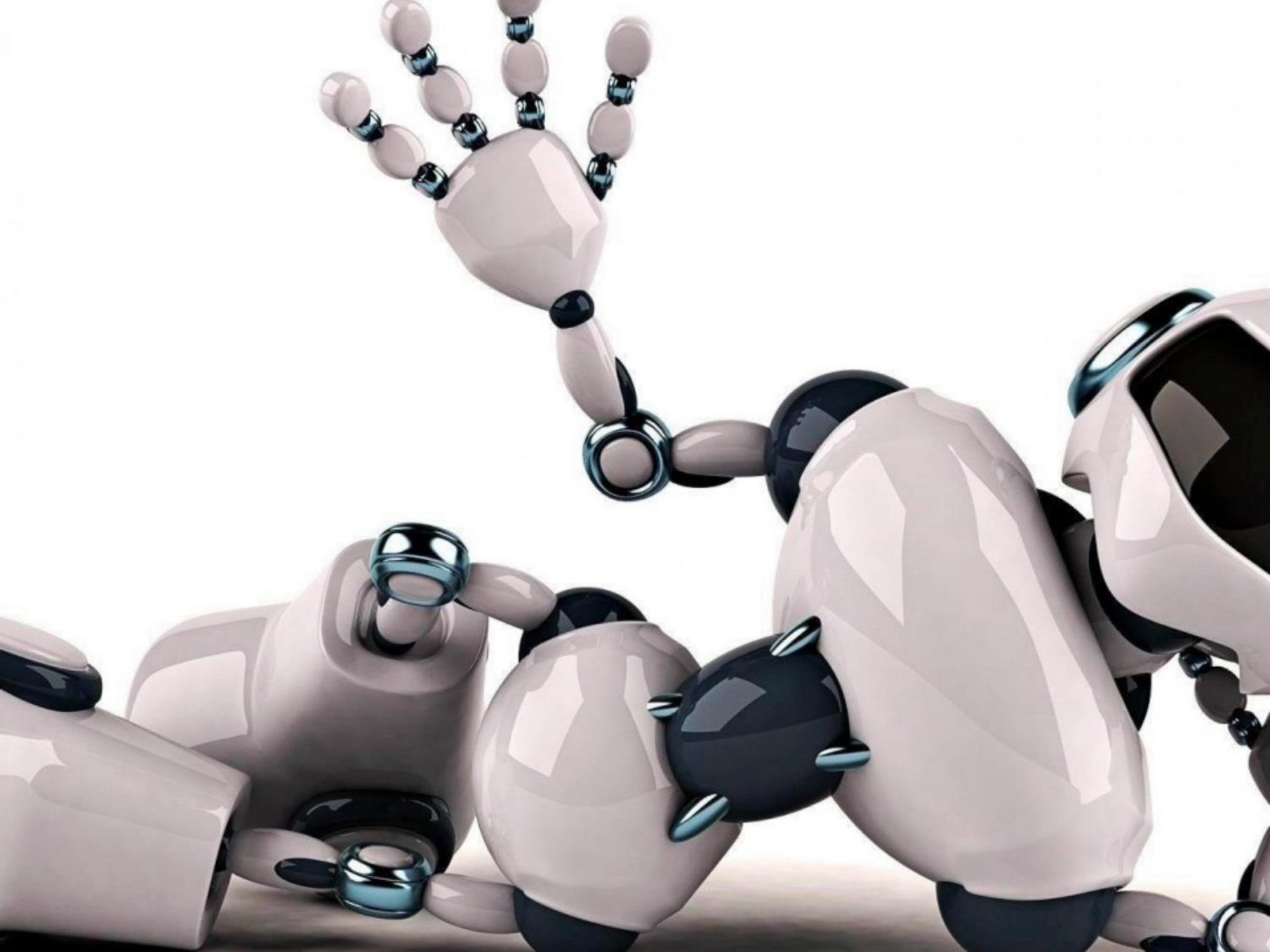

Thesis:

Synthesis of control of mobile robot with dynamic system for generating of control rules.

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Objective:

- The aim of the thesis is to model management system for a mobile robot through dynamic assignments that determine the behavior of mobile robot at a certain movement and changes in the environment.

Methods used to achieve the target :

Dynamical model:

Oyler- Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad (1.1)$$

Is transforming to:

$$a\ddot{\theta}_r + b\ddot{\theta}_l = M_r - k\dot{\theta}_r \qquad (1.2)$$

$$b\ddot{\theta}_r + a\ddot{\theta}_l = M_l - k\dot{\theta}_l$$

Transformation of the speed:

$$\ddot{\theta}_r = \dot{\omega}_r, \dot{\theta}_r = \omega_r, \ddot{\theta}_l = \dot{\omega}_l, \dot{\theta}_l = \omega_l, \dot{\theta} = \omega$$

Transformation into Hamilton equation:

$$\mathbf{H}\dot{\boldsymbol{\omega}} + \mathbf{K_{fc}}\boldsymbol{\omega} = \boldsymbol{\tau} \qquad (1.3)$$

Kinematic model:

„Unicycle“ model:

$$\dot{x} = v\cos(\theta)$$
$$\dot{y} = v\sin(\theta) \qquad (1.4)$$
$$\dot{\theta} = \omega$$

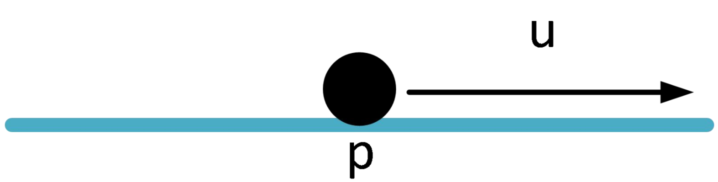
Differential mobile robot:

$$\dot{x} = \frac{R}{2}(\omega_r + \omega_l)\cos(\theta)$$
$$\dot{y} = \frac{R}{2}(\omega_r + \omega_l)\sin(\theta) \qquad (1.6)$$
$$\dot{\theta} = \frac{R}{l}(\omega_r - \omega_l)$$

Connection between the speed:

$$\omega_r = \frac{2v + l\omega}{2R}$$
$$(1.5)$$
$$\omega_l = \frac{2v - l\omega}{2R}$$

Generating the references of control behaviors:



$$\ddot{p} = u$$

Fig.1 Point behavior

State variables:

$$x_1 = p \qquad\qquad (1.7)$$
$$x_2 = \dot{p}$$

Dynamic of state variables:

$$\dot{x}_1 = x_2 \qquad\qquad (1.8)$$
$$\dot{x}_2 = u$$

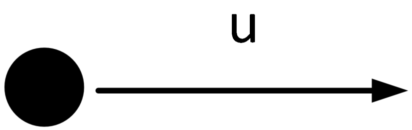
Vector matrix :

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ u \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad (1.9)$$

Stationary sistem:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \qquad\qquad (1.10)$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

Go to goal.



$$\dot{x} = u,\ x \in R^2$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{u}$$

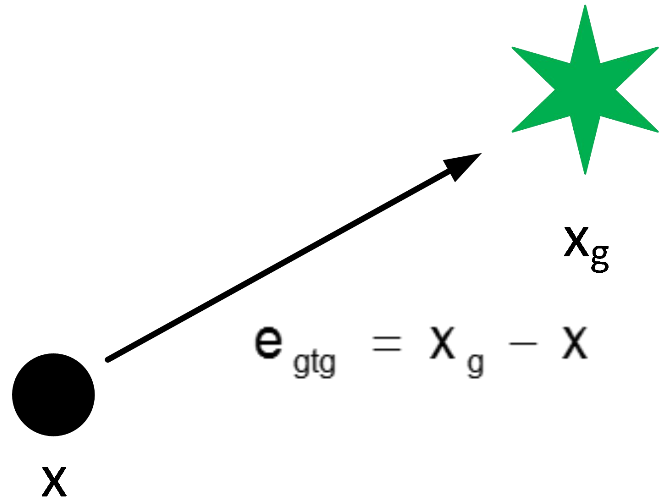


Fig.2 Go to goal behavior.

