Case Study 3

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1. Compiling and Running Instructions

To compile:

install GSL libs libgsl.so and libgslcblas.so under /home/username/lib.

g++ -L/home/username/lib -lgsl -lgslcblas case3. cpp -o case1

To run:

./case3

2. Pricing Engine

I choose fractional Fourier Transformation, because it enables us to choose lamda (the difference of log(K)) freely, so we can get more accurate result by setting smaller lamda. According to Case Study 1, the FrFFT method's pricing result is almost the same as BS Formula's result, when n=8. Therefore, I choose parameter lamda = 0.1 and n=8 for FrFFT model.

3. Loss Function and Optimization

For equally weighted calibration, the loss function I defined is sum of squared errors:

$$\sum (C_{Market} - C_{Model})^2$$

For calibration with weights inversely proportional to bid-ask spread, the loss function is

$$\sum \frac{(C_{Market} - C_{Model})^2}{C_{Ask} - C_{Bid}}$$

I used Downhill Simplex method C++ program written by Botao Jia (2010) to get model parameters that minimize the loss function. To speed up the calibration, I choose tolerance = 1e-3, and maximum number of iterations = 200.

4. Calibration Result:

Heston

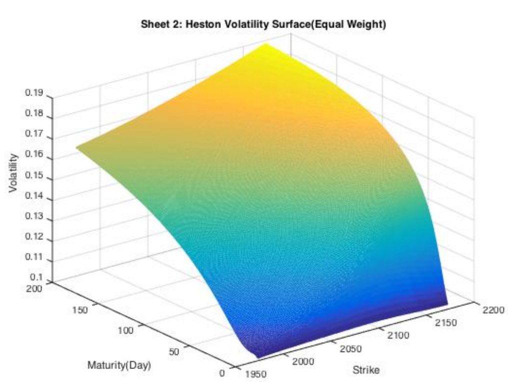
	Weight	κ	θ	σ	ρ	$\nu 0$	Object Value
sheet2	Equal	5.4170	0.0353	0.7088	0.3482	0.0100	1789.05
	Inverse	5.3922	0.0379	0.6950	-0.1528	0.0091	1501.36
sheet3	Equal	5.3247	0.0377	0.6616	0.2443	0.0087	1550.43
	Inverse	5.2487	0.0383	0.6975	-0.2813	0.0107	1471.97
sheet4	Equal	5.3407	0.0359	0.6818	0.2793	0.0105	1622.39
	Inverse	5.2747	0.0383	0.7080	-0.2541	0.0110	1526.81
sheet5	Equal	4.6647	0.0330	0.2471	-0.6878	0.0121	984.66
	Inverse	5.2739	0.0378	0.7039	-0.2582	0.0109	1420.54
sheet6	Equal	3.9953	0.0364	0.2425	-0.6472	0.0122	1272.92
	Inverse	5.2756	0.0382	0.6699	-0.2544	0.0108	1544.75
sheet7	Equal	3.9953	0.0364	0.2425	-0.6472	0.0122	1272.92
	Inverse	5.2756	0.0382	0.6699	-0.2544	0.0108	1544.75
sheet 8	Equal	5.2312	0.0349	0.6409	0.2155	0.0114	1436.27
	Inverse	4.4740	0.0360	0.4663	-0.9293	0.0143	1024.56

