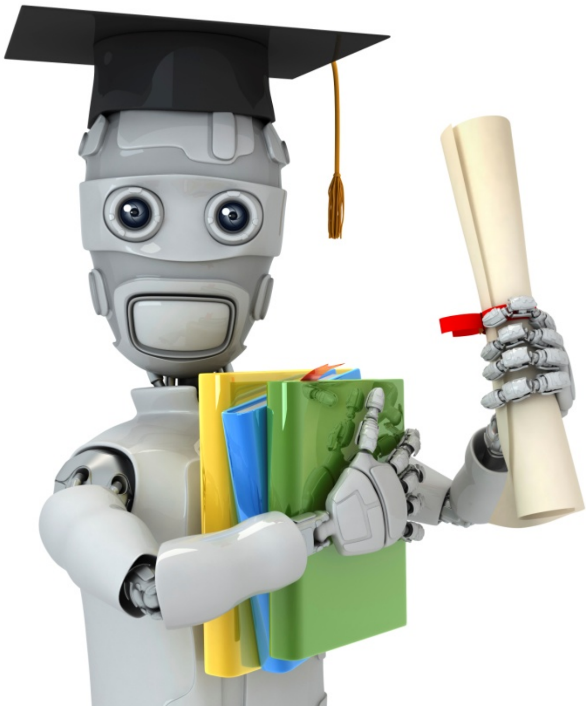


Machine Learning

异常检测

Anomaly detection

Problem motivation

Anomaly detection example 异常检测

Aircraft engine features:

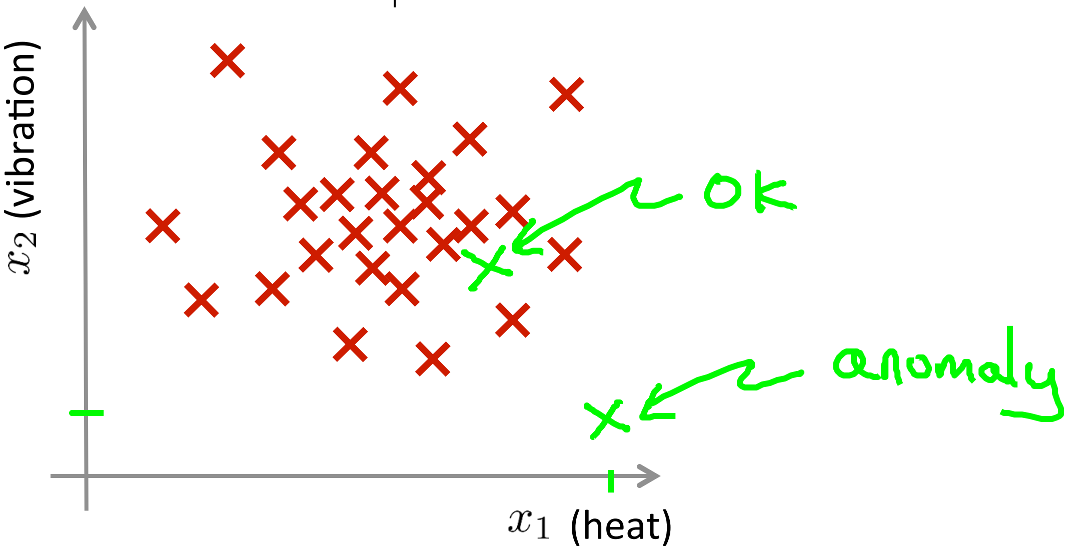
x_1 = heat generated

x_2 = vibration intensity

…

Dataset: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$

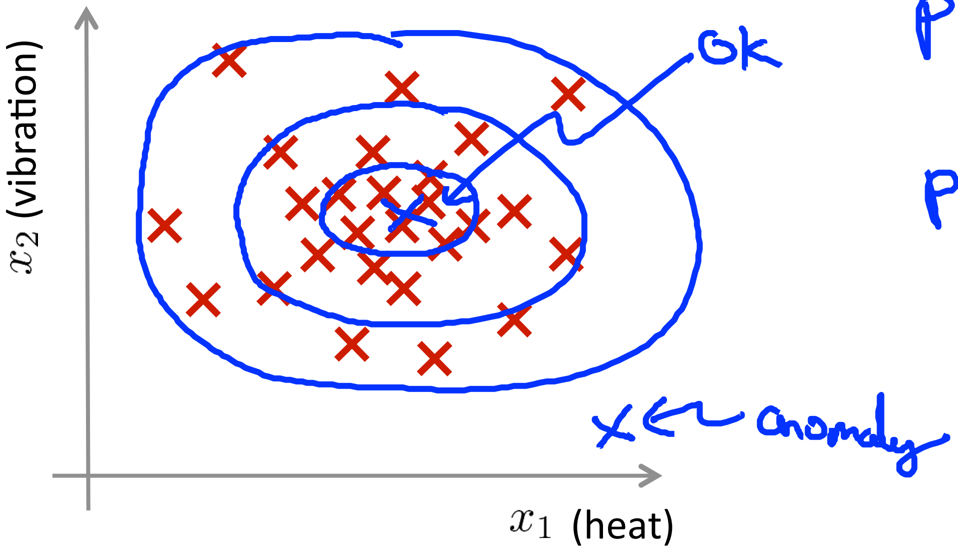
New engine: x_{test}



Density estimation

- Dataset: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$
- Is x_{test} anomalous?

Model $p(x)$ - probability



$p(x_{test}) < \varepsilon \rightarrow$ flag anomaly

$p(x_{test}) \geq \varepsilon \rightarrow$ OK

Anomaly detection example

- Fraud detection:
  - $x^{(i)}$ = features of user i's activities
  - Model $p(x)$ from data.
  - Identify unusual users by checking which have $p(x) < \varepsilon$

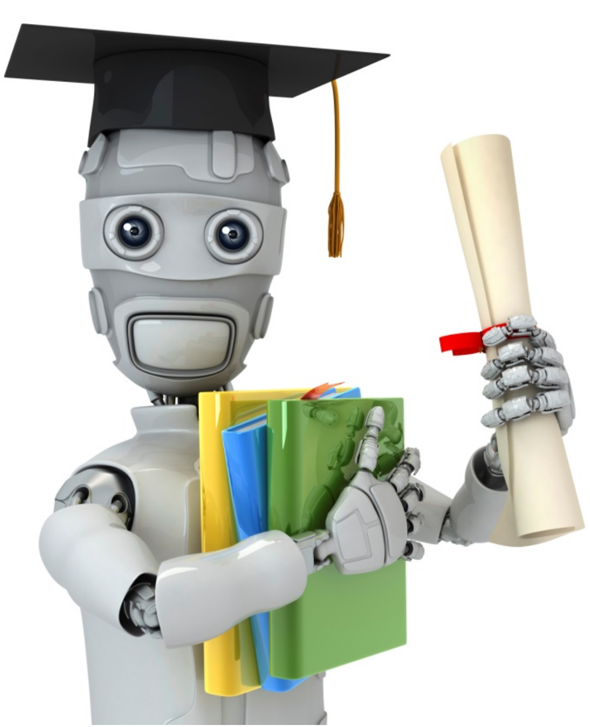
if there are too many Anomaly detected but actually not so, we should try to decrease epsilon.

x_1
x_2
x_3
x_4

$p(x)$

- Manufacturing
- Monitoring computers in a data center.
  - $x^{(i)}$ = features of machine i
  - x_1 = memory use, x_2 = number of disk accesses/sec,
  - x_3 = CPU load, x_4 = CPU load/network traffic.
  - …

$p(x) < \varepsilon$



Machine Learning

Anomaly detection

Gaussian distribution

Gaussian (Normal) distribution

two parameter

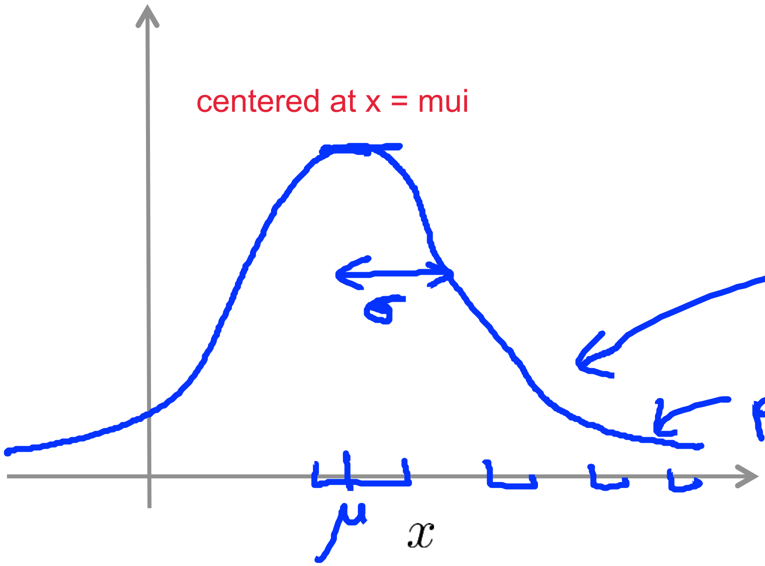
Say $x \in \mathbb{R}$. If x is a distributed Gaussian with mean μ , variance σ^2 .