Control law:

$$\mathbf{u}_{\mathbf{gtg}} = \mathbf{K}_{\mathbf{gtg}}\mathbf{e}_{\mathbf{gtg}} \qquad (1.11)$$

Avoiding obstacle:

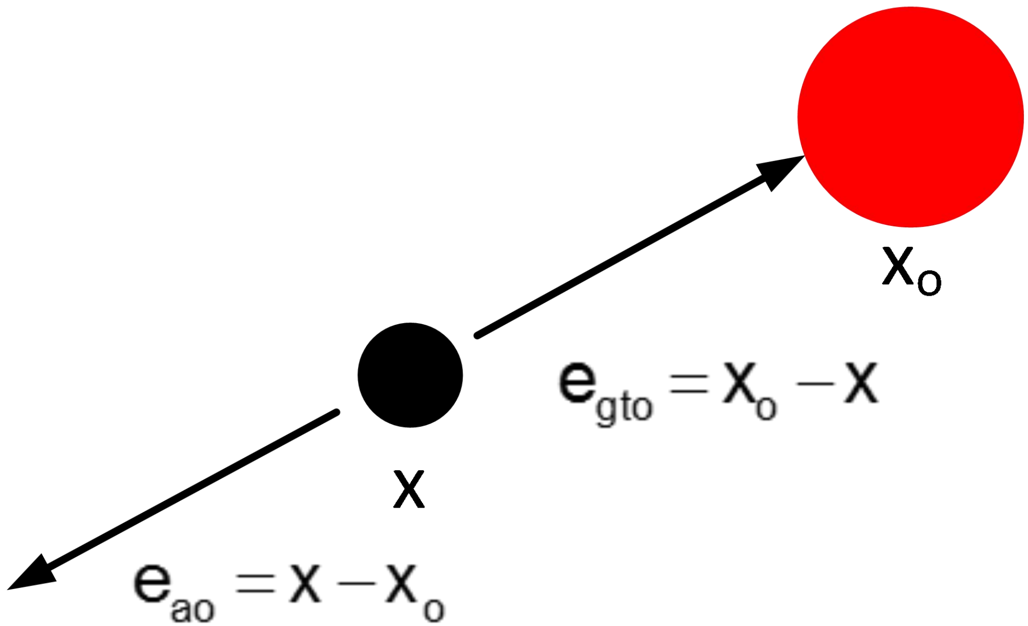


Fig.3 Avoiding obstacles behavior.

Control law:

$$\mathbf{u}_{ao} = \mathbf{K}_{ao}\mathbf{e}_{ao} \tag{1.12}$$

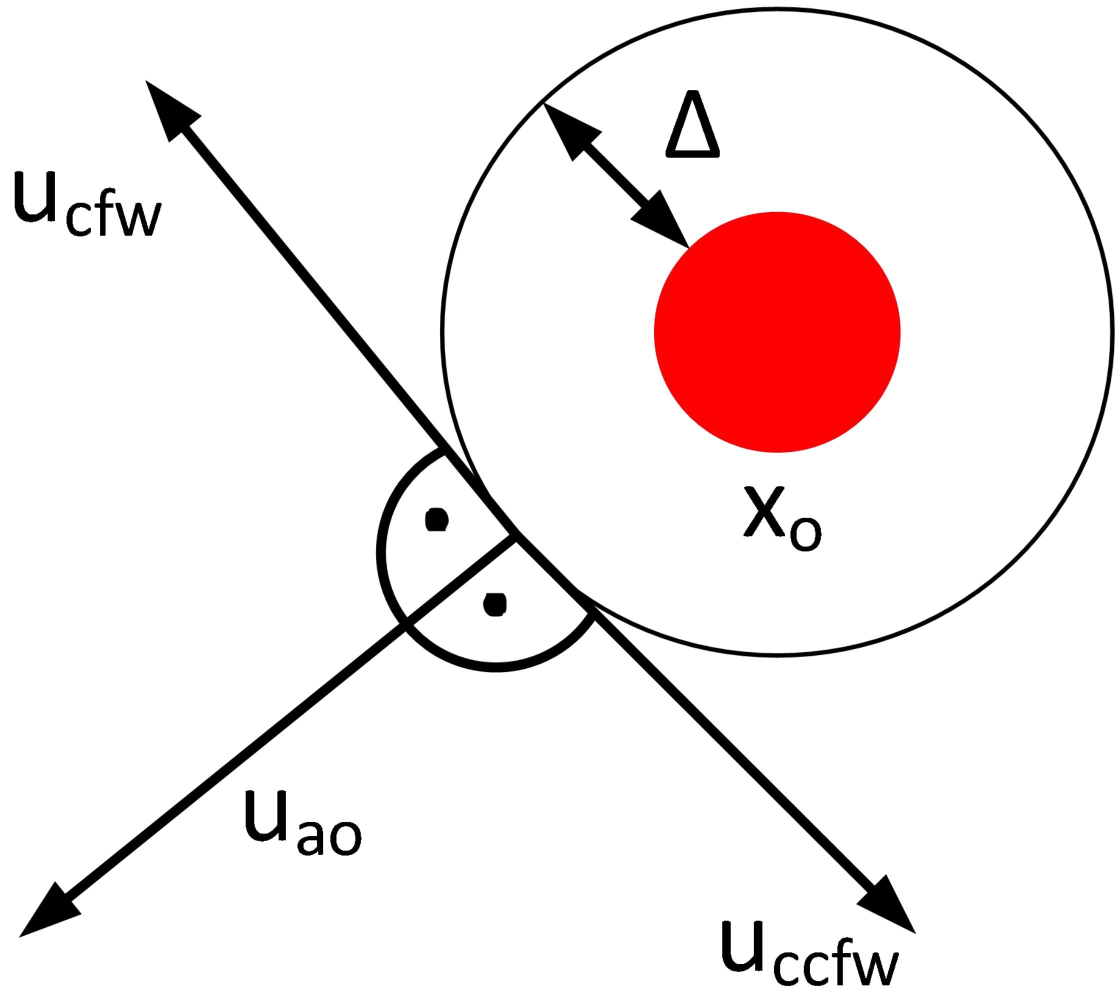


Fig.4 “Follow the wall” behavior.

