q_1(1-e^{-\lambda T}) & q_1e^{-\lambda T} \\ 1-q_1 & q_1(1-e^{-\lambda T}) & q_1e^{-\lambda T} \end{vmatrix}.$$

We define the new states

$$\eta_\nu'=0, \quad \text{if} \quad \begin{cases} \mu_\nu=0 \\ \mu_\nu=1 \end{cases}$$

$$\eta_\nu'=1, \quad \text{if} \quad \mu_\nu=2.$$

It can be shown that a good approximation to the new transition matrix is

$$\begin{vmatrix} 1-q_0' & q_0' \\ 1-q_1' & q_1' \end{vmatrix},$$

where

$$q_0'=\frac{e^{-\lambda T}[q_0(1-q_1)+q_0q_1(1-e^{-\lambda T})]}{[1-q_1+q_0(1-e^{-\lambda T})]},$$

$$q_1'=q_1e^{-\lambda T}.$$

8. Numerical example

To illustrate the application of the foregoing methods, daily precipitation data for Austin, Tex., will be used. Records are available for the period 1861–1967, with seven years missing.

a. Number of rainy days

The observed and theoretical distributions for the number of rainy days in the first 10-, 20- and 30-day periods in May are presented in Fig. 1. The theoretical values were computed for the binomial and Markov chain models. Sample values of the parameter $\hat{p}$ for the binomial model are shown on the figure and the sample values for the Markov chain parameters are shown in Table 1. All sample values are maximum likelihood estimates.

The Markov chain model gives a better approximation to the number of rainy days than does the binomial which is in agreement with previous studies (Gabriel and Neuman, 1962; Caskey, 1963).

b. Daily precipitation

It was assumed that the effective threshold $T=0$. Observed and computed exponential distribution functions are shown in Fig. 2. The exponential distribution fits the data very well for the first and second 10-day periods. For the third 10-day period, however, the observed frequency of rainfall amounts less than 0.4 inch is greater than the theoretical and the tail of the observed distribution is heavier than the exponential for each period.

c. n-day precipitation

The distribution functions for n-day precipitation were computed for the BE model [Eq. (21)] and the MCE model [Eq. (22)] for the first 10-, 20- and 30-day periods in May and are shown in Figs. 3 and 4, respectively. A Fortran program for the Markov chain model may be obtained from the authors.

The likelihood ratio test was used to determine if the MCE model resulted in an improved fit to the data as compared with the simpler BE model. For this purpose the null parameter space

$$\omega=\{\lambda, p=q_0=q_1\}$$

is tested against the alternative space

$$\Omega=\{\lambda, R, q_0, q_1\}.$$

If the actual parameter space is ω, the approximate large sample distribution of the likelihood ratio test statistic is chi-square with two degrees of freedom. The likelihood functions were computed using maximum likelihood estimates of parameters for the individual components of the model [rather than maximum

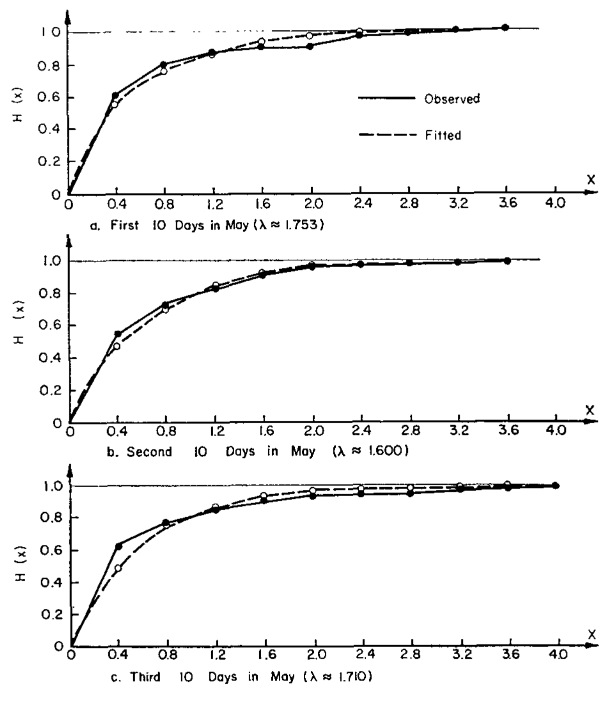

FIG. 2. Observed and theoretical (exponential) distributions of daily amounts of precipitation.