standard deviation

$x \sim \mathcal{N}(\mu, \sigma^2)$

"distributed as"

σ standard deviation



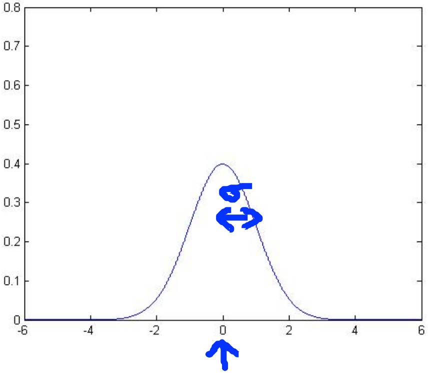
$p(x;\mu,\sigma^2)$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

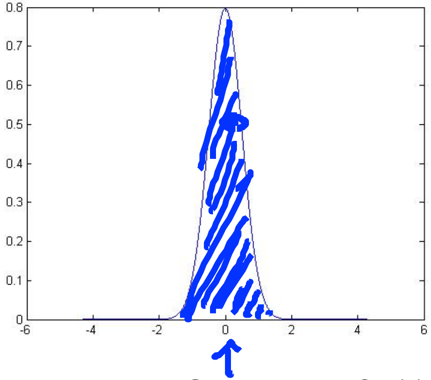
$p(x;\mu,\sigma^2)$

Gaussian distribution example

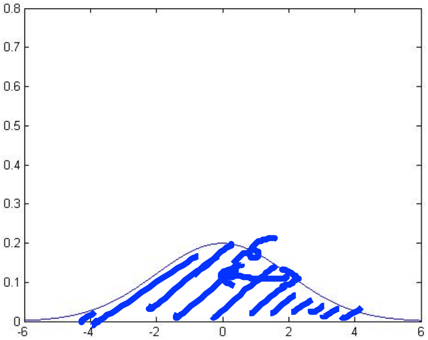
$\mu = 0, \sigma = 1$



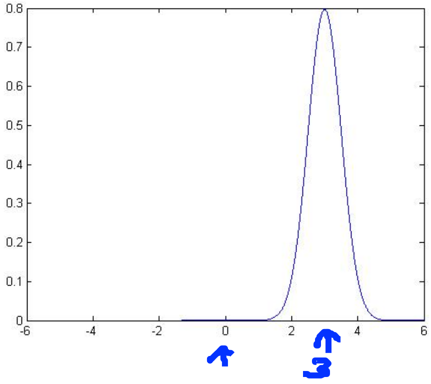
$\mu = 0, \sigma = 0.5$ $\quad \sigma^2 = 0.25$



$\mu = 0, \sigma = 2$



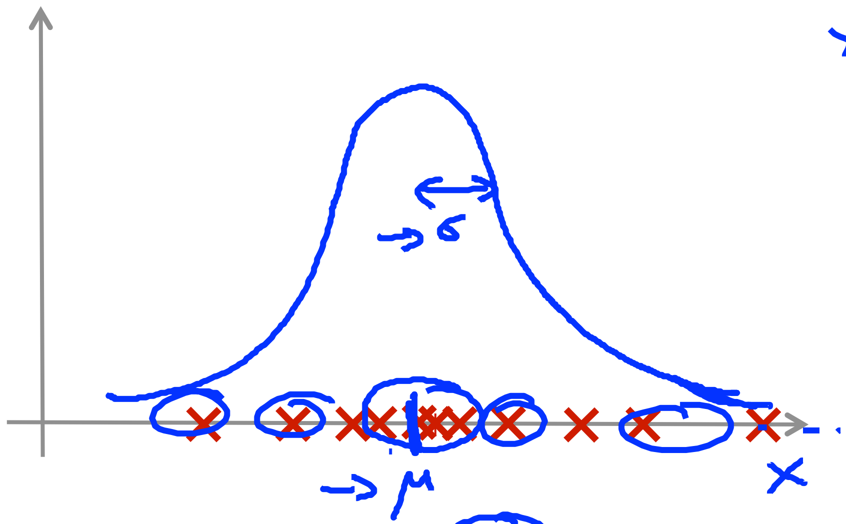
$\mu = 3, \sigma = 0.5$



Parameter estimation

Dataset: $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ $\quad x^{(i)} \in \mathbb{R}$

$x^{(i)} \sim \mathcal{N}(\mu, \sigma^2)$

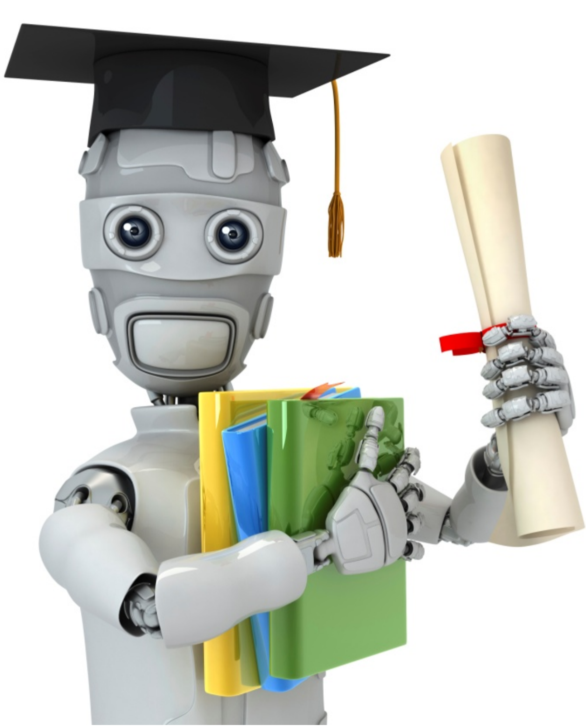


$\mu = \frac{1}{m} \sum_{i=1}^{m} x^{(i)}$

$\sigma^2 = \frac{1}{m} \sum_{i=1}^{m} \left(x^{(i)} - \mu\right)^2$

$m-1$ $\quad \frac{1}{m-1}$

Andrew Ng



Machine Learning

Anomaly detection

Algorithm

Density estimation

→ Training set: $\{x^{(1)}, \ldots, x^{(m)}\}$

Each example is $\underline{x \in \mathbb{R}^n}$

assume each feature apply to Gaussian Distribution

also assume all features are independent

$x_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$

$x_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$

$x_3 \sim \mathcal{N}(\mu_3, \sigma_3^2)$

$\rightarrow p(x)$

$= \boxed{p(x_1;\mu_1,\sigma_1^2)\,p(x_2;\mu_2,\sigma_2^2)\,p(x_3;\mu_3,\sigma_3^2)\cdots p(x_n;\mu_n,\sigma_n^2)}$

taking the product of all values

$= \boxed{\prod_{j=1}^{n}} \underline{p(x_j;\mu_j,\sigma_j^2)}$

$\sum_{i=1}^{n} i = 1+2+3+\ldots+n$

$\prod_{i=1}^{n} i = 1\times 2\times 3\times\ldots\times n$

Andrew Ng

Anomaly detection algorithm

1. Choose features x_i that you think might be indicative of anomalous examples.

$\{x^{(1)}, \ldots, x^{(m)}\}$

2. Fit parameters $\mu_1, \ldots, \mu_n, \sigma_1^2, \ldots, \sigma_n^2$

$$\boxed{\mu_j = \frac{1}{m} \sum_{i=1}^{m} x_j^{(i)}}$$

$$\sigma_j^2 = \frac{1}{m} \sum_{i=1}^{m} (x_j^{(i)} - \mu_j)^2$$

$p(x_j; \mu_j, \sigma_j^2)$

$\mu_1, \mu_2, \ldots, \mu_n$

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix} = \frac{1}{m} \sum_{i=1}^{m} x^{(i)}$$