Control methods for "follow the wall" behavior:

Clockwise follow:

$$u_{cfw} = \alpha\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} u_{ao} = \alpha\begin{bmatrix} \cos\left(-\frac{\pi}{2}\right) & -\sin\left(-\frac{\pi}{2}\right) \\ \sin\left(-\frac{\pi}{2}\right) & \cos\left(-\frac{\pi}{2}\right) \end{bmatrix} u_{ao} = \alpha\mathbf{R}\left(-\frac{\pi}{2}\right) u_{ao} \qquad (1.13a)$$

Counterclockwise follow:

$$u_{ccfw} = \alpha\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} u_{ao} = \alpha\begin{bmatrix} \cos\left(\frac{\pi}{2}\right) & -\sin\left(\frac{\pi}{2}\right) \\ \sin\left(\frac{\pi}{2}\right) & \cos\left(\frac{\pi}{2}\right) \end{bmatrix} u_{ao} = \alpha\mathbf{R}\left(\frac{\pi}{2}\right) u_{ao} \qquad (1.13b)$$

where **R** is the rotational matrix which have to make the following the wall to be at semicircle instead of line and α is constant which limiting the length of vector **u**.

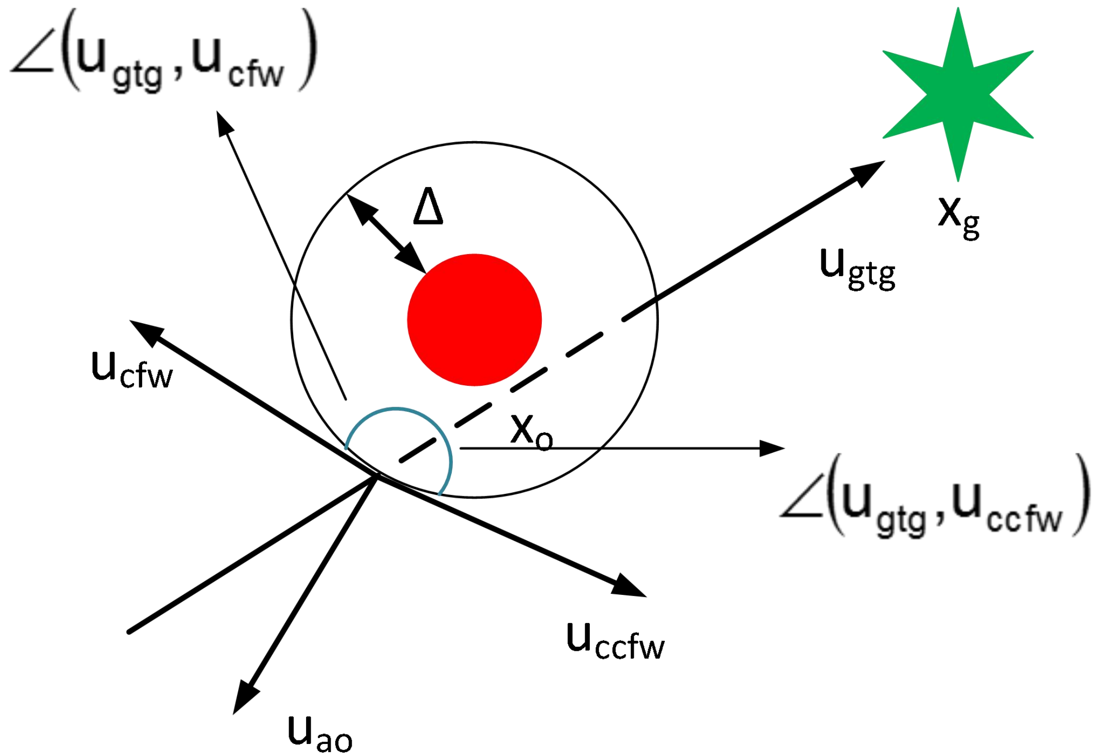


Fig. 5 Determine the direction of "following wall"

$$\langle u_{gtg}, u_{ccfw} \rangle = u_{gtg}{}^{T} u_{ccfw} = \| u_{gtg} \| \| u_{ccfw} \| \cos\left(\angle(u_{gtg}, u_{ccfw})\right) \qquad (1.14a)$$

$$\langle u_{gtg}, u_{ccfw} \rangle > 0 \Rightarrow u_{ccfw} \qquad (1.14b)$$

Determining the direction is made from the product between vector of following wall and the vector of go to goal behaviors.

Hybrid system:

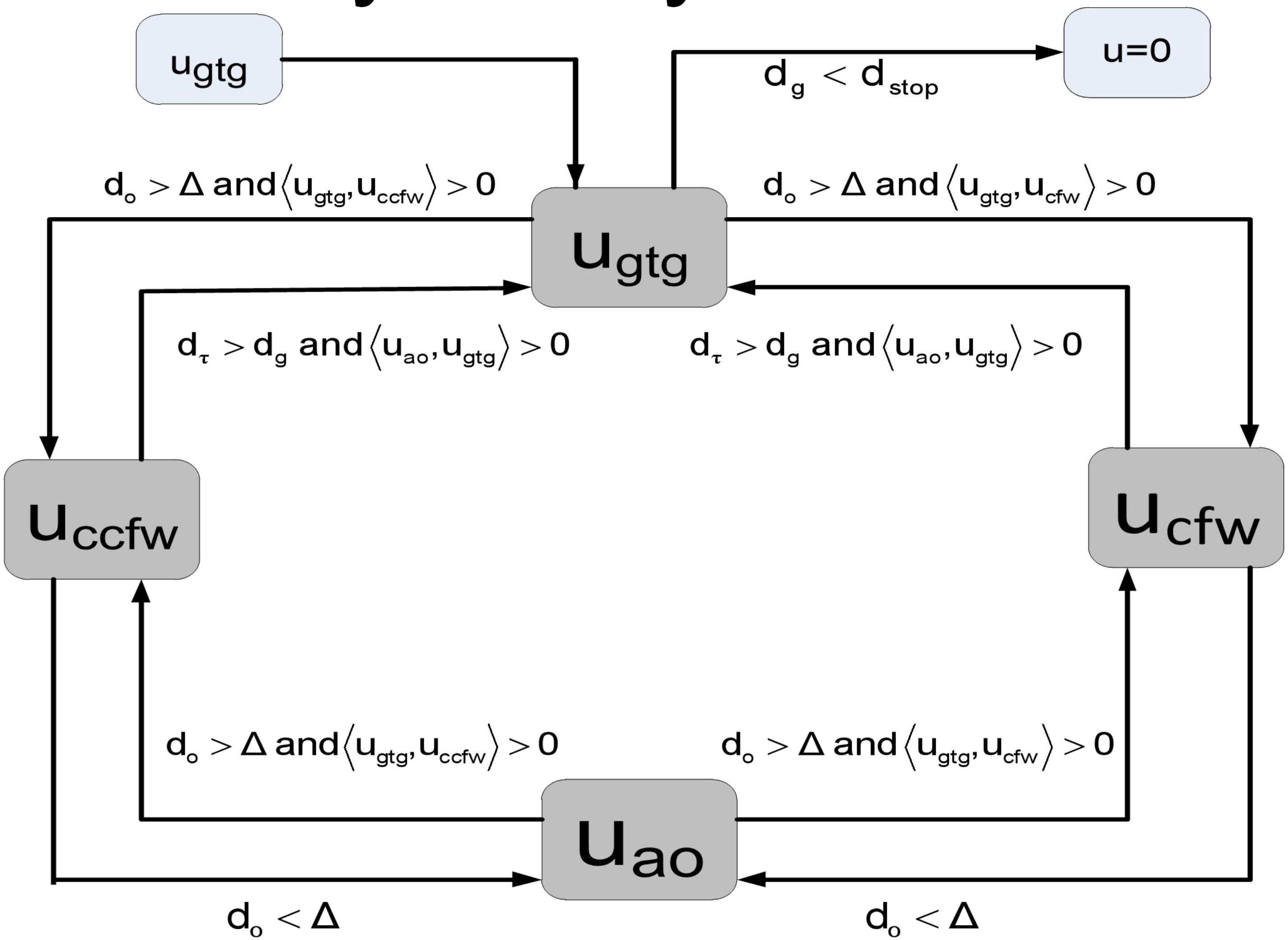


Fig. 6

Dynamic of new point:

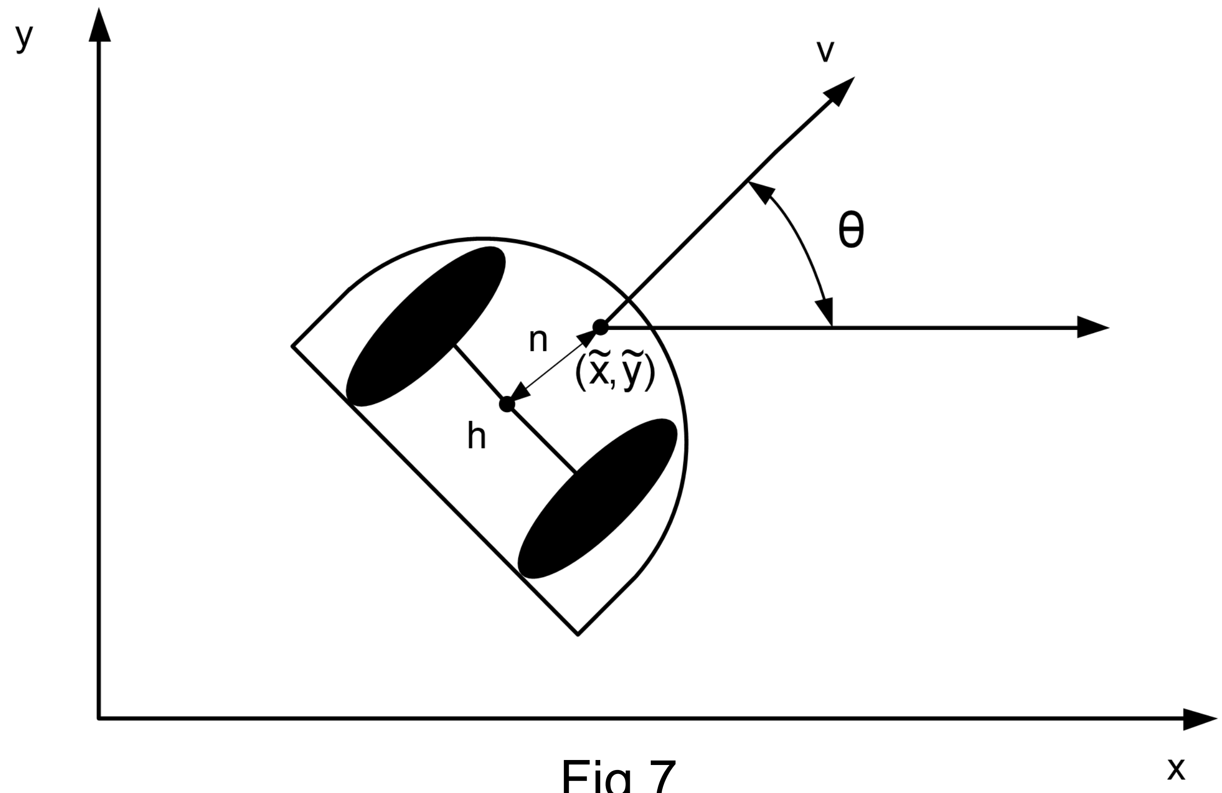


Fig.7

$$\dot{\tilde{x}} = v\cos(\theta) - n\omega\sin(\theta)$$
$$\dot{\tilde{y}} = v\sin(\theta) + n\omega\cos(\theta) \qquad (1.15)$$

Transforming of control methods to desired velocities.

$$\dot{\tilde{x}} = v\cos(\theta) - n\omega\sin(\theta) = u_1$$
$$\dot{\tilde{y}} = v\sin(\theta) + n\omega\cos(\theta) = u_2 \qquad (1.16)$$

Determining the desired velocities:

$$\begin{bmatrix} v_d \\ \omega_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n} \end{bmatrix} \mathbf{R}(-\theta) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (1.17)$$

where:

$$\mathbf{R}(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

Linearization of dynamic:

Dynamic of wheel velocities:

$$\dot{\boldsymbol{\omega}} = \mathbf{A}\boldsymbol{\omega} + \mathbf{B}\boldsymbol{\tau} \qquad (1.18) \qquad \text{където :} \quad \mathbf{A} = -\mathbf{H}^{-1}\mathbf{K}_{\mathbf{fc}}, \quad \mathbf{B} = \mathbf{H}^{-1}$$

Motor shaft transformation:

$$\frac{\omega}{\omega_m} = \frac{1}{r_\omega} \Rightarrow \omega_m = r_\omega \omega \qquad \frac{\boldsymbol{\tau}}{\boldsymbol{\tau}_{\mathbf{m}}} = r_{\boldsymbol{\tau}} \Rightarrow \boldsymbol{\tau}_m = \frac{1}{r_{\boldsymbol{\tau}}}\boldsymbol{\tau} \qquad (1.19)$$

Total ratio:

$$r = r_\omega r_{\boldsymbol{\tau}} \qquad (1.20)$$

Final equation:

$$\dot{\boldsymbol{\omega}}_{\mathbf{m}} = \mathbf{A}\boldsymbol{\omega}_{\mathbf{m}} + \mathbf{B}\mathbf{R}_r\boldsymbol{\tau}_{\mathbf{m}} \qquad (1.21) \qquad \text{където :} \quad \mathbf{R}_{\mathbf{r}} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$$

Cascade System for management speeds of the mobile robot :