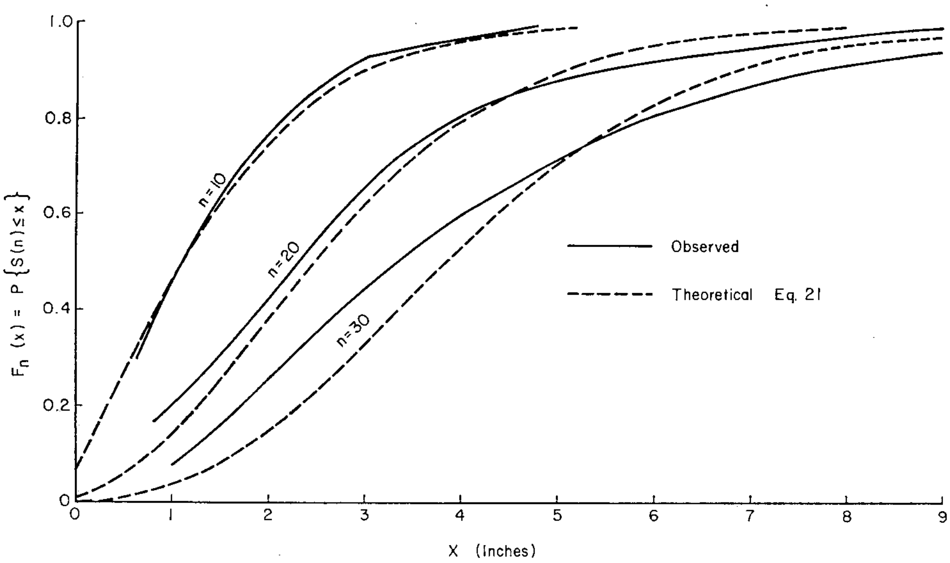

FIG. 3. Observed and theoretical (binomial-exponential) distributions of n-day precipitation.

likelihood estimates based upon (21) and (22)]. For the 10-day period the null hypothesis could not be rejected. For the 20- and 30-day periods the probability of exceeding the test statistic under the null hypothesis was less than 0.005.

These results suggest that the MCE model gives a better fit than the BE model for the distribution of n-day precipitation for the 20- and 30-day periods. Both models are less satisfactory for the 20- and 30-day periods than for the 10-day period possibly because of the less satisfactory fit of the exponential distribution to daily rainfall values for the second and third 10-day periods.

Observed and computed distributions of the n-day rainfall with $T=1.0$ inch are shown in Fig. 5. As might be expected from the structure of the models, there is very little difference between the BE and MCE models for a threshold this large. Both models give a satisfactory fit although the tails of the theoretical distributions are not as heavy as the observed distribution.

d. Extreme precipitation

Observed and computed distribution functions [Eqs. (19) and (20)] for the daily extreme precipitation in