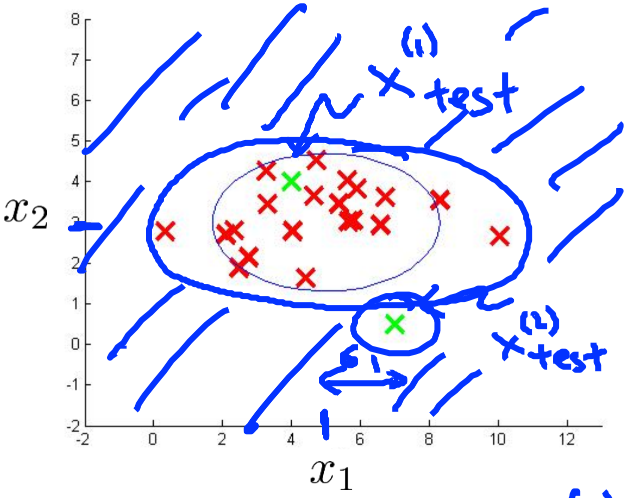
3. Given new example x, compute $p(x)$:

$$p(x) = \prod_{j=1}^{n} p(x_j; \mu_j, \sigma_j^2) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{(x_j - \mu_j)^2}{2\sigma_j^2}\right)$$

Anomaly if $p(x) < \varepsilon$

$\rightarrow p(x) = p(x_1; \mu_1, \sigma_1^2) \times p(x_2; \mu_2, \sigma_2^2)$

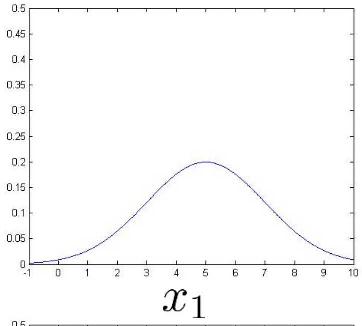
Anomaly detection example



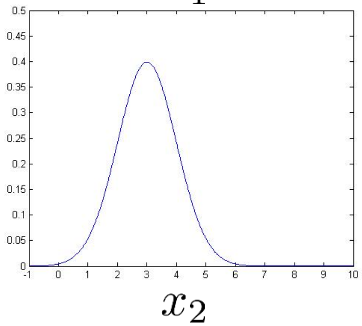
σ_1^2, σ_2^2

$\sigma_1^2 = 4$

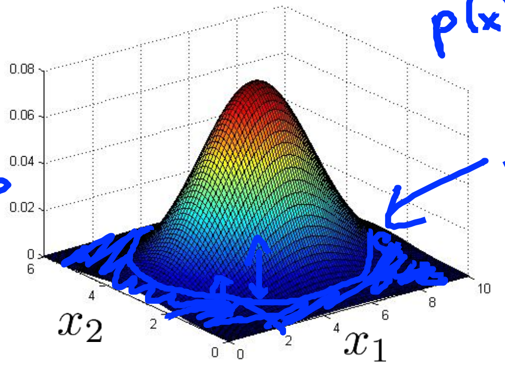
$$\boxed{\begin{aligned} \mu_1 &= 5, \sigma_1 = 2 \\ \mu_2 &= 3, \sigma_2 = 1 \end{aligned}}$$



$p(x_1; \mu_1, \sigma_1^2)$



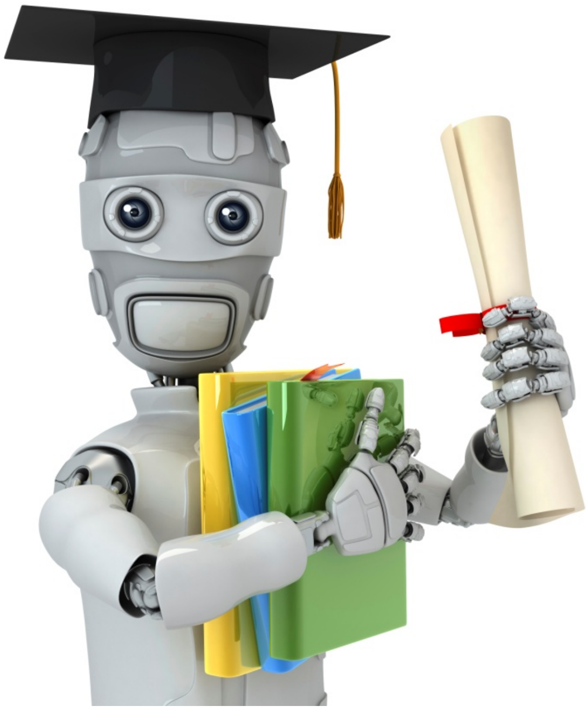
$p(x_2; \mu_2, \sigma_2^2)$



$\varepsilon = 0.02$

$p(x_{test}^{(1)}) = 0.0426 \quad \geq \varepsilon$

$p(x_{test}^{(2)}) = 0.0021 \quad < \varepsilon$



Machine Learning

Anomaly detection

Developing and evaluating an anomaly detection system

The importance of real-number evaluation

When developing a learning algorithm (choosing features, etc.), making decisions is much easier if we have a way of evaluating our learning algorithm.

- Assume we have some labeled data, of anomalous and non-anomalous examples. ($y = 0$ if normal, $y = 1$ if anomalous).

treat as unlabeled dataset

- Training set: $x^{(1)}, x^{(2)}, \dots, x^{(m)}$ (assume normal examples/not anomalous)

- Cross validation set: $(x_{cv}^{(1)}, y_{cv}^{(1)}), \dots, (x_{cv}^{(m_{cv})}, y_{cv}^{(m_{cv})})$
- Test set: $(x_{test}^{(1)}, y_{test}^{(1)}), \dots, (x_{test}^{(m_{test})}, y_{test}^{(m_{test})})$

$y=1$

Aircraft engines motivating example

10000 good (normal) engines

20 flawed engines (anomalous) 2-50 $y=1$

$\mu_1, \sigma_1^2, \ldots, \mu_n, \sigma_n^2$

Training set: 6000 good engines $(y=0)$ $p(x) = p(x_1; \mu_1, \sigma_1^2) \cdots p(x_n; \mu_n, \sigma_n^2)$

CV: 2000 good engines ($y = 0$), 10 anomalous ($y = 1$)

Test: 2000 good engines ($y = 0$), 10 anomalous ($y = 1$)

Alternative:

Training set: 6000 good engines

CV: 4000 good engines ($y = 0$), 10 anomalous ($y = 1$)

Test: 4000 good engines ($y = 0$), 10 anomalous ($y = 1$)

Algorithm evaluation

$(x_{test}^{(i)}, y_{test}^{(i)})$

- Fit model $p(x)$ on training set $\{x^{(1)}, \ldots, x^{(m)}\}$
- On a cross validation/test example x, predict

$$y = \begin{cases} 1 & \text{if } p(x) < \varepsilon \text{ (anomaly)} \\ 0 & \text{if } p(x) \geq \varepsilon \text{ (normal)} \end{cases}$$

$y = 0$

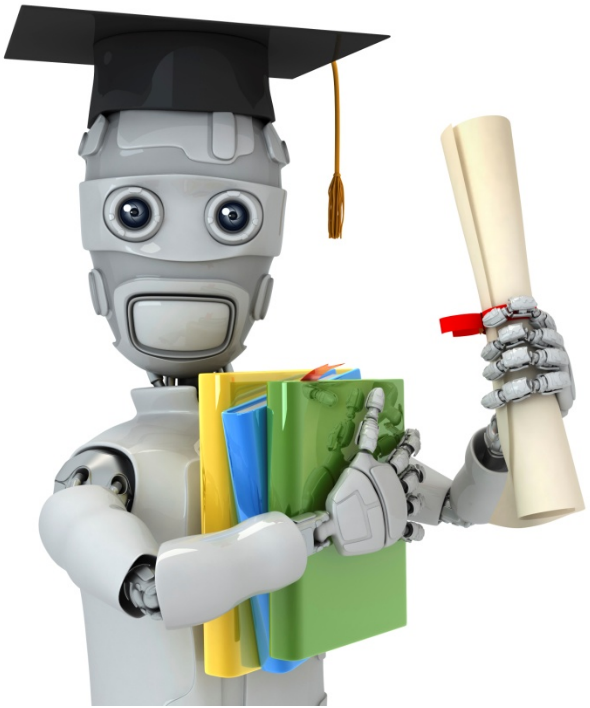
Possible evaluation metrics:

- True positive, false positive, false negative, true negative
- Precision/Recall
- F_1-score

CV

Test set

Can also use cross validation set to choose parameter ε



Machine Learning

Anomaly detection

Anomaly detection vs. supervised learning

Anomaly detection vs. Supervised learning

Anomaly detection

- Very small number of positive examples ($y = 1$). (0-20 is common).
- Large number of negative ($y = 0$) examples. $\boxed{p(x)}$
- Many different "types" of anomalies. Hard for any algorithm to learn from positive examples what the anomalies look like;
- future anomalies may look nothing like any of the anomalous examples we've seen so far.