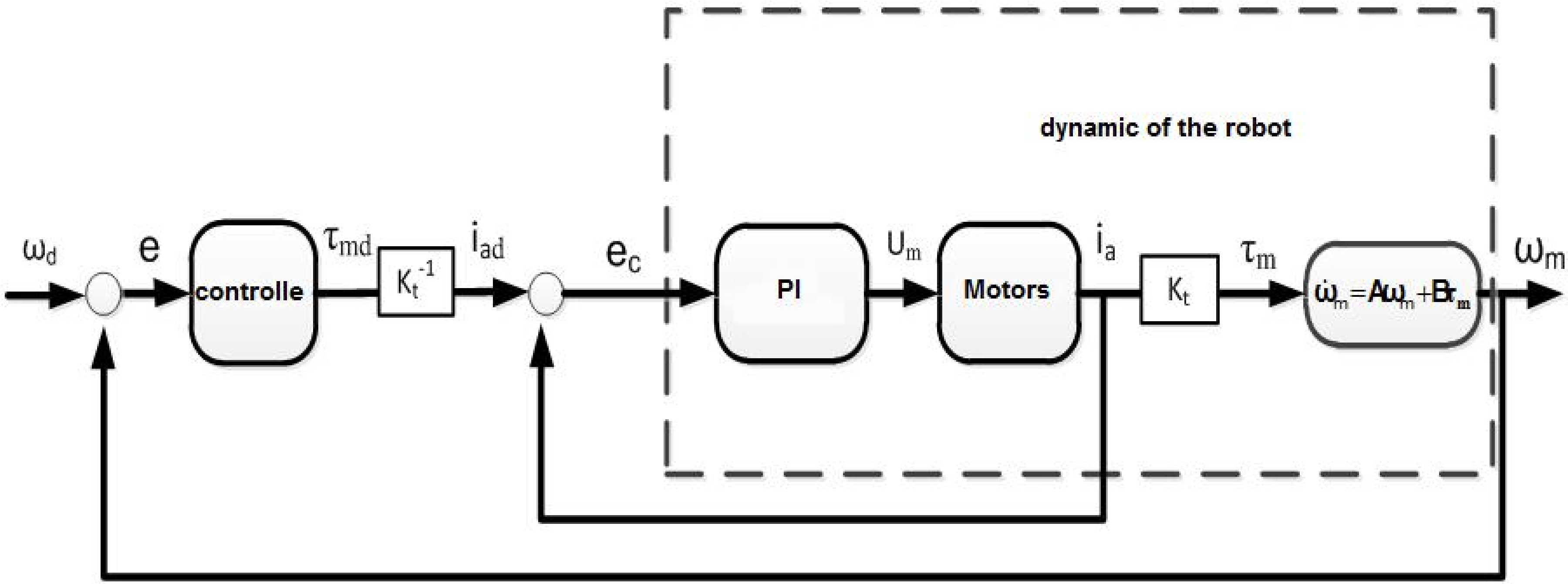


Fig.8

Overall management system of mobile robot :

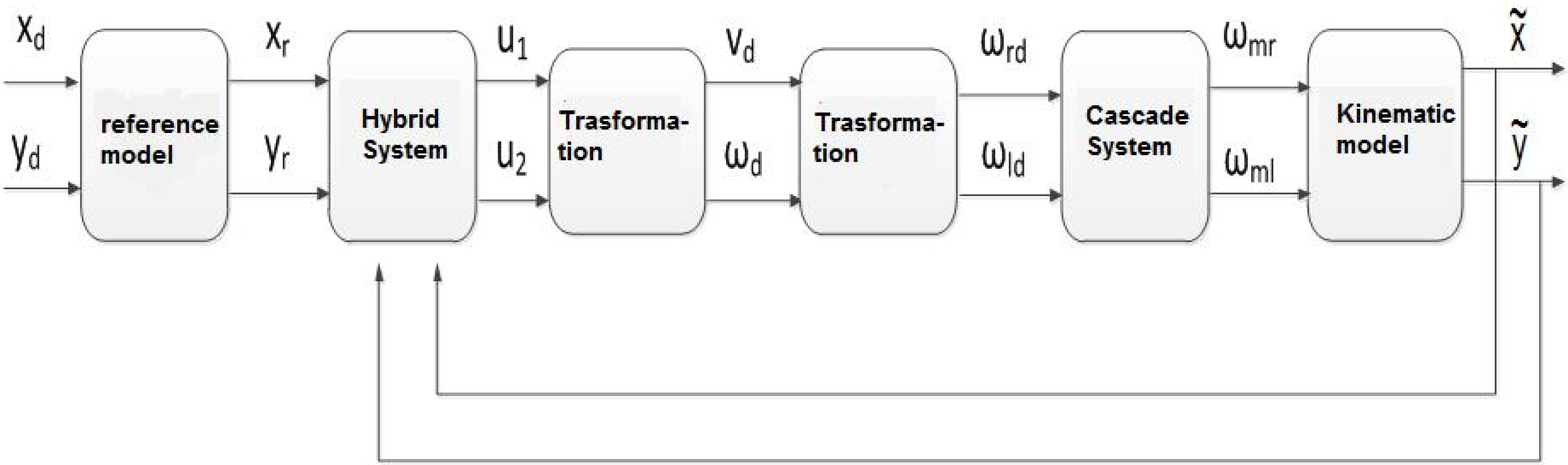


Fig.9

Experiment data :

Test 1. Coordinate system motion with one obstacle in the way of the robot.

x_g, [m]	y_g, [m]	Δ, [m]	ε, [m]	T_f, [s]	t_i	k_p	x_o, [m]	y_o, [m]
5	5	0.3	0.05	80	0.001	1	1.4	1.5

