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Application of Chebyshev polynomials for solving nonlinear Volterra-Fredholm integral equations system and convergence analysis

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Abstract
In this paper, we solve the nonlinear Volterra-Fredholm integral equations system by using the Chebyshev polynomials. First we introduce the Chebyshev polynomials and approximate functions via their application. Then, we use Chebyshev polynomials as a collocation basis to change the nonlinear Volterra-Fredholm integral equations system to a system of nonlinear algebraic equations. Finally, the convergence analysis is considered, and numerical examples given to illustrate the efficiency of this method.

Keywords: Volterra-Fredholm; System of integral equations; Chebyshev polynomials; Operational matrix.

Introduction

System of nonlinear Volterra-Fredholm integral equations are defined as follows:

$$f_i(s) = g_i(s) - \left(\sum_{j=0}^{n}\int_{-1}^{s} k_{ij}(s,t)[g_j(t)]^{p_j}\,dt\right) - \left(\sum_{j=0}^{n}\int_{-1}^{1} k'_{ij}(s,t)[g_j(t)]^{q_j}\,dt\right),$$

$$i = 0,1,2,...,n, \quad s \in [-1,1], \qquad (1)$$

where, for $i, j = 0,1,2,...,n$ the functions $f_i(s), k_{ij}(s,t)$ and $k'_{ij}(s,t)$ are known and $g_i(s)$ is the unknown functions to be determined, also $p_i, q_i \geq 1$ are positive integers. Equation (1) introduces a system of $n+1$ equations and $n+1$ unknowns.

Up to now several methods have been proposed for solving Volterra-Fredholm equations and it's systems. Yalsinbas (2002) used Taylor polynomials to approximate Volterra-Fredholm integral equations. Also Maleknejad and Mahmodi (2003) applied Taylor polynomials for solving high-order Volterra-Fredholm integro-differential equations. Rabbani *et al.,* (2007) solved Volterra-Fredholm integral equations system using an expansion method. Jumarhan and Mckee (1996) presented a numerical solution method based on integration to solve the nonlinear Volterra-Fredholm integral equations system. Solving the system of Volterra-Fredholm integral equations by Adomian decomposition method is considered in (Maleknejad & Fadaei Yami, 2006). Chuong and Tuan (1996) used Spline-collocation method for solving nonlinear Volterra-Fredholm equations system. Brunner (1990) solved the nonlinear Volterra-Fredholm integral equations by using collocation method. Maleknejad *et al.* (2007) solved nonlinear Volterra integral equations using Chebyshev polynomials. Also Cerdik-Yaslan & Akyuz-Dascioglu (2006) applied Chebyshev polynomials for solving Volterra-Fredholm integro-differential equations. Very recently, we used Chebyshev polynomials for solving nonlinear Volterra-Fredholm integral equations (Ezzati & Najafalizadeh, 2011).

Chebyshev polynomials of the first kind of degree n are defined as follows (Chihara, 1978):

$$T_n(s) = \cos(n\theta), \ \ \theta = \arccos(s), \quad n \geq 0.$$

Also we'll have the following recursive formula for these polynomials (Chihara, 1978):

$$T_0(s) = 1,$$

$$T_1(s) = s,$$

$$T_{n+1}(s) = 2sT_n(s) - T_{n-1}(s), \quad n = 1,2,3,... \qquad (2)$$

Inner product in the interval [-1,1] for Chebyshev polynomials is defined by (Chihara, 1978):

$$< T_i(s), T_j(s) >= \int_{-1}^{1} T_i(s)T_j(s)\omega(s)ds \qquad (3)$$

where

$$\omega(s) = (1-s^2)^{\frac{-1}{2}}.$$

With respect to the inner product which is defined in (3) Chebyshev polynomials are orthogonal (Chihara, 1978):

$$(T_i(s), T_j(s)) = \begin{cases} \pi, \ i = j, \\ \\ \dfrac{\pi}{2}\delta_{ij}, \ i \neq j. \end{cases} \qquad (4)$$

where

$$\delta_{ij} = \begin{cases} 1, \ i = j, \\ \\ 0, \ i \neq j. \end{cases}$$

In this paper, we approximate functions by using Chebyshev polynomials and we present operational matrices for integration of vectors. Then Chebyshev polynomials defined in (2) are used as a collocation basis to solve system (1) and reduce it to a system of algebraic equations. The generated algebraic system, which according to the type of system (1) would be either linear or nonlinear. Newton's iterative method can be used for solving nonlinear algebraic system. Finally, we introduce two theorems and proofs for convergence analysis.

