

# Case Study 1

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## 1. Compiling and Running Instructions

To compile:

install GSL libs libgsl.so and libgslcblas.so under /home/username/lib.

g++ -L/home/username/lib -lgsl -lgslcblas case1.cpp -o case1

To run:

./case1

## 2. FFT results

alpha	strike	9	11	13	15
		512	2048	8192	32768
0.4	2000	92.2742	92.2742	92.2742	92.2742
	2100	62.5725	62.5725	62.5725	62.5725
	2200	41.2995	41.2995	41.2995	41.2995
1.0	2000	92.1928	92.1928	92.1928	92.1928
	2100	62.4911	62.4911	62.4911	62.4911
	2200	41.2181	41.2181	41.2181	41.2181
1.4	2000	92.1928	92.1928	92.1928	92.1928
	2100	62.4911	62.4911	62.4911	62.4911
	2200	41.2181	41.2181	41.2181	41.2181
3.0	2000	92.1928	92.1928	92.1928	92.1928
	2100	62.4911	62.4911	62.4911	62.4911
	2200	41.2181	41.2181	41.2181	41.2181

From the pricing of FFT above, we can see that the option price is sensitive to alpha. For example, when alpha is 0.4, the result is inaccurate. In cases where alpha is bigger than one, FFT method gets almost the same option price as Black-Scholes Formula.

## 2. FrFFT results

alpha	n strike	6	7	8	9
		64	128	256	512
0.4	2000	92.6048	92.2776	92.2742	92.2742
	2100	63.2898	62.579	62.5725	62.5725
	2200	42.1562	41.3042	41.2995	41.2995
1.0	2000	92.2126	92.1934	92.1928	92.1928
	2100	62.9667	62.4965	62.4911	62.4911
	2200	41.9349	41.2237	41.2181	41.2181
1.4	2000	92.0072	92.1915	92.1928	92.1928
	2100	62.7895	62.4952	62.4911	62.4911
	2200	41.8126	41.2238	41.2181	41.2181
3.0	2000	91.3614	92.186	92.1928	92.1928
	2100	62.1077	62.4887	62.4911	62.4911
	2200	41.2364	41.2204	41.2181	41.2181

The table above shows the result of fractional FFT method. In this case, the choice of  $n$  is lower than FFT, so the option prices become sensitive to  $n$ , when  $n$  is 6, and 7. The option price is also sensitive to  $\alpha$ , and the result is inaccurate when  $\alpha$  is 0.4. When  $\alpha \geq 1$ ,  $n \geq 8$ , the frFFT get the same option price as BS Formula.

### 3. Fourier-Cosine Method Result

[a,b]	strike (n=9)		
	2000	2100	2200
-1, 1	92.1928	62.4911	41.2181
-4, 4	92.1928	62.4911	41.2181
-8, 8	92.1928	62.4911	41.2181
-12, 12	92.1928	62.4911	41.2181

Setting  $n = 9$  and different intervals of  $a$  and  $b$ , the table above shows that option prices calculated by Fourier Cosine Method are all accurate, and not sensitive to the choice of  $a$  and  $b$ .

[a,b]	strike (n=6)		
	2000	2100	2200
-1, 1	92.1928	62.4911	41.2181
-4, 4	92.1937	62.4922	41.218
-8, 8	106.479	234.75	317.869
-12, 12	-19586.1	16905	52347.9

When setting  $n = 6$ , the result is as above. We can conclude that the option price is sensitive to range of  $a$  and  $b$ . As the range  $a$  and  $b$  increases, the option price given by Fourier-Cosine method becomes inaccurate.