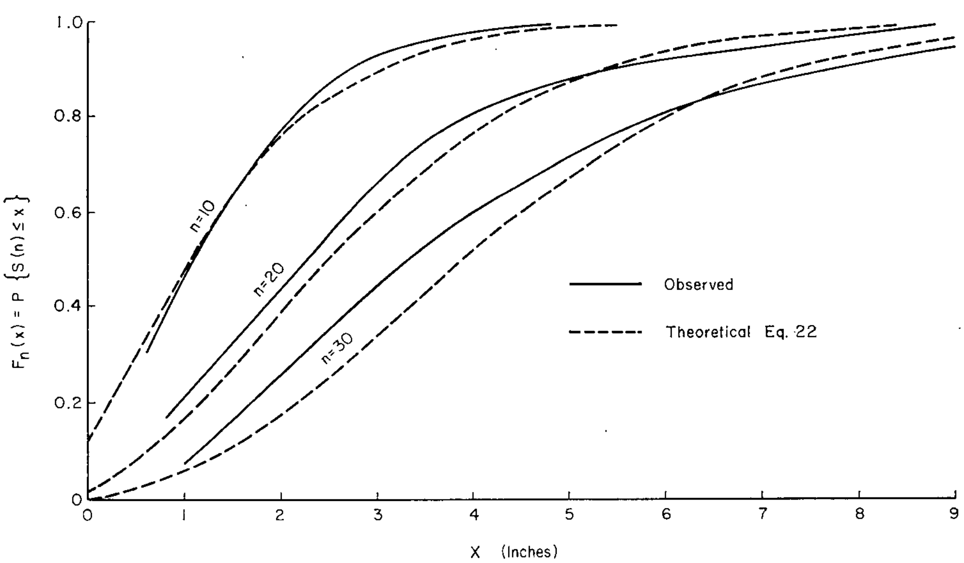

FIG. 4. Observed and theoretical (Markov chain-exponential) distributions of n-day precipitation.

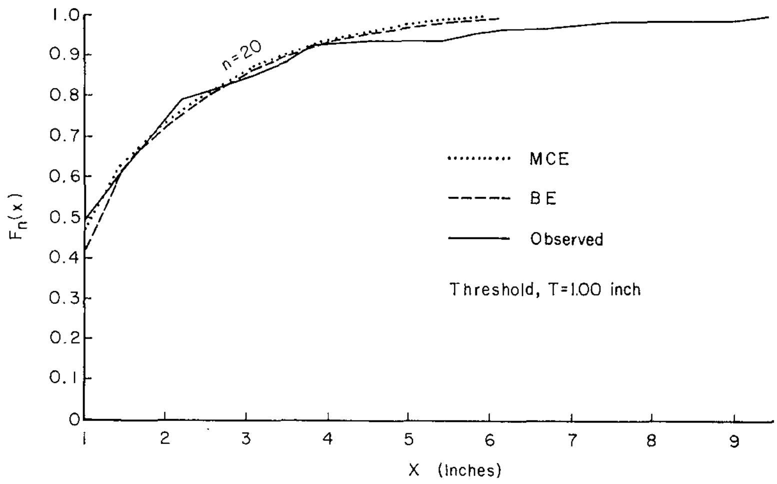

FIG. 5. Observed and theoretical distributions of n-day precipitation greater than a threshold T.

the first 10, 20 and 30 days of May are shown in Fig. 6. The agreement between the observations and both the BE and MCE models is satisfactory for the 10- and 20-day periods but is less satisfactory for the 30-day period. This deviation is probably caused by the inability of the exponential distribution to fit the tail of the distribution of daily precipitation.

9. Summary and conclusions

Based upon a stochastic model of n-day precipitation, general forms for the distribution function, mathematical expectation and variance for the largest daily amount of precipitation $\chi(n)$ and the total amount of precipitation $S(n)$ in an n-day period have been determined. Gringorten (1966) developed a stochastic model for weather phenomena that is similar in some respects to the models considered here; however, he had to resort to Monte Carlo methods to obtain distribution functions.

Two special cases of the general cases given by (6) and (7) were considered: the sequence of daily rainfall occurrences are (a) a sequence of independent, identically distributed random variables and (b) a Markov chain. Assuming that the distribution of daily rainfall amounts is exponential, expressions for the distribution functions of $\chi(n)$ and $S(n)$ are developed for case (a) using The Binomial-Exponential (BE) model and for case (b) the Markov Chain-Exponential (MCE) model, and a numerical example is given. For this example, the MCE model is superior to the BE model.