Approximation the function by using a series of Chebyshev polynomials

If $f(s)$ be a function in $[a,b]$ and $\{v_i\}_{i=0}^{\infty}$ be orthogonal on this interval, then $f(s)$ can be shown as follows:

$$f(s)=\sum_{i=0}^{\infty}\alpha_i v_i(s), \tag{5}$$

where α_i are Fourier coefficients that are as [11,12]:

$$\alpha_i=(f(s),v_i(s)), \tag{6}$$

As we mentioned above, we also can write the above series for the Chebyshev orthogonal basis, if $f(s)$ is defined in the interval [-1,1], by using Chebyshev polynomials of the first kind, relation (5) can be written as follows:

$$f(s)=\sum_{i=0}^{\infty}c_i T_i(s), \tag{7}$$

if the infinite series in (7) is truncated, then we'll have:

$$f(s)=\sum_{i=0}^{N}c_i T_i(s)=C^T T(s), \tag{8}$$

where C and T are $(N+1)\times 1$ definite vectors as follows:

$$C=[c_0,c_1,c_2,\ldots,c_N]^T, \tag{9}$$

$$T(s)=[T_0(s),T_1(s),T_2(s),\ldots,T_N(s)]^T. \tag{10}$$

Coefficients c_i are given as (6) where inner product with the weight function $\omega(s)=(1-s^2)^{-1/2}$ is:

$$c_i=(f(s),T_i(s))=\begin{cases}\dfrac{1}{\pi}\displaystyle\int_{-1}^{1}\omega(s)f(s)ds, & i=0,\\[2ex] \dfrac{2}{\pi}\displaystyle\int_{-1}^{1}\omega(s)T_i(s)f(s)ds, & i>0.\end{cases}$$

((11)

For the positive integer powers of a function $f(s)$, we have:

$$[f(s)]^p=[C^T T(s)]^p=C_p^{*T}T(s), \tag{12}$$

where C and T are defined vectors in (9), (10), and C_p^* is a column vector and it's elements are nonlinear combinations of the elements of vector C. C_p^* is called operational vector of p th power. Maleknejad *et al.* (2006) compute the second and third product operational vector by using Chebyshev polynomials as follows:

$$C_2^*=\frac{1}{2}\begin{pmatrix}2c_0^2+c_1^2+c_2^2+c_3^2\\ 4c_0c_1+2c_1c_2+2c_2c_3\\ c_1^2+4c_0c_2+c_1c_3\\ 2c_1c_2+4c_0c_3\end{pmatrix},$$

also

$$C_3^*=\frac{1}{4}\begin{pmatrix}4c_0^3+6c_0c_1^2+3c_1^2c_2+6c_0c_2^2+6c_1c_2c_3+6c_0c_3^2\\ 12c_0^2c_1+3c_1^3+12c_0c_1c_2+6c_1c_2^2+3c_1^2c_3+12c_0c_2c_3+3c_2^2c_3+6c_1c_3^2\\ 6c_0c_1^2+12c_0^2c_2+6c_1^2c_2+3c_2^3+12c_0c_1c_3+6c_1c_2c_3+6c_2c_3^2\\ c_1^3+12c_0c_1c_2+3c_1c_2^2+12c_0^2c_3+6c_1^2c_3+6c_2^2c_3+3c_3^3\end{pmatrix}.$$

Similarly, regarding a function $k(s,t)$, with two variables, which is defined on [-1,1], we'll have:

$$k(s,t)=\sum_{i=0}^{N}\sum_{j=0}^{N}T_i(s)k_{ij}T_j(t), \tag{13}$$

where

$$K_{ij}=(T_i(s),(k(s,t),T_j(t))). \tag{14}$$

By choosing $T(s)$ as (10) and K as a $(N+1)\times(N+1)$ matrix with elements of k_{ij}, equation (13) can be written as follows:

$$k(s,t)=T^T(s)KT(t). \tag{15}$$

The operational matrices for integration

In this section we present the operational matrix as P for computing the integral of vector (10), (Rao, 1983) We have the following relation about Chebyshev polynomial:

$$\int_{-1}^{s}T_{N-1}(t)dt=\frac{1}{2N}T_N(s)-\frac{1}{2(N-2)}T_{N-2}(s)+\frac{(-1)^{N-1}}{1-(N-1)^2}T_0(s),\quad N\geq 3.$$