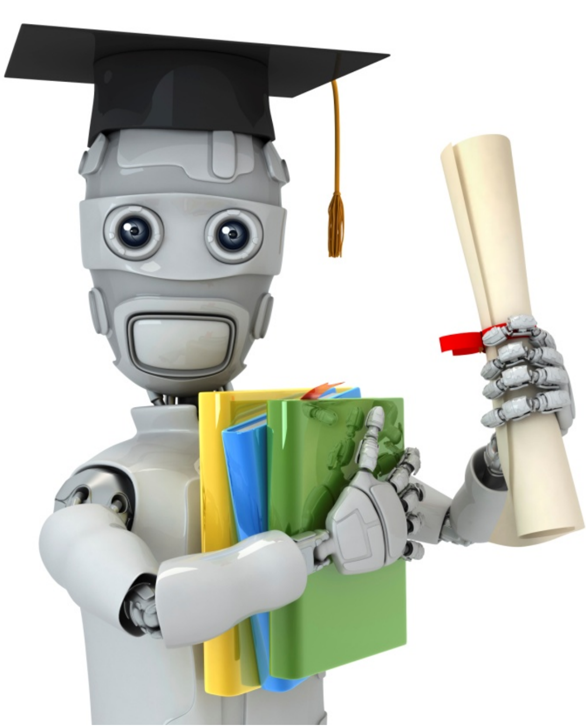
Supervised learning

- Large number of positive and negative examples.
- Enough positive examples for algorithm to get a sense of what positive examples are like, future positive examples likely to be similar to ones in training set.

Spam

Anomaly detection vs. Supervised learning

Anomaly detection	Supervised learning
• Fraud detection y=1	• Email spam classification
• Manufacturing (e.g. aircraft engines)	• Weather prediction (sunny/rainy/etc).
• Monitoring machines in a data center	• Cancer classification
⋮	⋮



Machine Learning

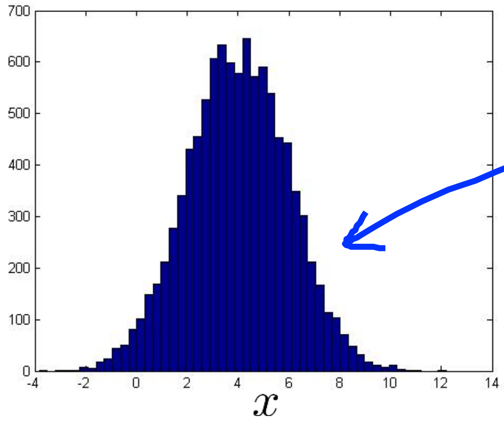
Anomaly detection

Choosing what features to use

Non-gaussian features

$p(x_i; \mu_i, \sigma_i^2)$

hist

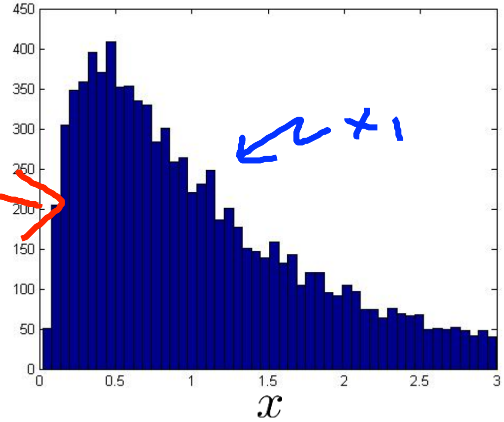


$x_1 \leftarrow \log(x_1)$

$x_2 \leftarrow \log(x_2+1)$ $\quad \log(x_2 + c)$

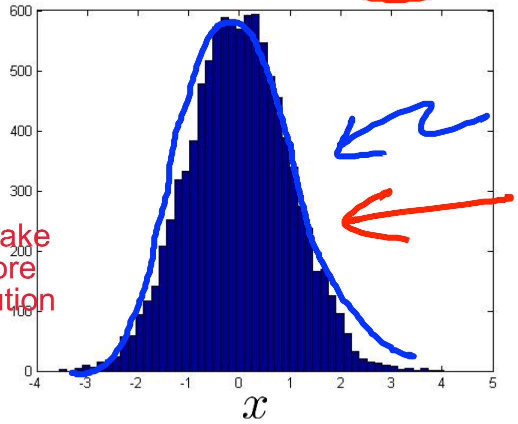
$x_3 \leftarrow \sqrt{x_3} = x_3^{\frac{1}{2}}$

$x_4 \leftarrow x_4^{\frac{1}{3}}$



$\log(x)$

transfer the data to make the histgram looks more like Gaussian Distribution



Error analysis for anomaly detection

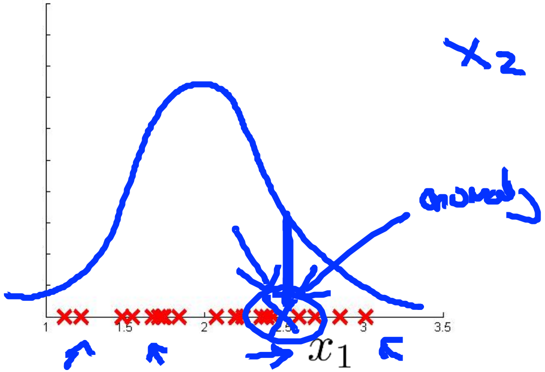
Want $p(x)$ large for normal examples x.

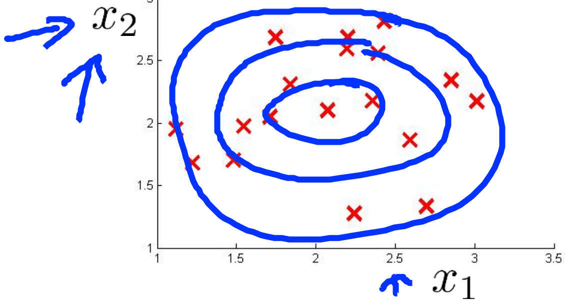
$p(x)$ small for anomalous examples x.

Most common problem:

$p(x)$ is comparable (say, both large) for normal and anomalous examples

see the anomaly that alogrithm
fail to detect and see if that inspires
you to creat new feature, find someth
unusual about the aircraft engine and
use that to creat new feature.





Monitoring computers in a data center

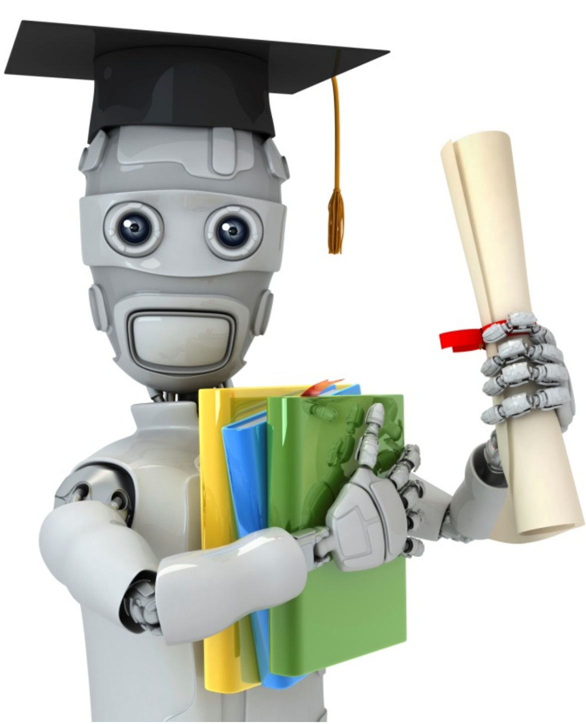
Choose features that might take on unusually large or small values in the event of an anomaly.

- x_1 = memory use of computer
- x_2 = number of disk accesses/sec
- x_3 = CPU load
- x_4 = network traffic

可以更好的捕捉到CPU 和network 之间的悬殊来定位异常

$$x_5 = \frac{\text{CPU load}}{\text{network traffic}} \qquad x_6 = \frac{(\text{CPU load})^2}{\text{network traffic}}$$

assume the anomaly situation is that computer get stuck into some infinite loop
that CPU load is large but network does not grow very huge.

