

# CS 615 Assignment3

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May 2020

## 1 Theory

### 1.1

$$\text{Given Matrix } X = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \text{Matrix } K = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

First we have to rotate K by 180 degrees to do  $X * K$ , so our new K would be:  $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

The matrix F would be of dimensions  $[\text{size}(X,1)-\text{size}(K,1)+1, \text{size}(X,2)-\text{size}(K,2)+1] = [3-3+1, 3-3+1] = [1,1]$ . To calculate F, we would be do element-wise multiplication between the X and this new rotated K, then sum all the elements. so, this would produce a single value F:

$$F = [1 * 9 + 8 * 2 + 3 * 7 + 4 * 6 + 5 * 5 + 6 * 4 + 3 * 7 + 2 * 8 + 1 * 9] = [165]$$

$$F_{ab} = (X_{ab} * K) = \sum_{i=-\frac{M}{2}}^{\frac{M}{2}} \sum_{j=-\frac{M}{2}}^{\frac{M}{2}} X_{(a-i,b-j)} K_{ij}$$

OR, F can calculated by using this formula:

where only possible a and b combination is a=2 and b=2,  $M = 3$

### 1.2

$$\text{Given Matrix } F = \begin{bmatrix} 1 & 2 & 3 & 4 & 1 & 3 \\ 4 & 5 & 6 & 0 & 0 & 12 \\ 7 & 8 & 9 & 1 & 0 & 4 \\ -100 & -100 & -100 & -100 & -100 & -100 \end{bmatrix}$$

#### 1.2.1 Max-Pooling width =2 stride = 2

In the max pooling, find the maximum values out of 2 X 2 matrix in F with moving by the stride of 2 in each iteration. The final matrix would be of dimensions  $[\text{size}(F,1)/\text{width}, \text{size}(F,2)/\text{width}] = [4/2, 6/2] = [2,3]$

$$\text{Matrix } h = \begin{bmatrix} \max(1, 2, 4, 5) & \max(3, 4, 6, 0) & \max(1, 3, 0, 12) \\ \max(7, -100, -100, 8) & \max(-100, -100, 1, 9) & \max(-100, -100, 0, 4) \end{bmatrix} = \begin{bmatrix} 5 & 6 & 12 \\ 8 & 9 & 4 \end{bmatrix}$$

#### 1.2.2 Mean-Pooling width =2 stride = 2

In the mean pooling, find the mean of the values in 2 X 2 matrix in F with moving by the stride of 2 in each iteration. The final matrix would be of dimensions  $[\text{size}(F,1)/\text{width}, \text{size}(F,2)/\text{width}] = [4/2, 6/2] = [2,3]$

$$\text{Matrix } h = \begin{bmatrix} 12/4 & 13/4 & 16/4 \\ -185/4 & -190/4 & -196/4 \end{bmatrix} = \begin{bmatrix} 3 & 3.25 & 4 \\ -46.25 & -47.5 & -49 \end{bmatrix}$$

### 1.3

Find K in:  $X * K = X$

Now Let replace the convolution multiplication with the matrix product and replace the 'K' with 'Y'. Later to find the final solution, we just have to rotate our Y by 180 degrees.

In order to generalize our solution for all types of 'X' including non-inversible matrix (Not a square matrix), multiplying the both side by  $X^T$  as  $X^T X$  would always be a square matrix. Then, the equation would be:

$$X^T X Y = X^T X$$

$$Y = (X^T X)^{-1} (X^T X)$$

Using the matrix property of  $A^{-1} A = I$  where I is an identity matrix, equation would be:

$$Y = I$$

Rotating the Identity matrix by 180 degrees would result in Identity matrix itself so,