(16)

also for $T_0(s)$ and $T_1(s)$ we have:

$$\int_{-1}^{s}T_0(t)dt=T_0(s)+T_1(s),$$

$$\int_{-1}^{s}T_1(t)dt=\frac{-1}{4}T_0(s)+\frac{1}{4}T_2(s). \tag{17}$$

Equations (16) and (17) allow us to write:

$$\int_{-1}^{s}T(t)dt=PT(s), \tag{18}$$

$$\int_{-1}^{1}T(t)dt=PT(1), \tag{19}$$

where P is the $(N+1)\times(N+1)$ operational matrix as follows:

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$$P=\begin{pmatrix} 1 & 1 & 0 & 0 & . & . & . & 0 & 0 \\ -\frac{1}{4} & 0 & \frac{1}{4} & 0 & . & . & . & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{2} & 0 & \frac{1}{6} & . & . & . & 0 & 0 \\ \frac{1}{8} & 0 & -\frac{1}{4} & 0 & . & . & . & 0 & 0 \\ . & . & . & . & . & & & . & . \\ . & . & . & . & & . & & . & . \\ . & . & . & . & & & . & . & . \\ \frac{(-1)^{N-1}}{1-(N-1)^2} & 0 & 0 & 0 & . & . & . & 0 & \frac{1}{2N} \\ \frac{(-1)^N}{1-N^2} & 0 & 0 & 0 & . & . & . & -\frac{1}{2(N-1)} & 0 \end{pmatrix}. \quad (20)$$

For Chebyshev polynomials we have:

$$T(s)T^T(s)C=\tilde{C}^T T(s), \quad (21)$$

where C is a vector in (9) and $\tilde{C}$ is a $(N+1)\times(N+1)$ square matrix as follows:

$$\tilde{C}=\frac{1}{2}\begin{pmatrix} 2c_0 & c_1 & . & . & . & c_i & . & . & . & c_{N-1} & c_N \\ 2c_1 & 2c_0+c_2 & . & . & . & c_{i-1}+c_{i+1} & . & . & . & c_{N-2}+c_N & c_{N-1} \\ . & . & . & & & . & & & & . & . \\ . & . & & . & & . & . & . & . & . & . \\ . & . & & & . & . & & & & . & . \\ 2c_i & c_{i-1}+c_{i+1} & . & . & . & 2c_0+c_{2i} & . & . & . & c_{N-i-1} & c_{N-i} \\ . & . & & & & . & . & & & . & . \\ . & . & . & . & . & & & . & & . & . \\ . & . & & & & . & & & . & . & . \\ 2c_{N-1} & c_{N-2}+c_N & . & . & . & c_{N-i-1} & . & . & . & 2c_0 & c_1 \\ 2c_N & c_{N-1} & . & . & . & c_{N-i} & . & . & . & c_1 & 2c_0 \end{pmatrix},$$

((22)

where $i=[\frac{N}{2}]$.

Description of the method

In this section, we solve the nonlinear Volterra-Fredholm integral equations system by using the Chebyshev polynomials of the first kind.

With respect to the method of Section 2 for $i,j=0,1,2,...,n$ we have:

$$g_i(s)=T^T(s)G_i,$$

$$[g_j(s)]^m=T^T(s)G^*_{jm}, \quad for \ \ m=p_j,q_j,$$

$$k'_{ij}(s,t)=T^T(s)K'_{ij}T(t),$$

$$k_{ij}(s,t)=T^T(s)K_{ij}T(t), \quad (23)$$

where $G^*_{jp_j}$ and $G^*_{jq_j}$ are operational vectors defined in Section 2 and G_i is a $(N+1)\times 1$ vector

$$G_i=[g_{i0},g_{i1},g_{i2},...,g_{iN}]^T. \quad (24)$$

Now, with substituting equation (23) in system (1) we'll have:

$$f_i(s)=T^T(s)G_i-(\sum_{j=0}^{n}\int_{-1}^{s}T^T(s)K_{ij}T(t)T^T(t)G^*_{jp_j}\,dt)$$