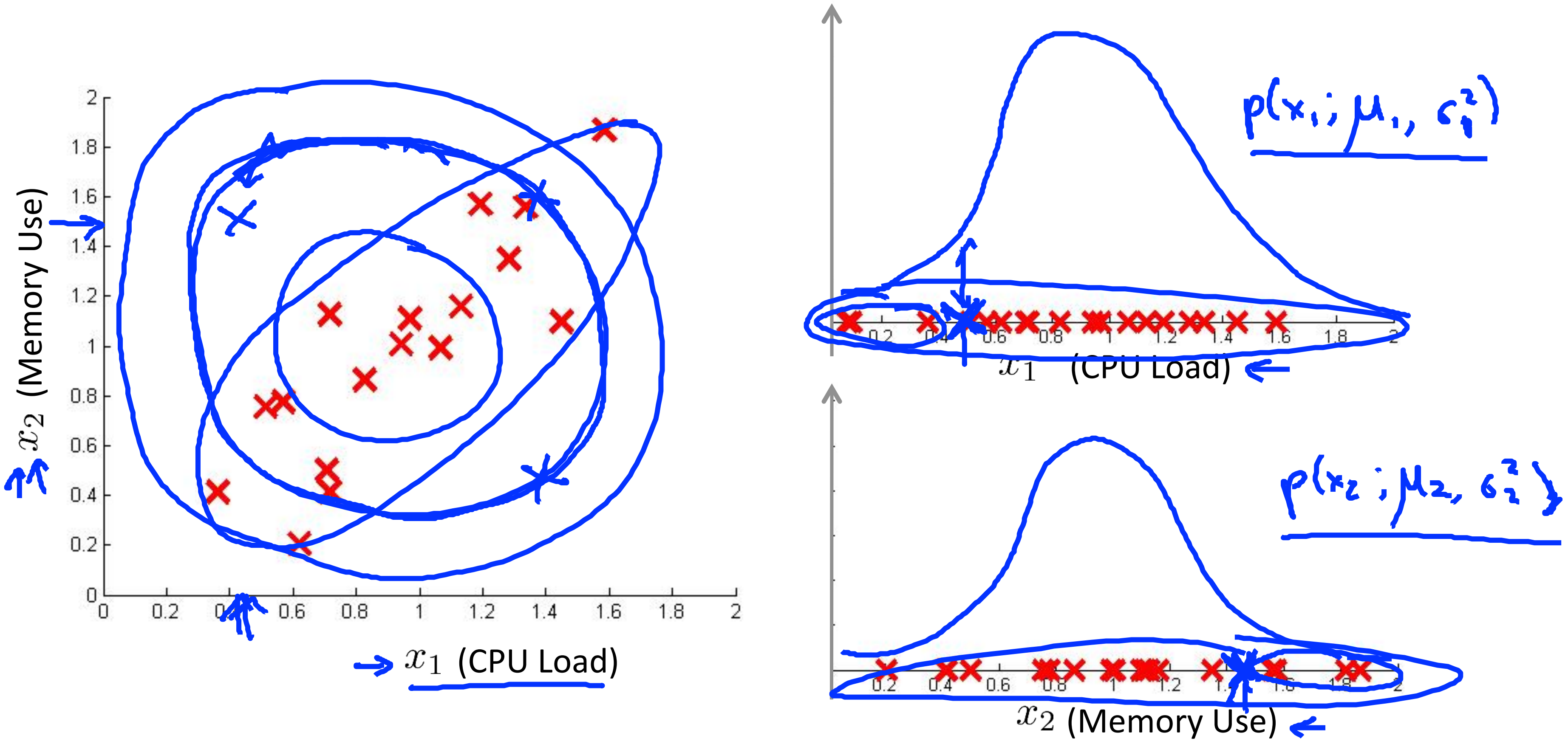


Machine Learning

Anomaly detection

Multivariate Gaussian distribution

Motivating example: Monitoring machines in a data center



Multivariate Gaussian (Normal) distribution

$x \in \mathbb{R}^n$. Don't model $p(x_1), p(x_2), \ldots,$ etc. separately.

Model $p(x)$ all in one go.

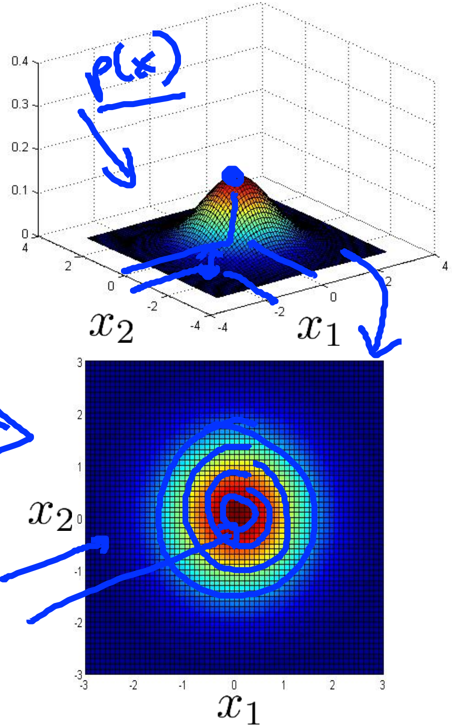
Parameters: $\mu \in \mathbb{R}^n$, $\boxed{\Sigma \in \mathbb{R}^{n \times n}}$ (covariance matrix)

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

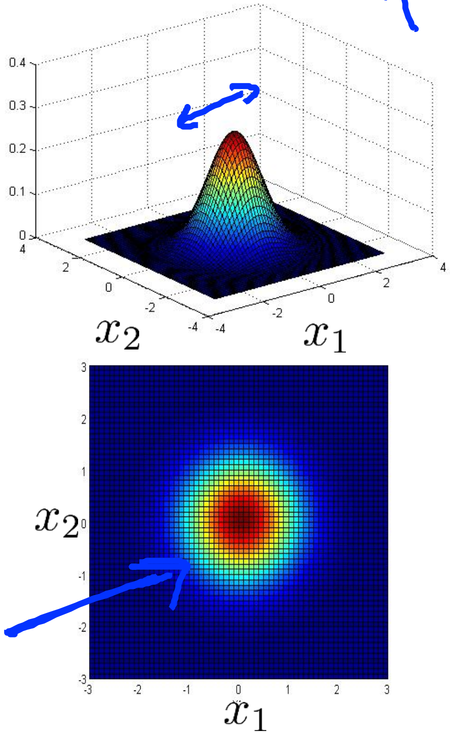
$|\Sigma|$ = determinant of Σ | det(Sigma)

Multivariate Gaussian (Normal) examples

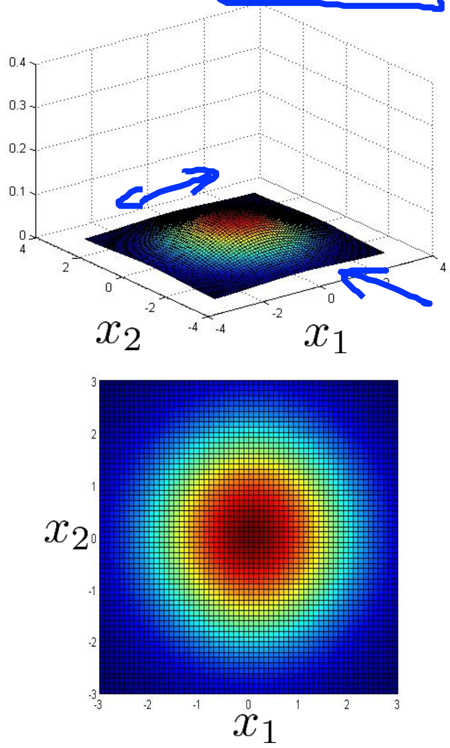
$$\boxed{\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \; \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$



$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \; \boxed{\Sigma = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}}$$



$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \; \boxed{\Sigma = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}}$$