$$K = I$$

## 2 CNN for LSE Classification

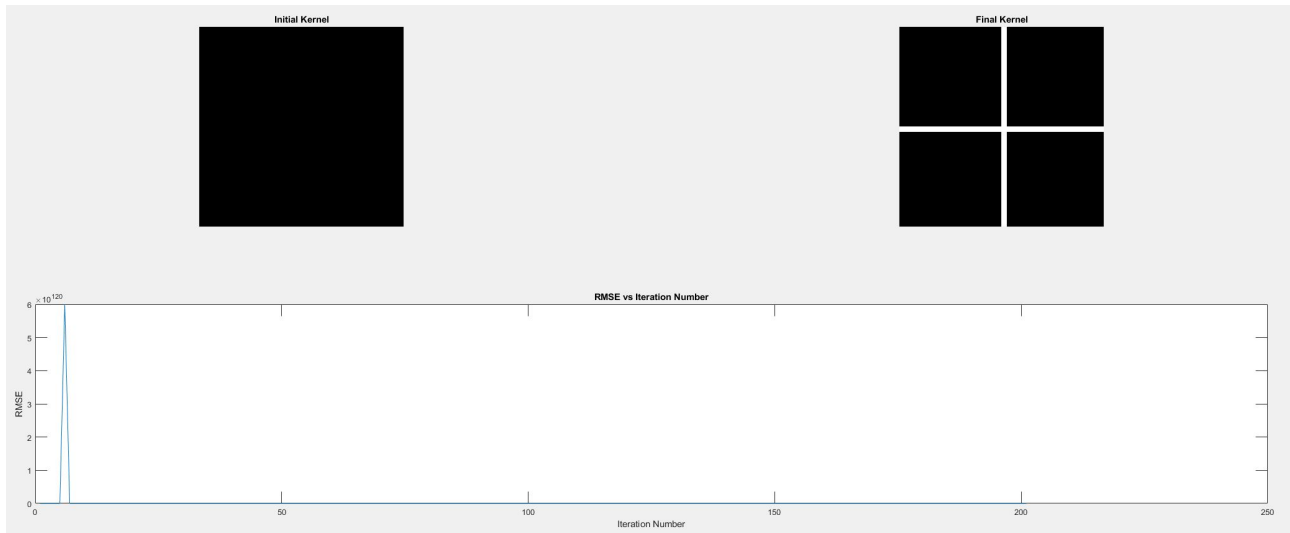
Hyper-parameters:

L2 regularization = 0.000001

Learning rate = 0.55

Initialization of kernels and theta layer by randomly picking numbers from  $[0, 0.000001]$

Termination criteria is max iteration = 200



### 3 CNN for MLE Classification

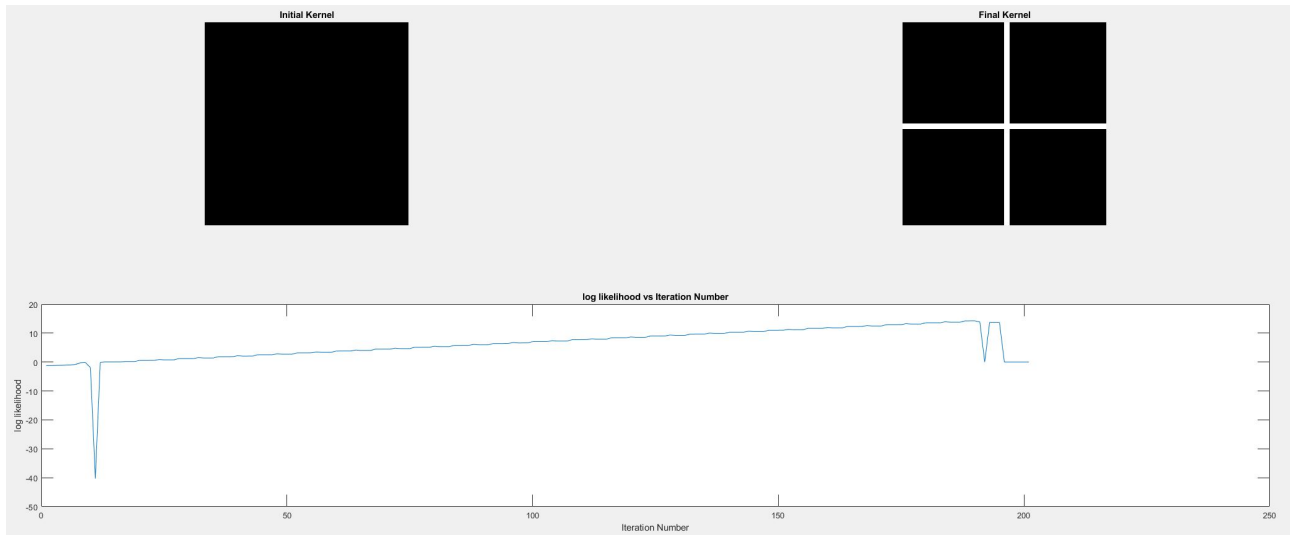
Hyper-parameters:

L2 regularization = 0.001

Learning rate = 0.013

Initialization of kernels and theta layer by randomly picking numbers from  $[0, 0.000001]$