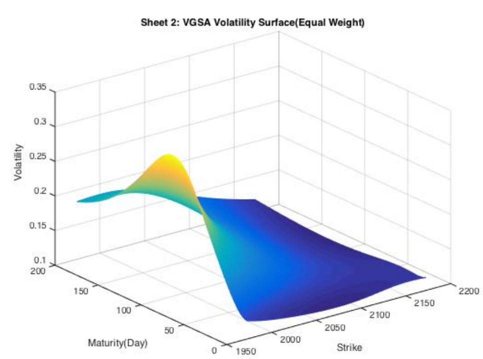
VGSA

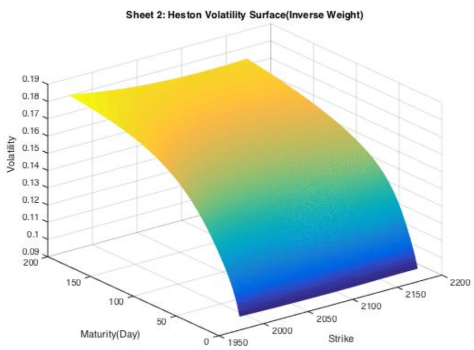
	Weight	σ	ν	θ	κ	η	λ	Object Value
sheet2	Equal	0.0367	0.1230	-0.0611	7.1711	3.1561	10.4070	801.68
	Inverse	0.0132	0.0861	-0.0719	10.6559	3.1457	12.1714	1163.59
sheet3	Equal	0.0410	0.1316	-0.0547	7.5423	3.4477	12.5541	658.46
	Inverse	0.0267	0.0669	-0.0490	7.8923	5.9795	9.4802	1298.64
sheet4	Equal	0.0365	0.1096	-0.0504	7.2670	3.4678	12.6120	721.23
	Inverse	0.0259	0.0882	-0.0561	7.8855	2.8958	11.9395	1263.28
sheet5	Equal	0.0365	0.1148	-0.0549	7.4390	3.4233	11.6355	686.32
	Inverse	0.0190	0.0558	-0.0489	10.2194	5.8509	12.3756	1166.22
sheet6	Equal	0.0461	0.1634	-0.0652	6.6670	2.8771	10.8248	995.85
	Inverse	0.0233	0.0653	-0.0475	7.3039	2.4051	12.2265	1320.90
sheet7	Equal	0.0461	0.1634	-0.0652	6.6670	2.8771	10.8248	995.85
	Inverse	0.0233	0.0653	-0.0475	7.3039	2.4051	12.2265	1320.90
sheet 8	Equal	0.0483	0.1624	-0.0557	7.3443	3.2428	12.1489	629.40
	Inverse	0.0249	0.0660	-0.0517	8.3474	3.9885	10.3674	995.11

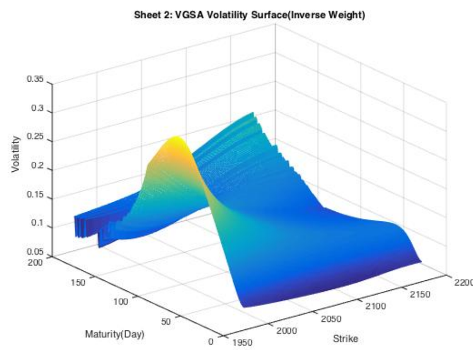
5. Volatility Surface:

Sheet 2:









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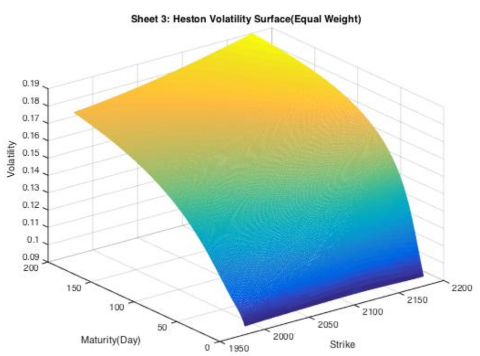