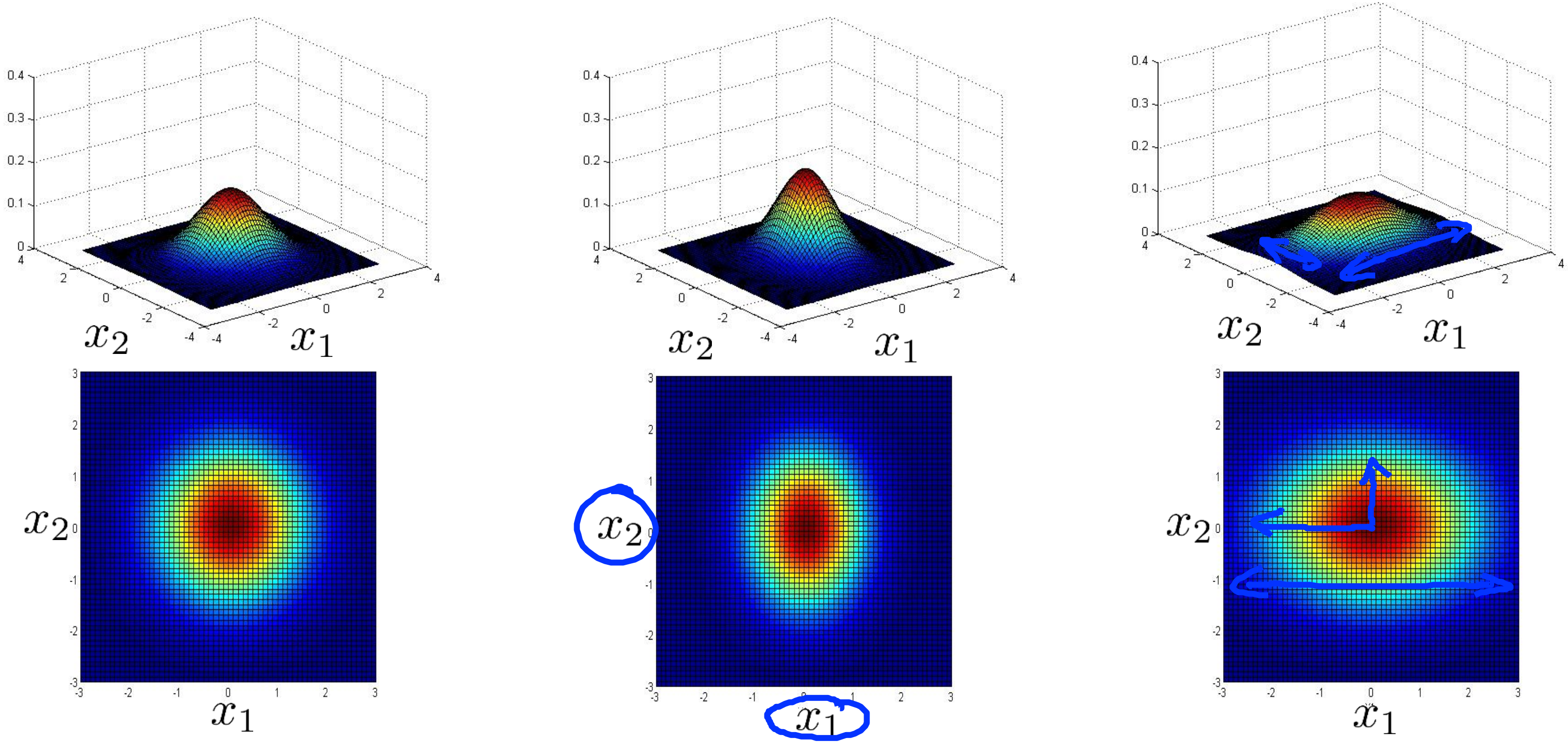


Multivariate Gaussian (Normal) examples

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

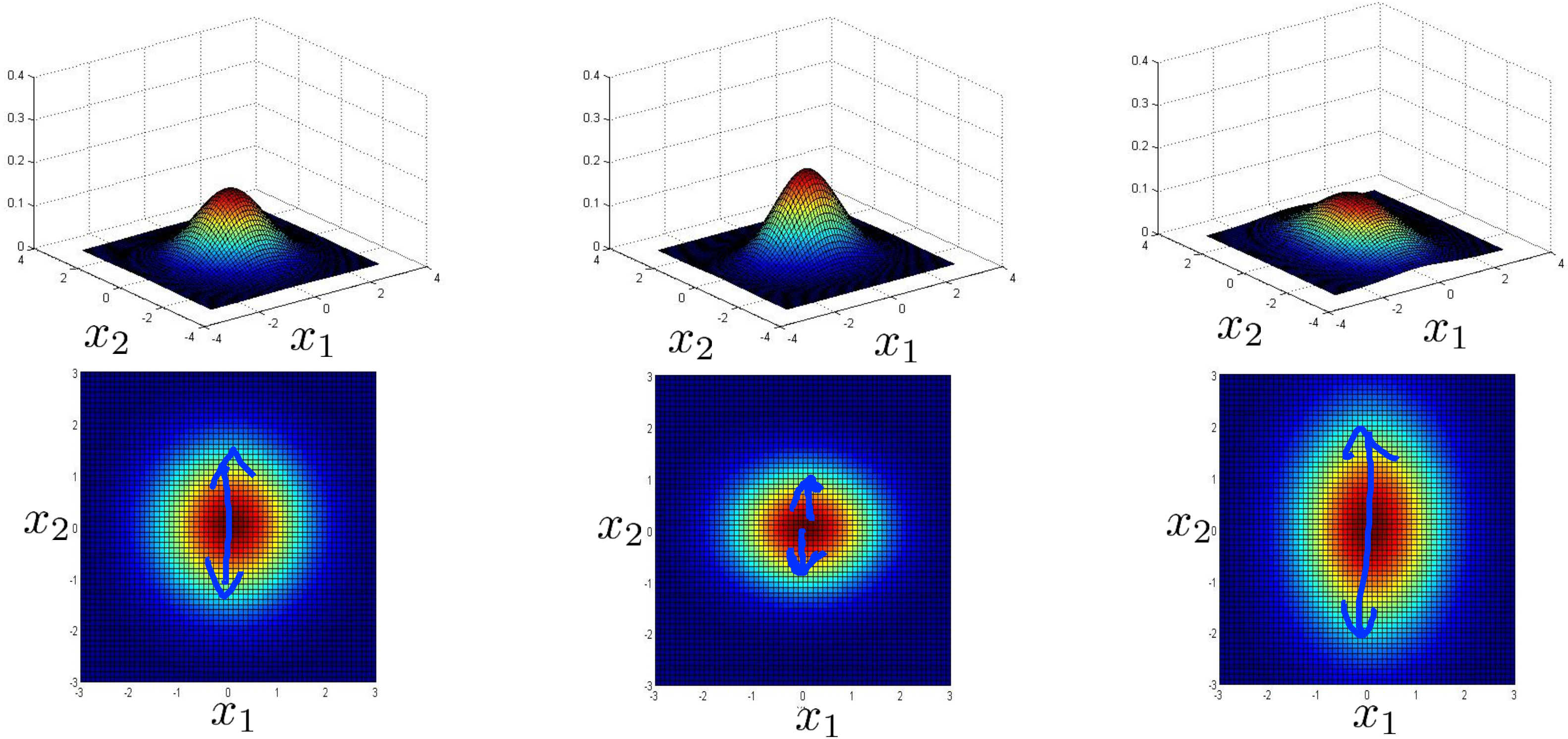
$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 0.6 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$



Multivariate Gaussian (Normal) examples

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 0.6 \end{bmatrix} \qquad \mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