$$-(\sum_{j=0}^{n}\int_{-1}^{1}T^T(s)K'_{ij}T(t)T^T(t)G^*_{jq_j}\,dt),$$

$$=T^T(s)G_i-(\sum_{j=0}^{n}T^T(s)K_{ij}\int_{-1}^{s}T(t)T^T(t)G^*_{jp_j}\,dt)-(\sum_{j=0}^{n}T^T(s)K'_{ij}\int_{-1}^{1}T(t)T^T(t)G^*_{jq_j}\,dt),$$

$$=T^T(s)G_i-(\sum_{j=0}^{n}T^T(s)K_{ij}\tilde{G}^*_{jp_j}\int_{-1}^{s}T(t)dt)-(\sum_{j=0}^{n}T^T(s)K'_{ij}\tilde{G}^*_{jq_j}\int_{-1}^{1}T(t)dt).$$

(25)

$$f_i(s)=T^T(s)G_i-(\sum_{j=0}^{n}T^T(s)K_{ij}\tilde{G}^*_{jp_j}PT(s))$$

$$-(\sum_{j=0}^{n}T^T(s)K'_{ij}\tilde{G}^*_{jq_j}PT(1)), \ \ i=0,1,2,...,n. \quad (26)$$

Hence Equation (26) represent a system with $(n+1)$ equations and $(n+1)\times(N+1)$ unknowns, so we rewrite each equation of the system at the collocation points of $\{s_k\}_{k=0}^{\infty}$ in the interval [-1,1]. Then we'll have a system with $(n+1)\times(N+1)$ equations and $(n+1)\times(N+1)$ unknowns:

$$f_i(s_k)=T^T(s_k)G_i-(\sum_{j=0}^{N}T^T(s_k)K_{ij}\tilde{G}^*_{jp_j}PT(s_k))-(\sum_{j=0}^{N}T^T(s_k)K'_{ij}\tilde{G}^*_{jq_j}PT(1)),$$

$$for \ \ i=0,1,2,...,n \ \ and \ \ k=0,1,2,...,N. \quad (27)$$

Relation (27) leads to a linear or nonlinear system of equations such that the unknown coefficients can be found.

Convergence analysis

We can show the nonlinear terms in equation (1) by $F(g_j)=[g_j(t)]^{p_j}$ and $F'(g_j)=[g_j(t)]^{q_j}$. Let $(C[-1,1],\|.\|)$ be the Banach space of all continuous functions on interval [-1,1] with norm $\|f\|_\infty=\max_{\forall s\in[-1,1]}|f(s)|$. Suppose the nonlinear terms $F(u)$ and $F'(u)$ are satisfied in Lipschitz condition

$$|F(u)-F(v)|\le L_1|u-v|,$$

and

$$|F'(u)-F'(v)|\le L_2|u-v|.$$

We also assume for all $i,j=0,1,2,...,n,\ |k_{ij}(s,t)|\le M$ and $|k'_{ij}(s,t)|\le M'$. We show exact solutions of the nonlinear Volterra-Fredholm integral equations system by $g_j(s)$ and approximate solutions of the nonlinear Volterra-Fredholm integral equations system for N by

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$\bar{g}_{jN}(s)$. Moreover, we define $\alpha = ML_1(s+1)+2M^{'}L_2$. So, we are ready for presenting two theorems about convergence analysis.

Theorem 5.1 *For $n=0$ the solution of the nonlinear Volterra-Fredholm integral equation by using Chebyshev polynomials is convergent if $0<\alpha<1$.*

Proof.

$$\|g_0-\bar{g}_{0N}\|_\infty = \max_{\forall s\in[-1,1]} |g_0(s)-\bar{g}_{0N}(s)|$$

$$= \max_{\forall s\in[-1,1]} |(\sum_{j=0}^{n}\int_{-1}^{s}k_{00}(s,t)(F(g_0)-F(\bar{g}_{0N}))dt)$$

$$-(\sum_{j=0}^{n}\int_{-1}^{1}k_{00}^{'}(s,t)(F^{'}(g_0)-F^{'}(\bar{g}_{0N}))dt)|$$

$$\le ML_1(s+1)\|g_0-\bar{g}_{0N}\|_\infty + 2M^{'}L_2\|g_0-\bar{g}_{0N}\|_\infty$$

$$=\alpha\|g_0-\bar{g}_{0N}\|_\infty.$$

$$\Rightarrow \|g_0-\bar{g}_{0N}\|_\infty < \alpha\|g_0-\bar{g}_{0N}\|_\infty. \quad (28)$$

By selection $0<\alpha<1$ we'll have:

$$N\to\infty,\quad \|g_0-\bar{g}_{0N}\|_\infty \to 0,$$

so the proof is completed.

