CS383 Assignment 3

Himanshu Gupta

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Question 1 Part 1

The data given is presented by matrix X and labels are presented by matrix Y.

$$TheMatrixX = \begin{bmatrix} -2 \\ -5 \\ -3 \\ 0 \\ -8 \\ -2 \\ 1 \\ 5 \\ -1 \\ 6 \end{bmatrix} \quad TheMatrixY = \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

Now adding the bias feature in the first column of Matrix X, the new X would become $= \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix}$

The derived formula for calculating the coefficients θ used in linear regression $= (X^T X)^{-1} X^T Y$

$$X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -5 & -3 & 0 & -8 & -2 & 1 & 5 & -1 & 6 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 1+1+1+1+1+1+1+1+1+1 & -2-5-3+0-8-2+1+5-1+6 \\ -2-5-3+0-8-2+1+5-1+6 & 4+25+9+0+64+4+1+25+1+36 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 10 & -9 \\ -9 & 169 \end{bmatrix}^{-1} \quad = \frac{\begin{bmatrix} 169 & 9 \\ 9 & 10 \end{bmatrix}}{det(X^T X)} \quad = \frac{\begin{bmatrix} 169 & 9 \\ 9 & 10 \end{bmatrix}}{(1690 - 81)} \quad = \begin{bmatrix} 0.1050 & 0.0056 \\ 0.0056 & 0.0062 \end{bmatrix}$$

$$(X^T X)^{-1} X^T = \begin{bmatrix} 0.1050 & 0.0056 \\ 0.0056 & 0.0062 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -5 & -3 & 0 & -8 & -2 & 1 & 5 & -1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0938 & 0.0771 & 0.0883 & 0.1050 & 0.0603 & 0.0938 & 0.1106 & 0.1330 & 0.0994 & 0.1386 \\ -0.0068 & -0.0255 & -0.0131 & 0.0056 & -0.0441 & -0.0068 & 0.0118 & 0.0367 & -0.0006 & 0.0429 \end{bmatrix}$$

$$(X^T X)^{-1} X^T Y =$$

$$\begin{bmatrix} 0.0938 & 0.0771 & 0.0883 & 0.1050 & 0.0603 & 0.0938 & 0.1106 & 0.1330 & 0.0994 & 0.1386 \\ -0.0068 & -0.0255 & -0.0131 & 0.0056 & -0.0441 & -0.0068 & 0.0118 & 0.0367 & -0.0006 & 0.0429 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

$$\theta = (X^T X)^{-1} X^T Y = \begin{bmatrix} 1.0286 \\ -0.4127 \end{bmatrix}$$