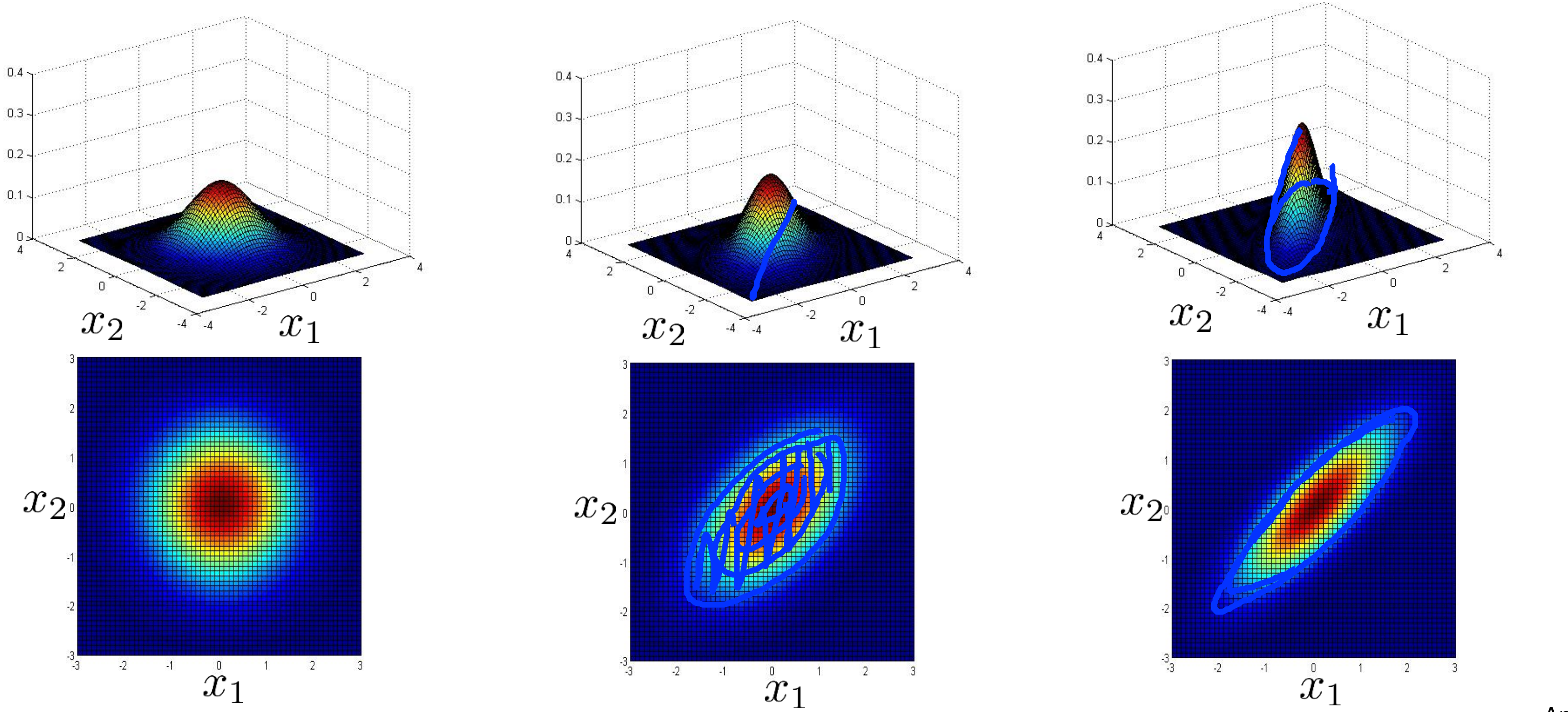


Multivariate Gaussian (Normal) examples

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Sigma = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