Termination criteria is max iteration = 200



## 4 CNN for LCE Classification

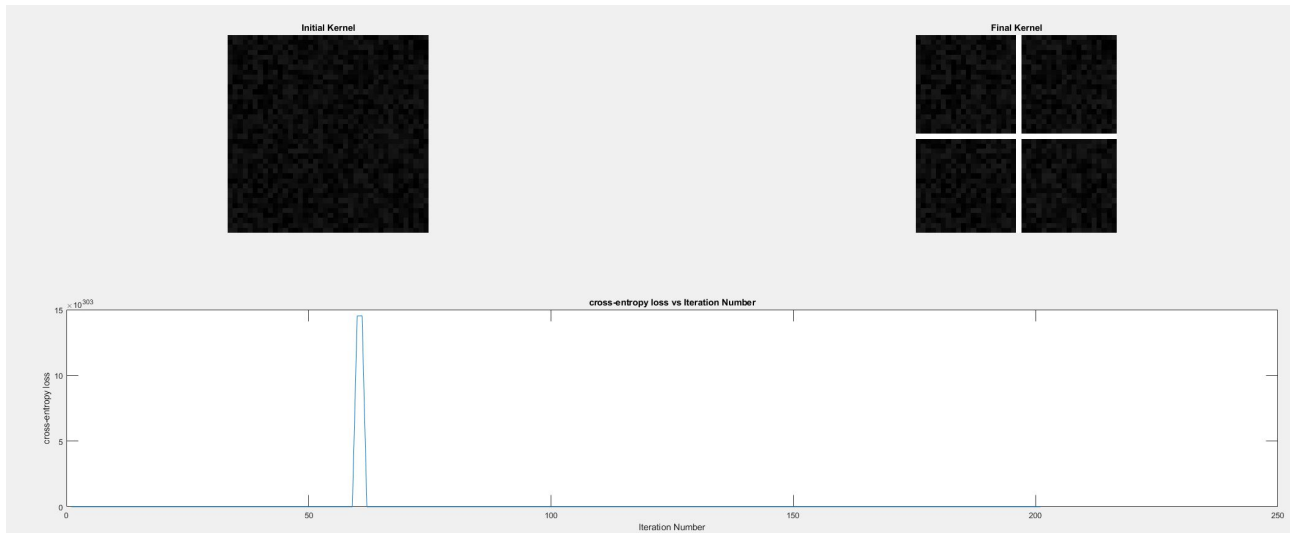
Hyper-parameters:

L2 regularization = 0.001

Learning rate = 0.7

Initialization of kernels and theta layer by randomly picking numbers from  $[0,0.1]$

Termination criteria is max iteration = 200



## 5 CNN With Multiple Kernels

Activation function is linear activation function

Objective function is RMSE (Root mean squared error)

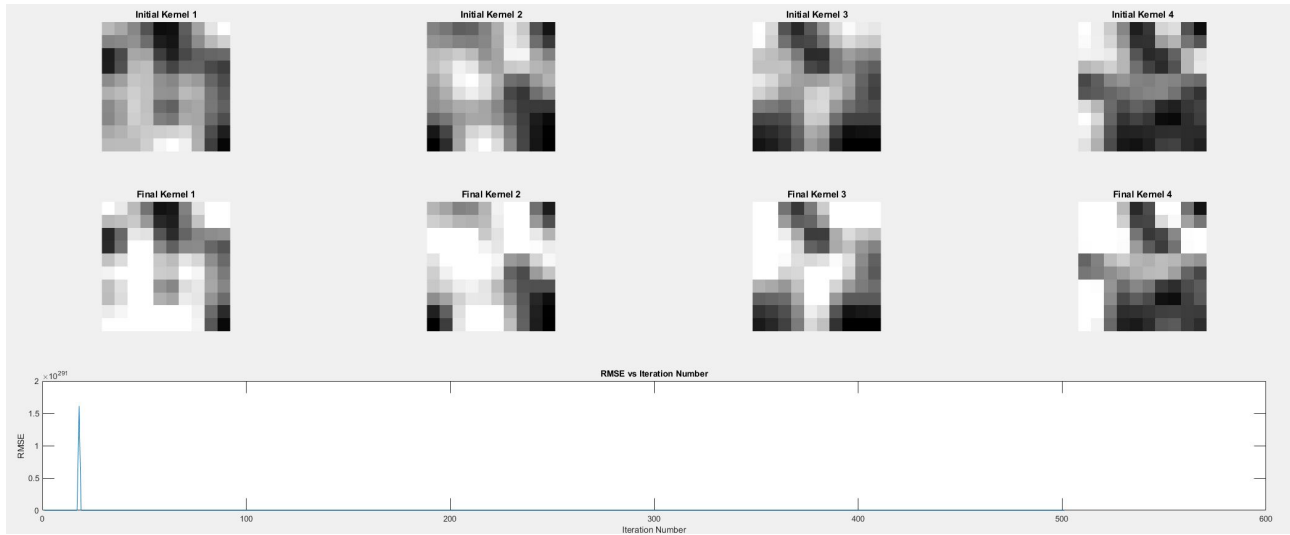
Hyper-parameters:

L2 regularization = 0.1

Learning rate = 0.013

Initialization of kernels and theta layer by randomly picking numbers from  $[0,1]$