Theorem 5.2 *For $n\ge 1$ the solution of the nonlinear Volterra-Fredholm integral equations system by using Chebyshev polynomials is convergent if*

$$0<\alpha<\frac{1}{1+n}.$$

Proof. Let us consider the following norm for the i th equation of system (1.1):

$$\|g_i-\bar{g}_{iN}\|_\infty = \max_{\forall s\in[-1,1]} |g_i(s)-\bar{g}_{iN}(s)|$$

$$= \max_{\forall s\in[-1,1]} |(\sum_{j=0}^{n}\int_{-1}^{s}k_{ij}(s,t)(F(g_j)-F(\bar{g}_{jN}))dt)$$

$$-(\sum_{j=0}^{n}\int_{-1}^{1}k_{ij}^{'}(s,t)(F^{'}(g_j)-F^{'}(\bar{g}_{jN}))dt)|$$

$$\le \max_{\forall s\in[-1,1]} (\sum_{j=0}^{n}\int_{-1}^{s}|k_{ij}(s,t)||(F(g_j)+F(\bar{g}_{jN})|dt$$

$$-\sum_{j=0}^{n}\int_{-1}^{1}|k_{ij}^{'}(s,t)||(F^{'}(g_j)-F^{'}(\bar{g}_{jN}))|dt)$$

$$\le \sum_{j=0}^{n}ML_1\int_{-1}^{s}\|g_j-\bar{g}_{jN}\|_\infty dt + \sum_{j=0}^{n}M^{'}L_2\int_{-1}^{1}\|g_j-\bar{g}_{jN}\|_\infty dt$$

$$=\sum_{j=0}^{n}(ML_1(s+1)+2M^{'}L_2)\|g_j-\bar{g}_{jN}\|_\infty,$$

$$\Rightarrow \|g_j-\bar{g}_{jN}\|_\infty \le \sum_{j=0}^{n}\alpha\|g_j-\bar{g}_{jN}\|_\infty. \quad (29)$$

If we rewrite relation (29) for $i=0,1,2,...,n$ and then add up the obtained unequal extremes, we'll have:

$$\Rightarrow \sum_{i=0}^{n}\|g_i-\bar{g}_{iN}\|_\infty \le \sum_{j=0}^{n}(n+1)\alpha\|g_j-\bar{g}_{jN}\|_\infty, \quad (30)$$

so

$$\Rightarrow \sum_{i=0}^{n}(1-(1+n)\alpha)\|g_i-\bar{g}_{iN}\|_\infty < 0, \quad (31)$$

according to relation (31) by selecting $0<\alpha<\frac{1}{1+n}$ we'll have:

$$N\to\infty,\quad \|g_i-\bar{g}_{iN}\|_\infty \to 0,$$

so the proof is completed.

Examples

In this section, the efficiency of the presented method is shown in following three examples. In examples 1 and 2, we use Newton's iterative method for solving the generated nonlinear system. Mathematica 5.2 software is applied in computing examples.

Example 1. As a first example we have the following system with 2 equations and 2 unknowns:

$$f_1(s) = g_1(s)-\int_{-1}^{1}st^2g_1(t)dt-\int_{-1}^{1}(st-1)g_2(t)dt,$$

$$f_2(s) = g_2(s)-\int_{-1}^{s}(s-t)g_1(t)dt-\int_{-1}^{s}2tg_2(t)dt, \quad (32)$$

where $f_1(s)=2s+1$ and $f_2(s)=-\frac{2}{3}s^3+1$. The exact solutions of the above system are $g_1(s)=2s-1$ and $g_2(s)=s+1$. Table 1 illustrates the numerical results for $N=9$ and $N=11$.

Example 2. Consider the following nonlinear integral equations system:

$$f_1(s) = g_1(s)-\int_{-1}^{s}16st[g_1(t)]^2dt-\int_{-1}^{1}s^2t^3g_2(t)dt,$$

Table 1. The Numerical results of Example 6.1.