RMSE = 3.7013

Question 1 Part 2

The code for the program is given in Q1.m

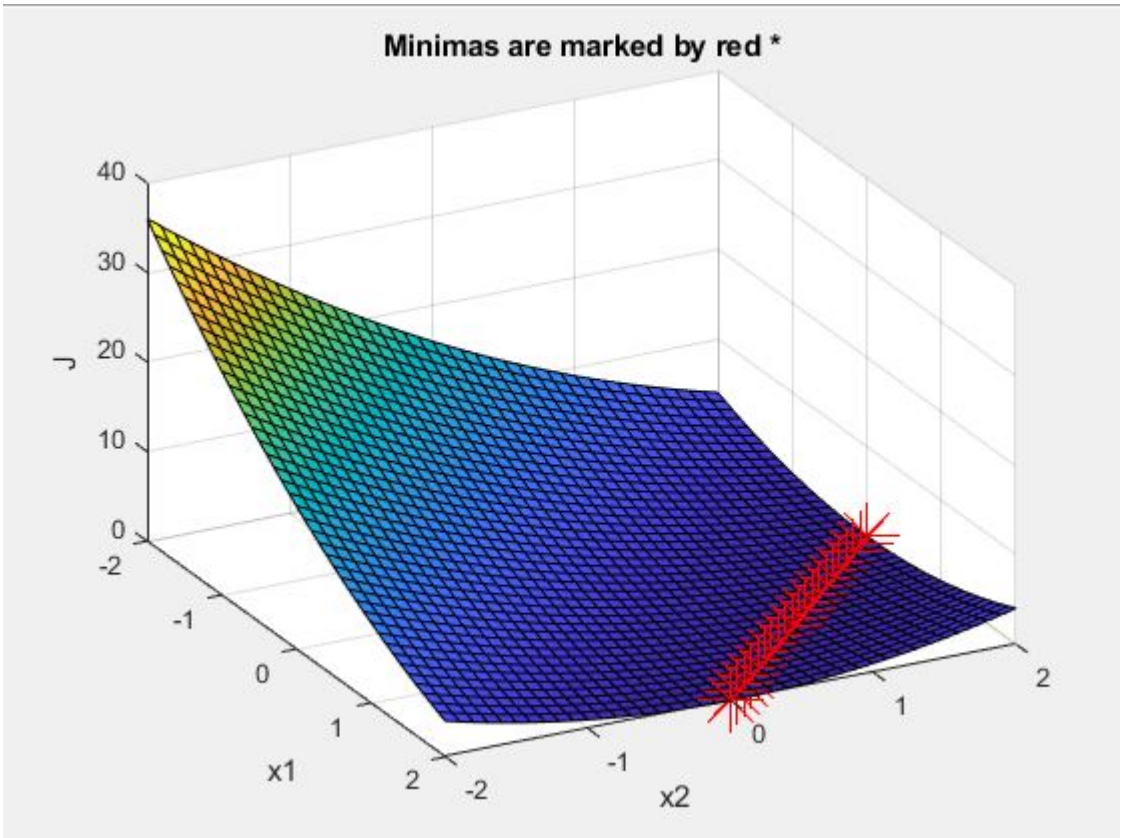
$$\text{Given J} = (x_1 + x_2 - 2)^2$$

a

$$\frac{\partial J}{\partial x_1} = 2(x_1 + x_2 - 2)$$
$$\frac{\partial J}{\partial x_2} = 2(x_1 + x_2 - 2)$$

b

3D plot of $x_1$vs $x_2$vs J



c

The minimas are computed when both the partial derivatives are zero simultaneously which happens to be on the line

$$x_1 + x_2 - 2 = 0$$

which is clearly shown in the figure plotted in part-b.

Also, the minimizing values of $x_1 =$

$$\begin{bmatrix} 0 \\ 0.1000 \\ 0.2000 \\ 0.3000 \\ 0.4000 \\ 0.5000 \\ 0.6000 \\ 0.7000 \\ 0.8000 \\ 0.9000 \\ 1.0000 \\ 1.1000 \\ 1.2000 \\ 1.3000 \\ 1.4000 \\ 1.5000 \\ 1.6000 \\ 1.7000 \\ 1.8000 \\ 1.9000 \\ 2.0000 \end{bmatrix}$$

and $x2 =$

$$\begin{bmatrix} 2.0000 \\ 1.9000 \\ 1.8000 \\ 1.7000 \\ 1.6000 \\ 1.5000 \\ 1.4000 \\ 1.3000 \\ 1.2000 \\ 1.1000 \\ 1.0000 \\ 0.9000 \\ 0.8000 \\ 0.7000 \\ 0.6000 \\ 0.5000 \\ 0.4000 \\ 0.3000 \\ 0.2000 \\ 0.1000 \\ 0 \end{bmatrix}$$