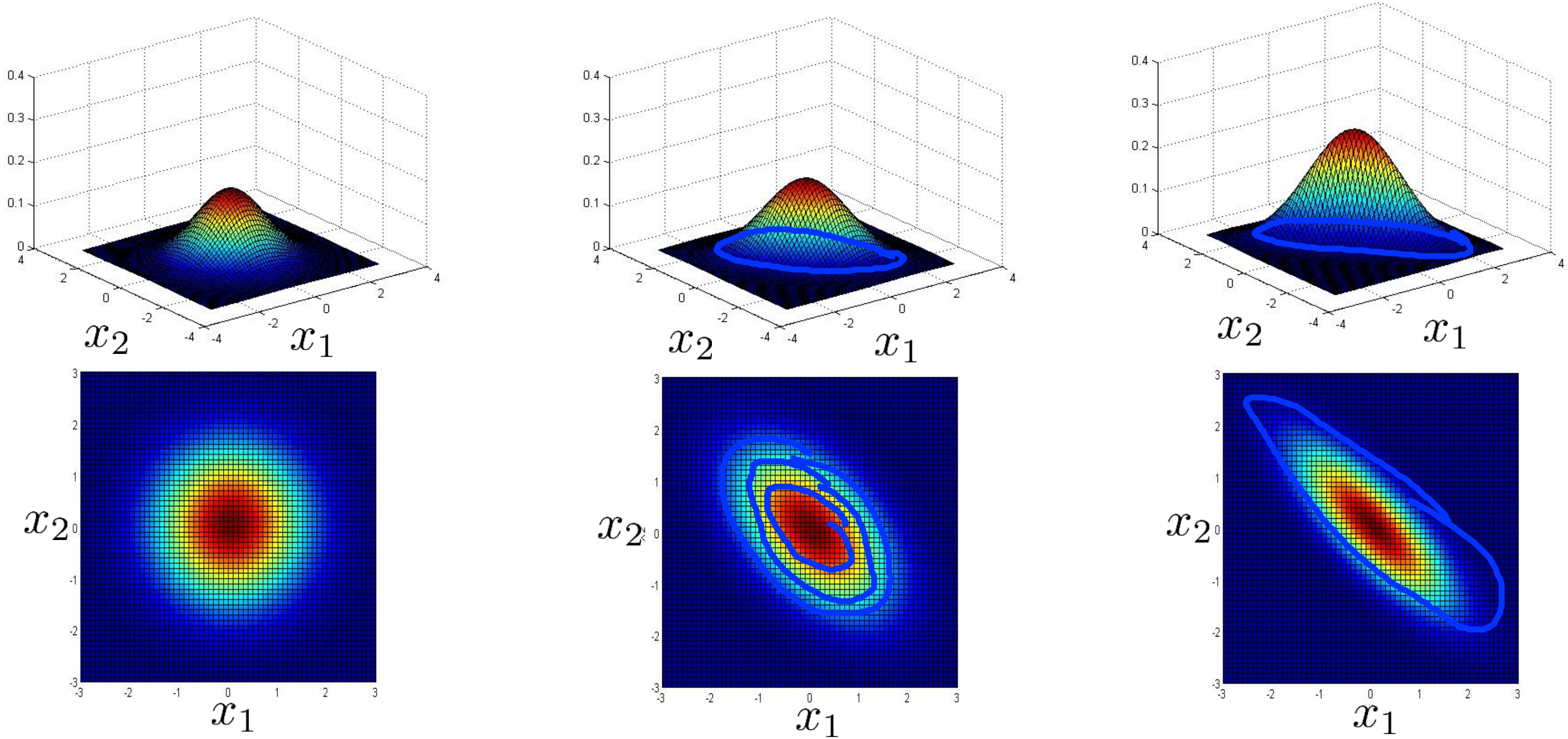


Multivariate Gaussian (Normal) examples

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix}$$

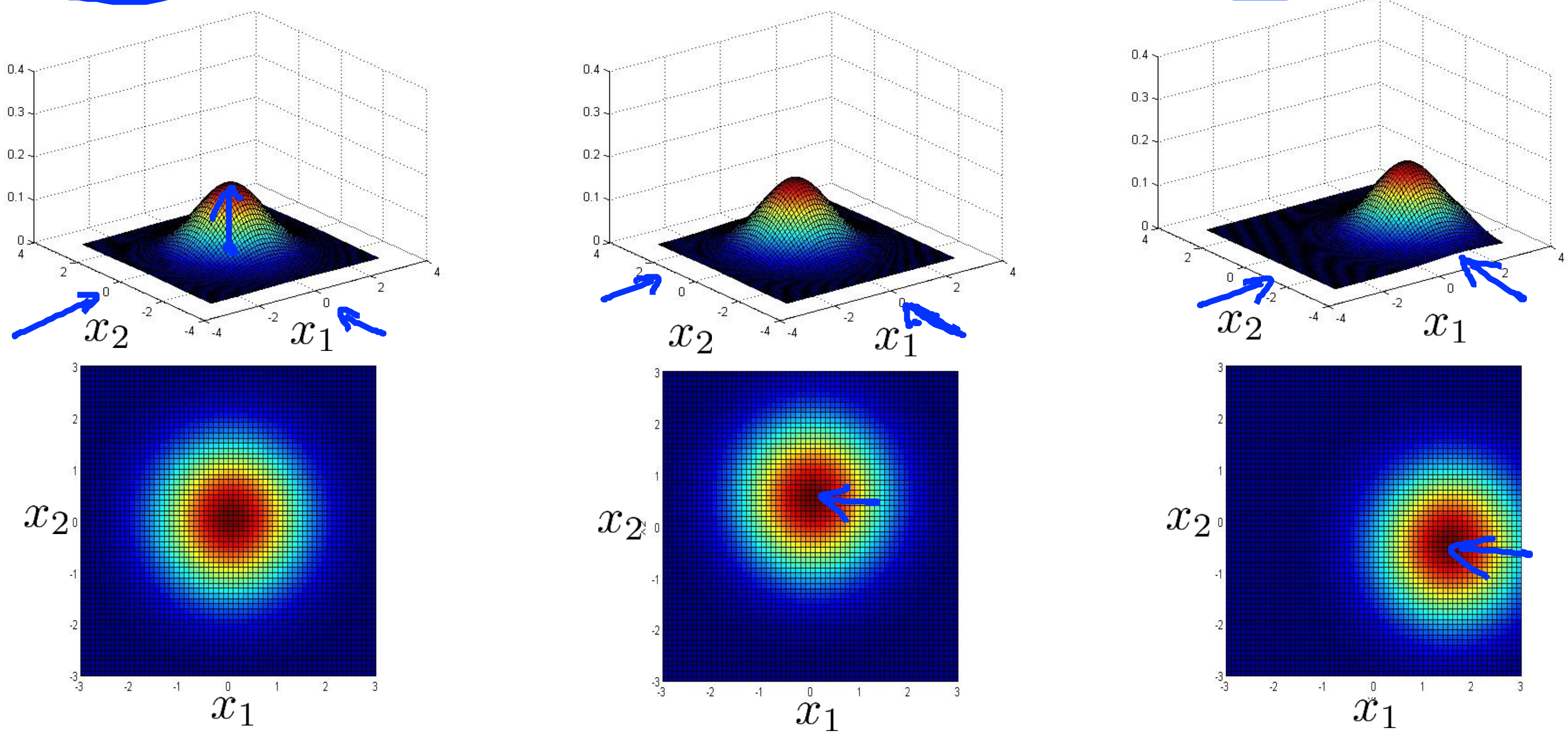


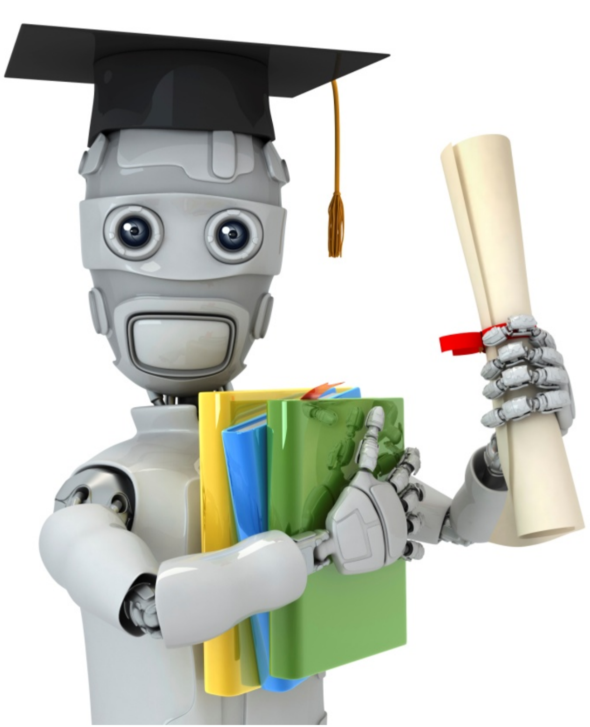
Multivariate Gaussian (Normal) examples

$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\mu = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\mu = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$





Machine Learning

Anomaly detection

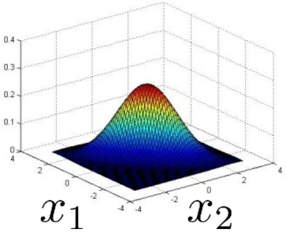
Anomaly detection using the multivariate Gaussian distribution

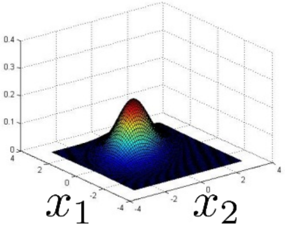
Multivariate Gaussian (Normal) distribution

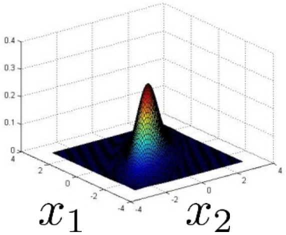
$\mu \in \mathbb{R}^n$ $\Sigma \in \mathbb{R}^{n \times n}$

Parameters μ, Σ

$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}}\exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$







Parameter fitting:

Given training set $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$ $x \in \mathbb{R}^n$

$$\boxed{\mu} = \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \qquad \boxed{\Sigma} = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)}-\mu)(x^{(i)}-\mu)^T$$

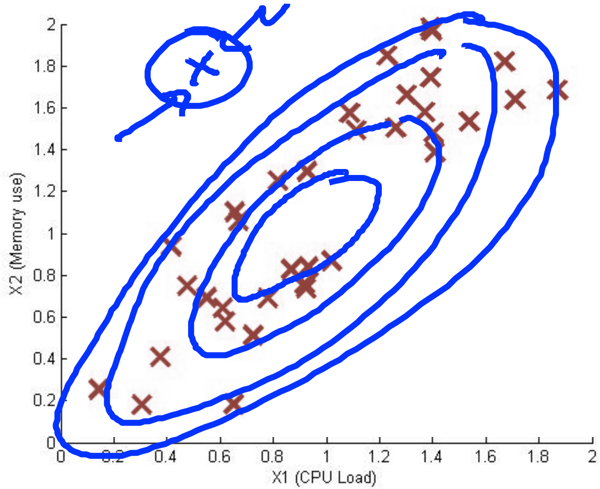
Andrew Ng

Anomaly detection with the multivariate Gaussian

1. Fit model $p(x)$ by setting

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$$

$$\Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu)(x^{(i)} - \mu)^T$$



2. Given a new example x, compute

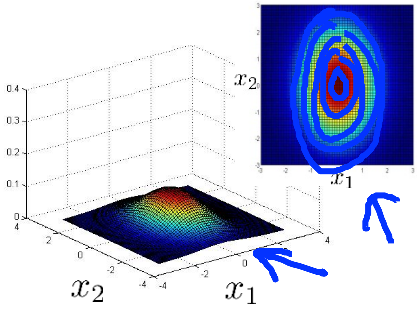
$$p(x) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}}\exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

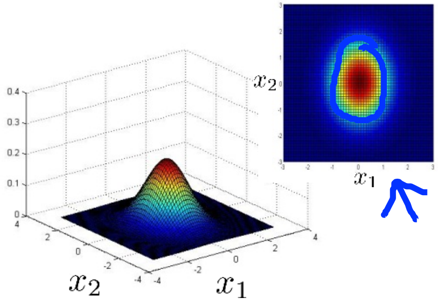
Flag an anomaly if $p(x) < \varepsilon$

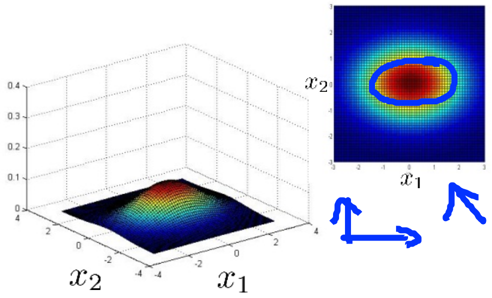
Relationship to original model

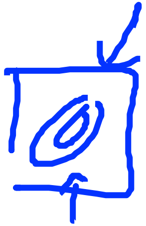
Original model: $p(x) = p(x_1;\mu_1,\sigma_1^2) \times p(x_2;\mu_2,\sigma_2^2) \times \cdots \times p(x_n;\mu_n,\sigma_n^2)$

p(x)









$\Sigma = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$

Corresponds to multivariate Gaussian

$$\boxed{p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)}$$

where $\Sigma =$

Original model vs. Multivariate Gaussian

Original model

$$p(x_1; \mu_1, \sigma_1^2) \times \cdots \times p(x_n; \mu_n, \sigma_n^2)$$

Manually create features to capture anomalies where x_1, x_2 take unusual combinations of values.

$$\boxed{x_3 = \frac{x_1}{x_2} = \frac{\text{CPU load}}{\text{memory}}}$$

Computationally cheaper (alternatively, scales better to large m)

$n = 10{,}000$, $n = 100{,}000$

OK even if m (training set size) is small

Multivariate Gaussian

$$p(x; \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

Automatically captures correlations between features

$\Sigma \in \mathbb{R}^{n \times n}$ $\quad$ Σ^{-1}

Computationally more expensive

$\Sigma \sim \frac{n^2}{2}$

$x_1 = \cancel{x_2}$

$\cancel{x_3} = x_4 + x_5$

Must have $\boxed{m > n}$ or else Σ is non-invertible.

$\boxed{m \geq 10n}$