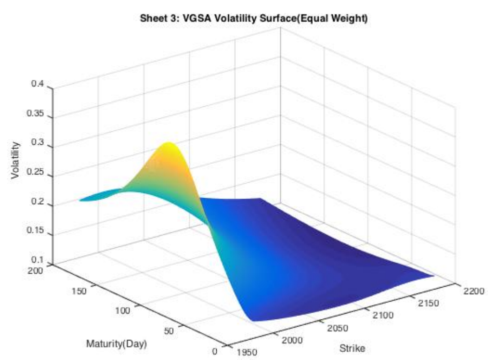

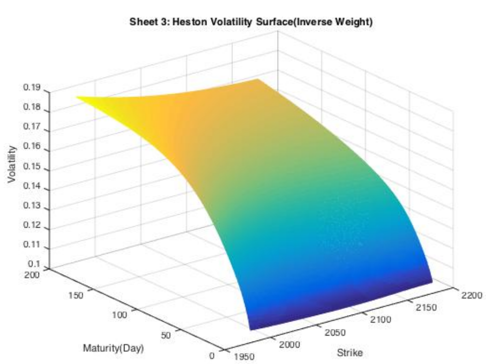

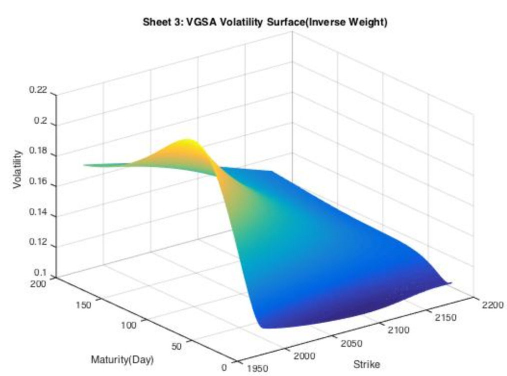

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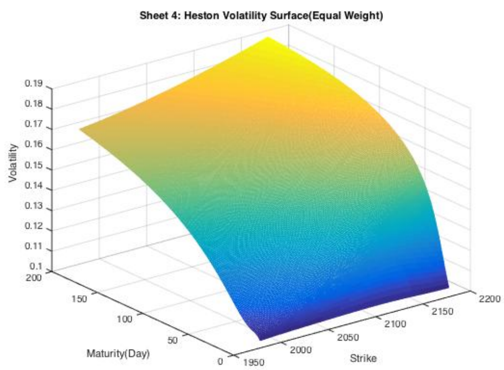
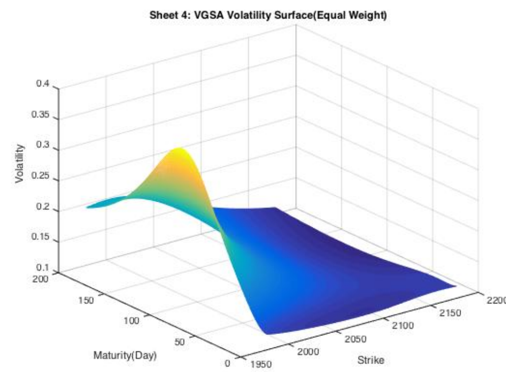
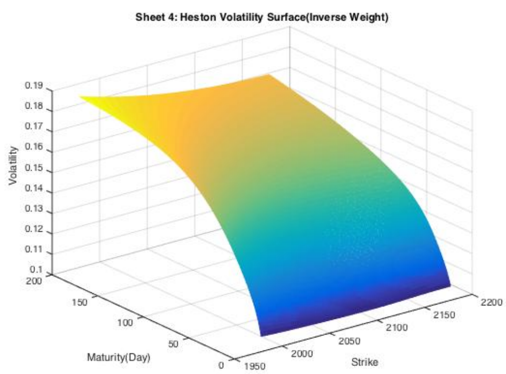
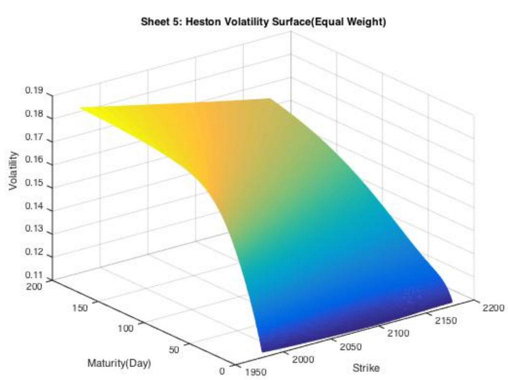