A model using a distribution for daily precipitation

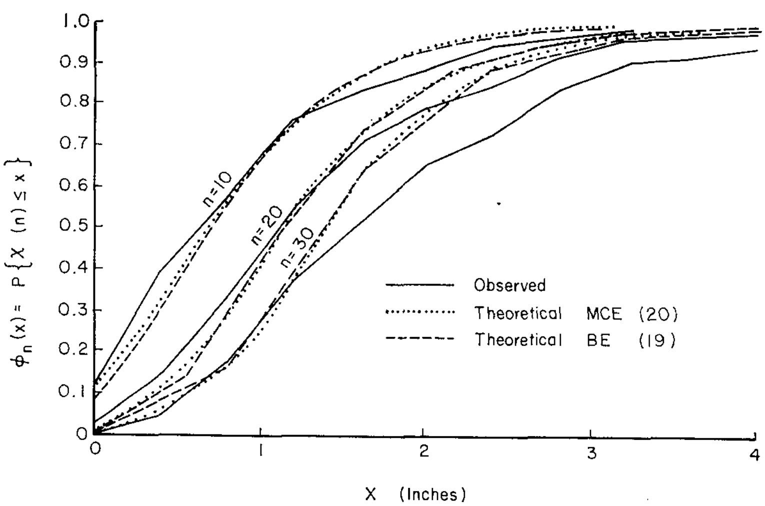

FIG. 6. Observed and theoretical distributions of extreme daily precipitation in an n-day period.

other than the exponential would be more general. This will be investigated in a subsequent paper.

The stochastic model for n-day precipitation proposed here seems preferable to previous approaches [for examples, see Dingens and Steyaert (1971) or Skees and Shenton (1974)] for two reasons: 1) by specifying a logical probabilistic structure, expressions can be derived for several distributions, i.e., n-day precipitation, precipitation above a threshold, extremes, etc., that have common parameters; and 2) these parameters can be readily interpreted in terms of the length of period, the threshold, the nature of the counting process for the number of rainy days, and the distribution of daily rainfall amounts.

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APPENDIX

List of Symbols

a	first integer not smaller than $\frac{1}{2}(c-1)$
b	first integer not smaller than $\frac{1}{2}c$
c	summation index
c_0	upper limit of summation, defined by Eq. 16
c_1	upper limit of summation, defined by Eq. 13
d	q_1-q_0
$E\{X\}$	mathematical expectation of the random variable X
$F_n(x)$	distribution function of $S(n)$
j	day index
k	number of wet days
n	number of days in period under consideration
N_n	number of wet days in the n-day period
q_0	conditional probability that the νth day is wet provided the $(\nu-1)$th day was dry
q_1	conditional probability that the νth day is wet provided the $(\nu-1)$th day was wet
R	$P(\eta_0=1)$
$S(n)$	total amount of precipitation of the n-day period
X_k	total amount of precipitation of k wet days
$\Gamma(\nu)$	the gamma function $(\nu-1)!$
η_j	random variable equal to 1 if the jth day is wet and 0 if the jth day is dry
ξ_ν	amount of precipitation of the νth rainy day in the n-day period
$\phi_n(x)$	distribution function of $\chi(n)$
$\chi(n)$	largest daily amount of precipitation in the n-day period
$\Psi_0(\nu,n)$	probability that in the n-day period, ν days will be wet provided the day preceding the n-day period was dry
$\Psi_1(\nu,n)$	conditional probability of having ν wet days in the n-day period provided the day preceding the n-day period was wet.

REFERENCES

Caskey, James, E., Jr., 1963: A Markov chain model for the probability of precipitation occurrence in intervals of various lengths. *Mon. Wea. Rev.*, **101**, 298–301.

Dingens, P., and H. Steyaert, 1971: Distribution for K-day rainfall totals. *Bull. Intern. Assoc. Sci. Hydrol.*, **16**, No. 3, 19–24.

Gabriel, K. R., 1959: The distribution of the number of successes in a sequence of dependent trials. *Biometrika*, **96**, 959–960.

——, and J. Neuman, 1962: A Markov chain model for daily rainfall in Tel Aviv. *Quart. J. Roy. Meteor. Soc.*, **88**, 90–95.

Gringorten, Irvin I., 1966: A stochastic model of the frequency and duration of weather events. *J. Appl. Meteor.*, **5**, 606–624.

Skees, P., and L. R. Shenton, 1974: Comments on the statistical distribution of rainfall per period under various transformations. *Proc. Symp. Statistical Hydrology*, USDA Misc. Publ. 1275, 172–196.

Todorovic, P., and D. A. Woolhiser, 1974: Stochastic model of daily rainfall. *Proc. Symp. Statistical Hydrology*, USDA Misc. Publ. 1275, 232–246.