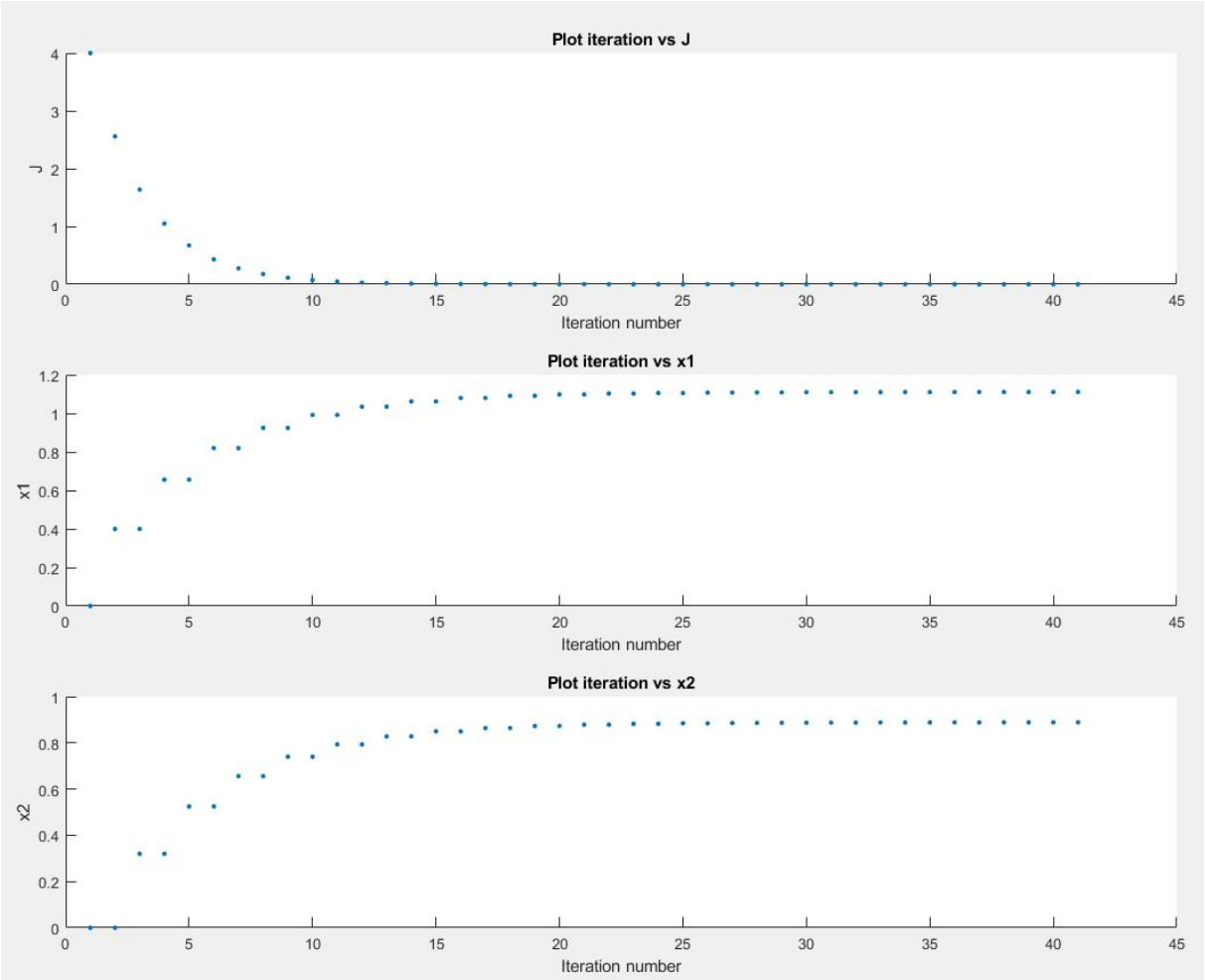
where $J_{min} =$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

These values are over the range of [-2,2] for each variable x_1 and x_2

Question 2

The code for the program is given in Q2.m
All the three plots are shown in a single figure:



Question 3

The code for the program is given in Q3.m

1

$$\text{The final model is y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

where $\theta_0 = -138.1994, \theta_1 = 4.1207, \theta_2 = 0.0330, x_1 =$ Temperature of Water, $x_2 =$ Length of fish and y = Age of the fish

$$\text{The final model is y} = -138.1994 + 4.1207x_1 + 0.0330x_2$$

2

$$\text{RMSE} = 21.0463$$

Question 4

The code for the program is given in Q4.m

1

The average and standard deviation of the root mean squared error for S = 2 over the 20 different seed values are 21.6858 and 1.5325 respectively.

2

The average and standard deviation of the root mean squared error for S = 4 over the 20 different seed values are 21.2291 and 0.8516 respectively.

3

The average and standard deviation of the root mean squared error for S = 22 over the 20 different seed values are 20.9993 and 0.2088 respectively.

4

The average and standard deviation of the root mean squared error for S = N = 44 over the 20 different seed values are 21.0046 and 7.1520e-15 respectively.