Sheet 4: Heston Volatility Surface(Equal Weight)
Volatility
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200


Sheet 4: VGSA Volatility Surface(Equal Weight)
Volatility
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200


Sheet 4: Heston Volatility Surface(Inverse Weight)
Volatility
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200


Sheet 4: VGSA Volatility Surface(Inverse Weight)
Volatility
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200

Sheet 5: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Volatility
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
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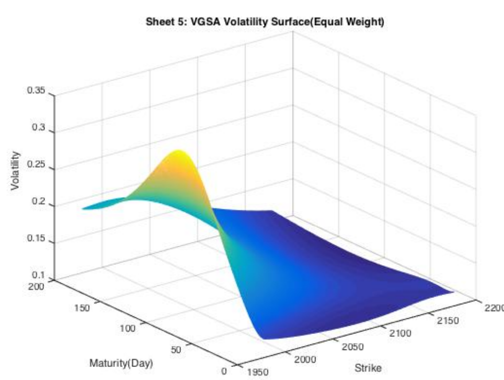
2150
2200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Volatility
0.1
0.15
0.2
0.25
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Maturity(Day)
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Strike
1950
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2150
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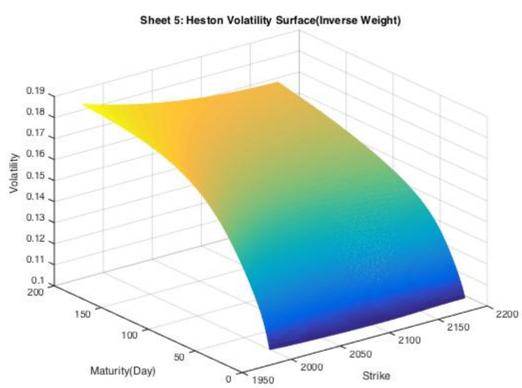
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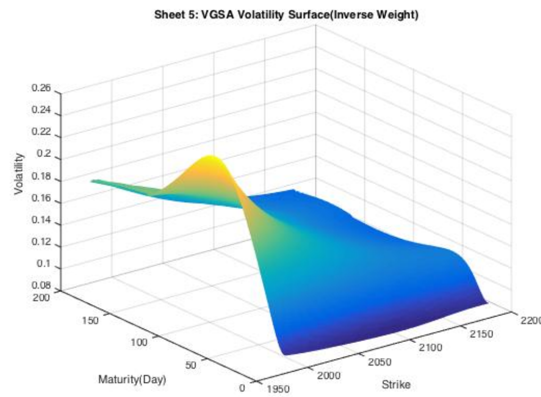
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200


Sheet 5: VGSA Volatility Surface(Inverse Weight)
Volatility
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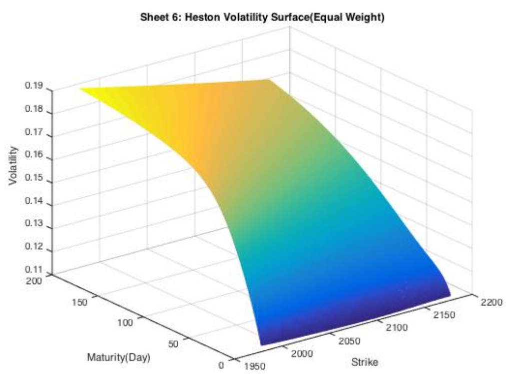
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200