Termination criteria is max iteration = 500



## 6 Multi-Kernel CNN For Image Classification

### 6.1 Activation function: logistic (sigmoid) function, Objective Function: log likelihood, max-pooling with width 2 and stride 2, and kernels dimension = 5 X 5 X 4

Hyper-parameters:

L2 regularization = 0.1

Learning rate = 0.02

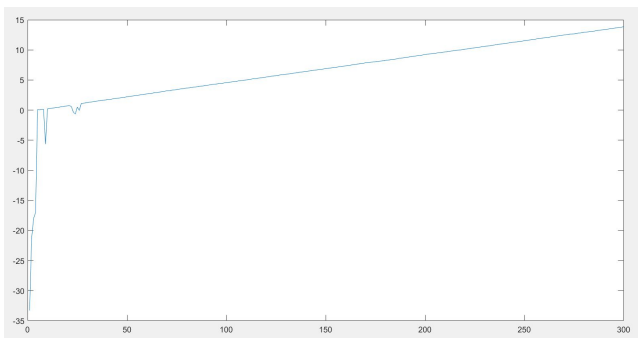
Initialization of kernels and theta layer by randomly picking numbers from  $[0,1]$

Termination criteria is max iteration =

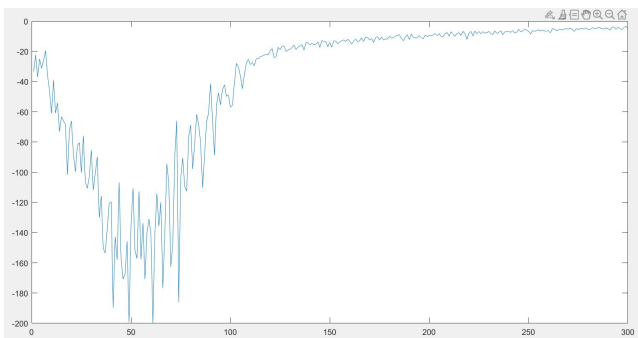
Training Accuracy = 99.61%

Testing Accuracy = 79.61%

Log likelihood vs iterations for Training



Log likelihood vs iterations for Testing



### 6.2 Activation function: logistic (sigmoid) function, Objective Function: log likelihood, max-pooling with width 2 and stride 2, and kernels dimension = 5 X 5 X 2

Hyper-parameters:

L2 regularization = 0.1

Learning rate = 0.01

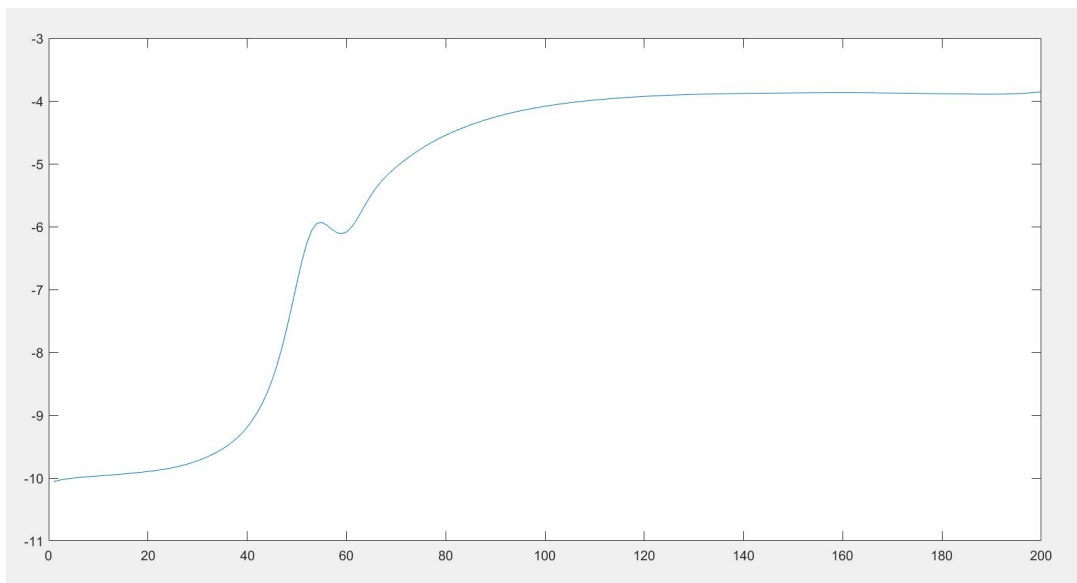
Initialization of kernels and theta layer by randomly picking numbers from  $[0,0.01]$

Termination criteria is max iteration =

Training Accuracy = 99.51%

Testing Accuracy = 80.58%

Log likelihood vs iterations for Training



Log likelihood vs iterations for Testing