	Exact solution		Approximation solution with N=9		Approximation solution with N=11	
x_i	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$
-1	-3	0	-2.99998	-0.00977	-3	0.00011
-0.75	-2.5	.25	-2.49998	0.24262	-2.5	0.25005
-0.5	-2	.5	-1.99998	0.49506	-2	0.50002
-0.25	-1.5	.75	-1.49998	0.74751	-1.5	0.75
0	-1	1	-0.99998	1	-1	1
0.25	-0.5	1.25	-0.49998	1.25	-0.5	1.25
0.5	0	1.5	0.00002	1.50506	0	1.49999
0.75	.5	1.75	0.50002	1.75763	0.5	1.74997
1	1	2	1.00002	2.01023	1	1.99999

$$f_2(s) = g_2(s) - \int_{-1}^{s}(3s^2 - 2t)[g_2(t)]^2 dt - \int_{-1}^{1}(2t-4)g_1(t)dt, \quad (33)$$

where $f_1(s) = s^5 - \frac{2}{5}s^2 - \frac{s}{2}$, $f_2(s) = -s^5 - \frac{5}{2}s^4 - \frac{5}{3}s^5 + s + \frac{1}{6}$, and the exact solutions $g_1(s) = \frac{s}{2}$ and $g_2(s) = s+1$. Table 2 illustrates the numerical results.

Table 2. The Numerical results of Example 5.2.

	Exact solution		Approximation solution with N=10		Approximation solution with N=12	
x_i	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$
-1	-0.5	0	-0.48378	-0.03068	-0.49910	-0.00045
-0.75	-0.375	.25	-0.36599	0.22503	-0.37468	0.24956
-0.5	-0.25	.5	-0.24563	0.48074	-0.24999	0.49957
-0.25	-0.125	.75	-0.12360	0.73645	-0.12505	0.74959
0	0	1	0	0.99216	0	0.9996
0.25	0.125	1.25	0.12516	1.24788	0.12505	1.24961
0.5	0.25	1.5	0.25188	1.50359	0.24999	1.49963
0.75	0.375	1.75	0.380278	1.75930	0.37468	1.74964
1	0.5	2	0.51122	2.01501	0.49910	1.99965

Example 3. As a last example, we have the following nonlinear Volterra-Fredholm integral equations system:

$$f_1(s) = g_1(s) - \int_{-1}^{s}(t^2 - s)g_1(t)dt - \int_{-1}^{1}st^2 g_1(t)dt - \int_{-1}^{1}(t+1)s[g_2(t)]^2 dt,$$

$$f_2(s) = g_2(s) - \int_{-1}^{s}2g_2(t)dt - \int_{-1}^{1}3s[g_1(t)]^2 dt, \quad (34)$$

where $f_1(s) = -\frac{s^4}{4} + \frac{5s^3}{6} - s^2 - \frac{s}{10} - \frac{5}{12}$ and $f_2(s) = -\frac{2}{3}s^3 + 2s^2 - 9s - \frac{5}{2}$.

The exact solution of above system is $g_1(s) = s-1$ and $g_2(s) = s^2 - s$. Table 3 shows the numerical results for N=10 and N=12.

Table 3. The Numerical results of Example 5.3.

	Exact solution		Approximation solution with N=10		Approximation solution with N=12	
x_i	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$
-1	-2	2	-2.01013	1.98970	-2.00010	2.00057
-0.75	-1.75	1.3125	-1.75755	1.30476	-1.75007	1.31291
-0.5	-1.5	0.75	-1.50501	0.74482	-1.50005	0.75025
-0.25	-1.25	0.3125	-1.25249	0.30989	-1.25002	0.31261
0	-1	0	-1	-0.00002	-1	-0.00001
0.25	-0.75	0.1875	-0.74754	-0.18493	-0.74998	-0.18762
0.5	-0.5	-0.25	-0.49511	-0.24483	-0.49996	-0.25022
0.75	-0.25	-0.1875	-0.24270	-0.17972	-0.24994	-0.18780
1	0	0	0.00967	0.01040	0.00008	-0.00039

Conclusion

In this paper, we solved a system of Volterra-Fredholm integral equations by using Chebyshev collocation method. The properties of Chebyshev polynomials are used to reduce the system of Volterra-Fredholm integral equations to a system of nonlinear algebraic equations. Computations are excuted using Mathematica 5.2 software. Three numerical examples demonstrate the validity and efficiency of proposed method.

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