Sheet 6: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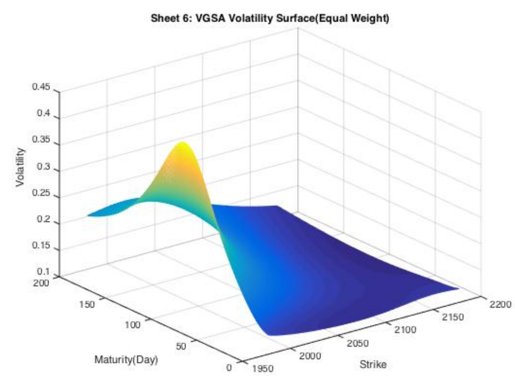
Volatility
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200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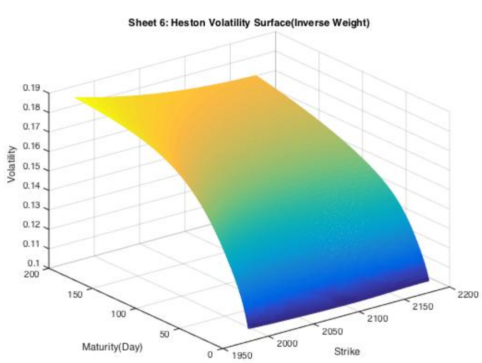
Volatility
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Maturity(Day)
0
50
100
150
200
Strike
1950
2000
2050
2100
2150
2200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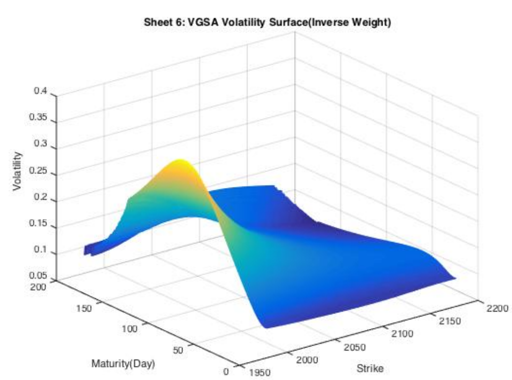
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Volatility
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Maturity(Day)
0
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100
150
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Strike
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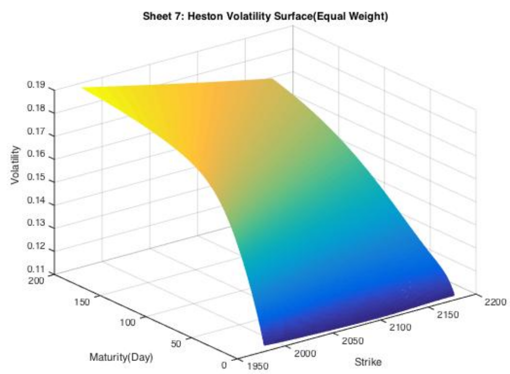
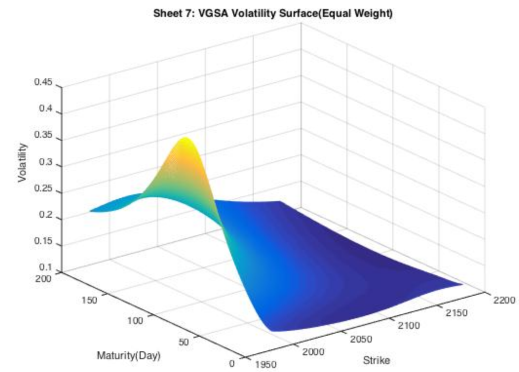
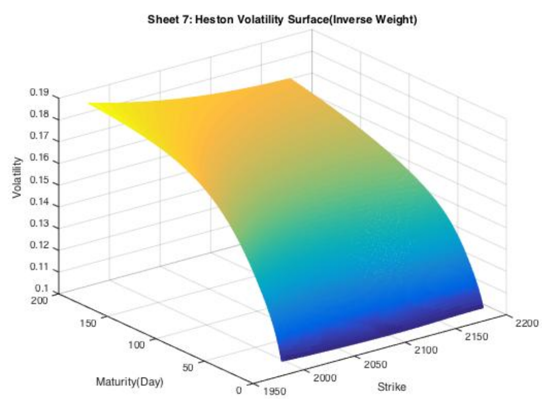
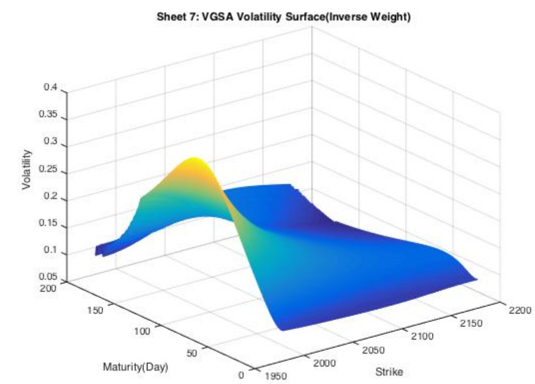
2050
2100
2150
2200

Sheet 7:









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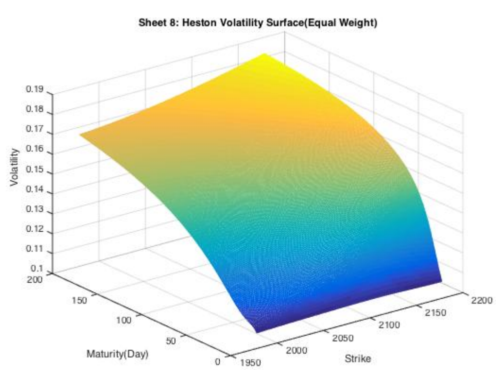

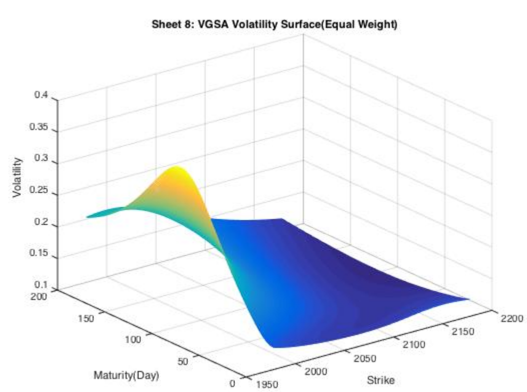

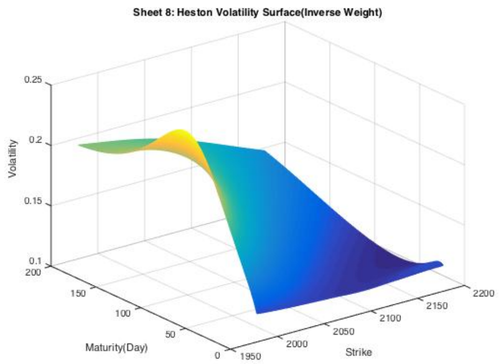

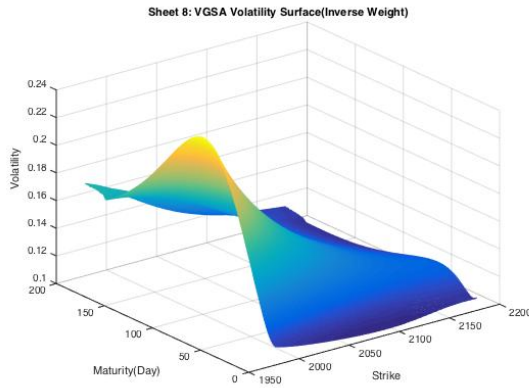

6. Observations:

From the volatility surface, we can find that Heston Model gives us smoother volatility surface, and the local volatility is positively correlated with maturity. It might because the future volatility is increasing. When ρ<0, local volatility is negatively correlated with strike, and when ρ>0, local volatility is positively correlated with strike. The surface of VGSA model shows that the local volatility reaches the highest at certain maturity and then decreases. The relationship between strike and volatility is uncertain and varies with different maturity.