Table 1

Trajectory in coordinate system

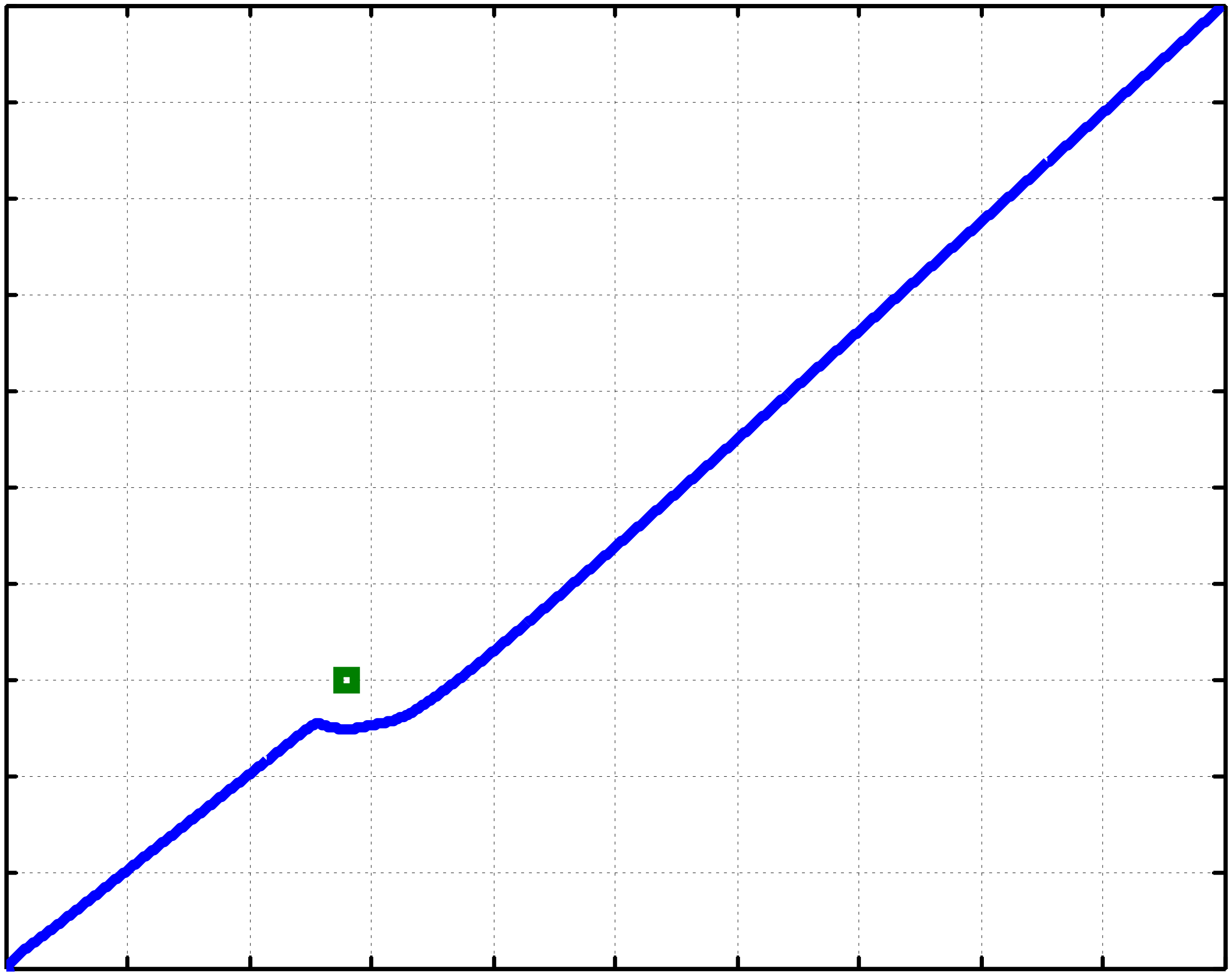


Fig.10

Angular velocities of right and left wheels:

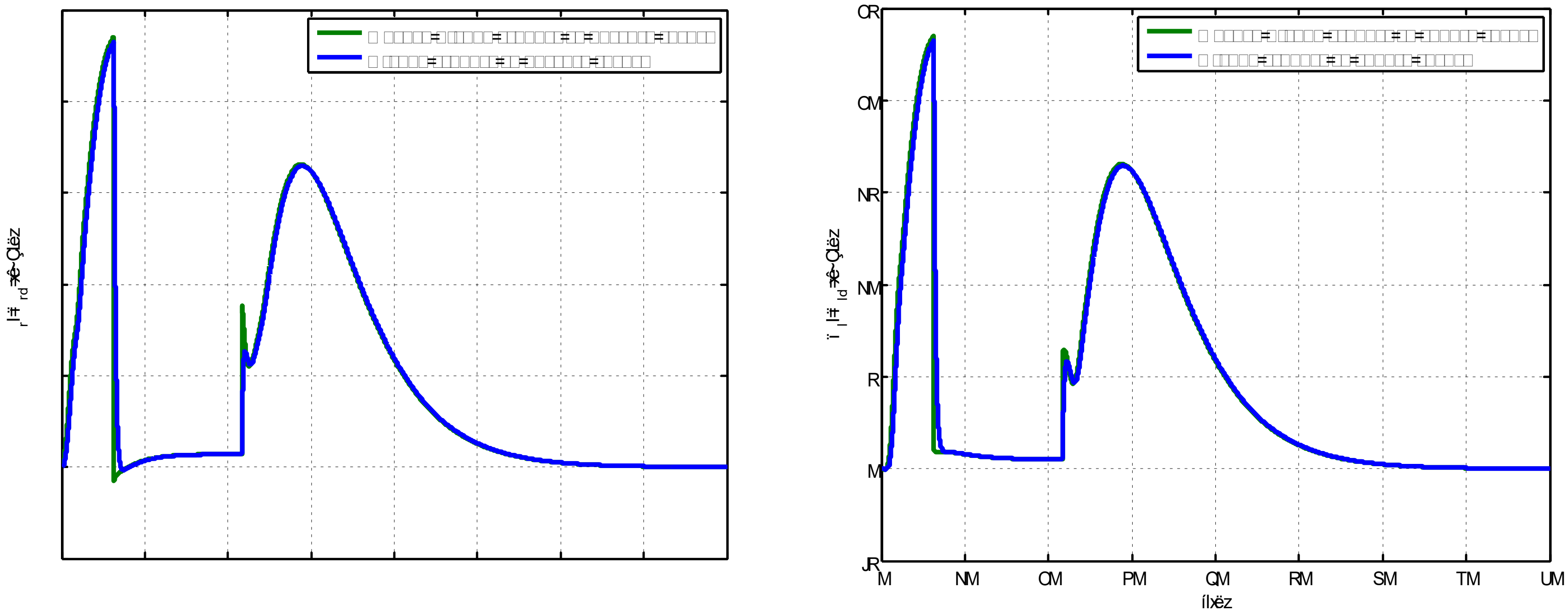


Fig.11

Torques of right and left motors

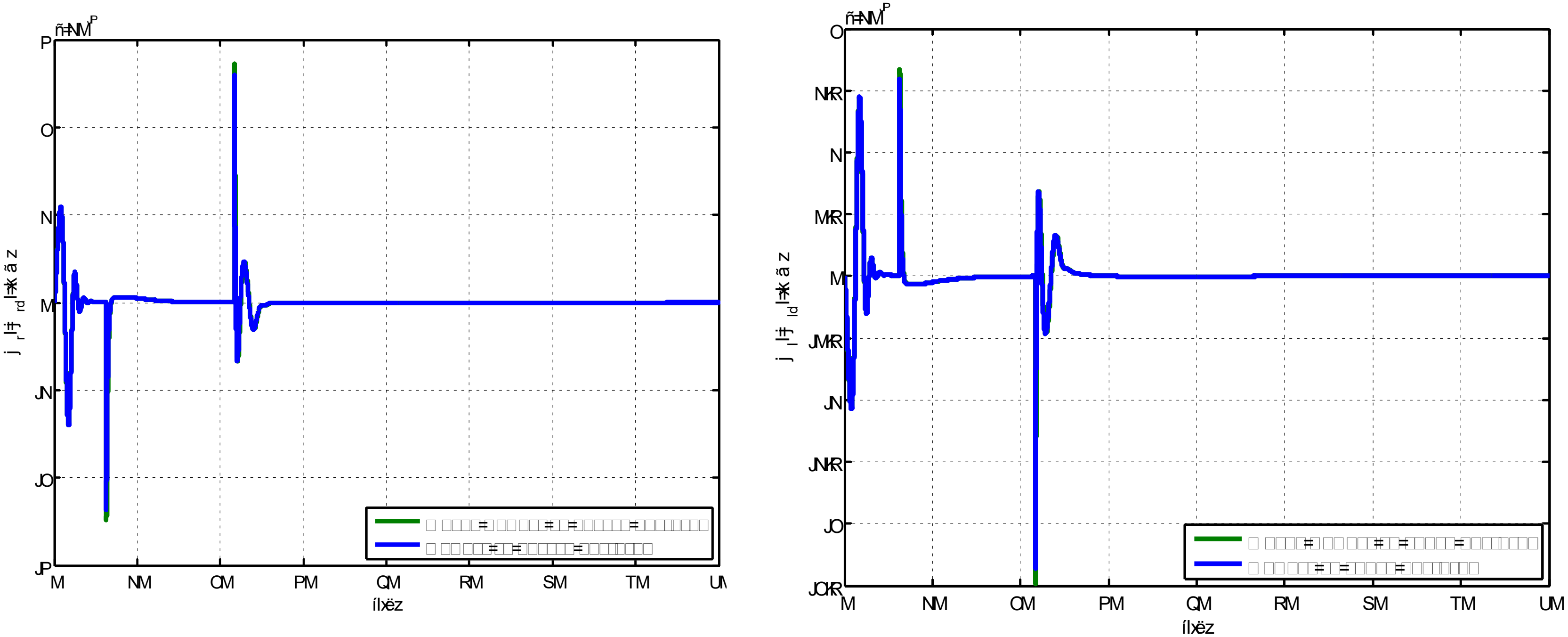


Fig.12

Right and left motor currents:

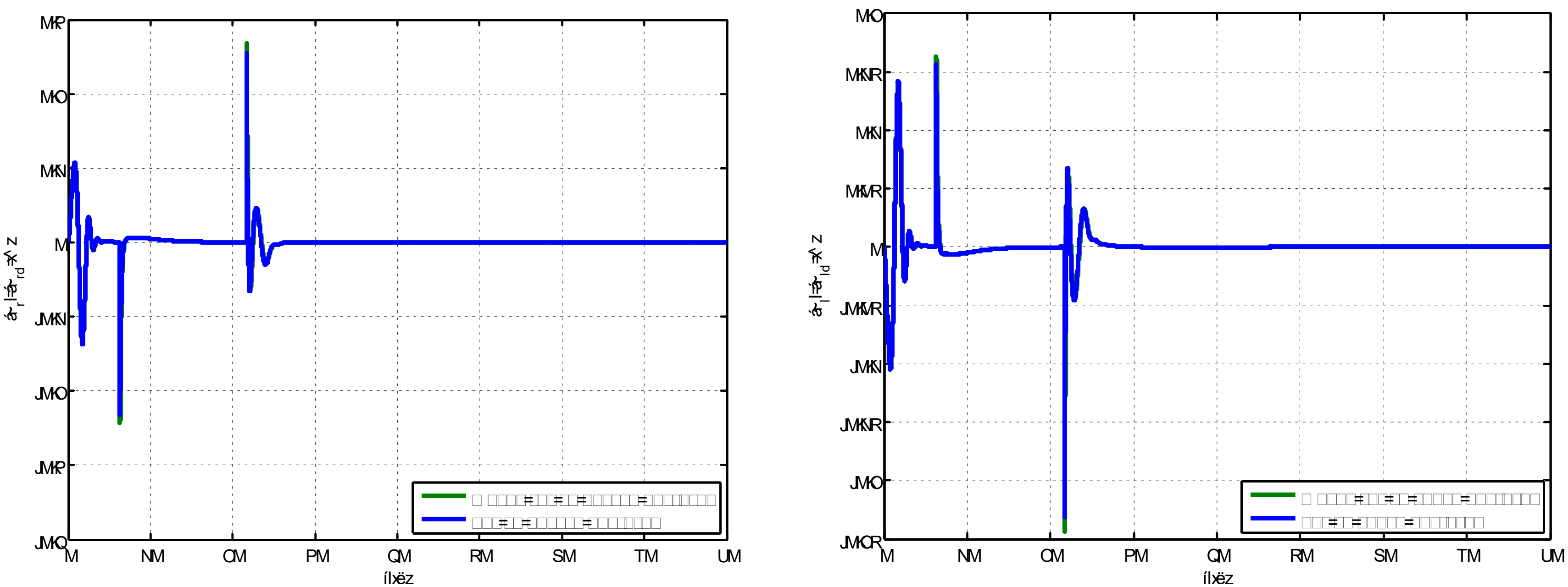


Fig.13

Test 2. Coordinate system motion with two obstacles in the way of the robot.

x_g, [m]	y_g, [m]	Δ, [m]	ε, [m]	T_f, [s]	t_i	k_p	x_o, [m]	y_o, [m]	x_{o2}, [m]	y_{o2}, [m]
5	5	0.3	0.05	100	0.001	0.5	1.4	1.5	3.2	2.9

Table 2

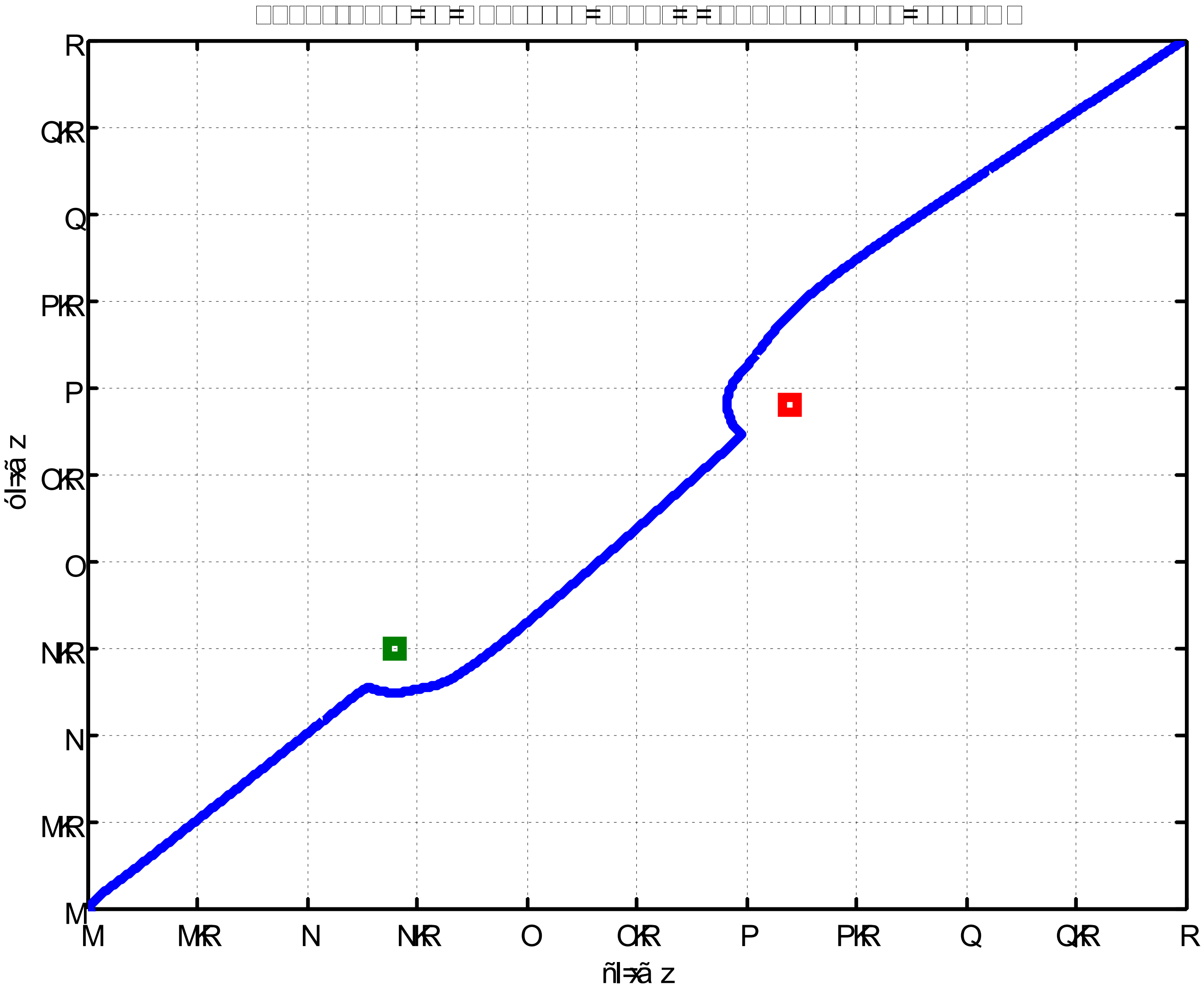


Fig.14 Trajectory

Angular velocities of right and left wheels:

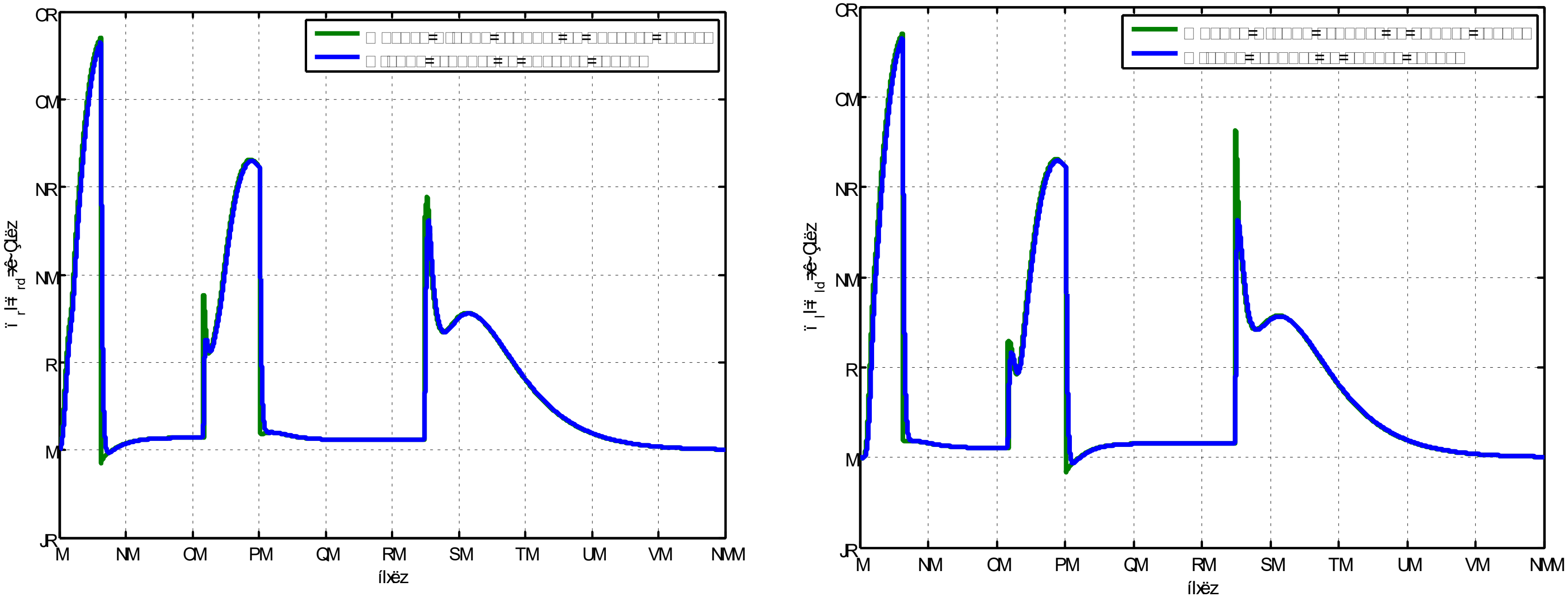


Fig.15

Torques of right and left motors

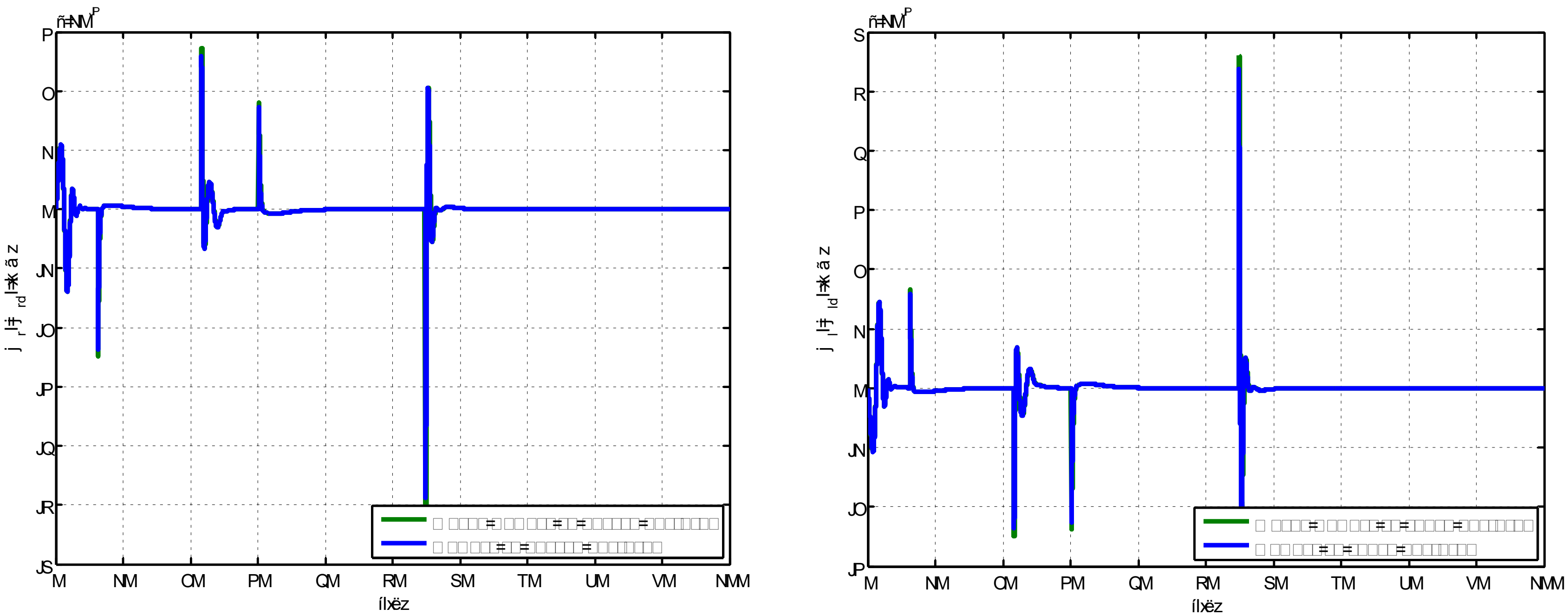
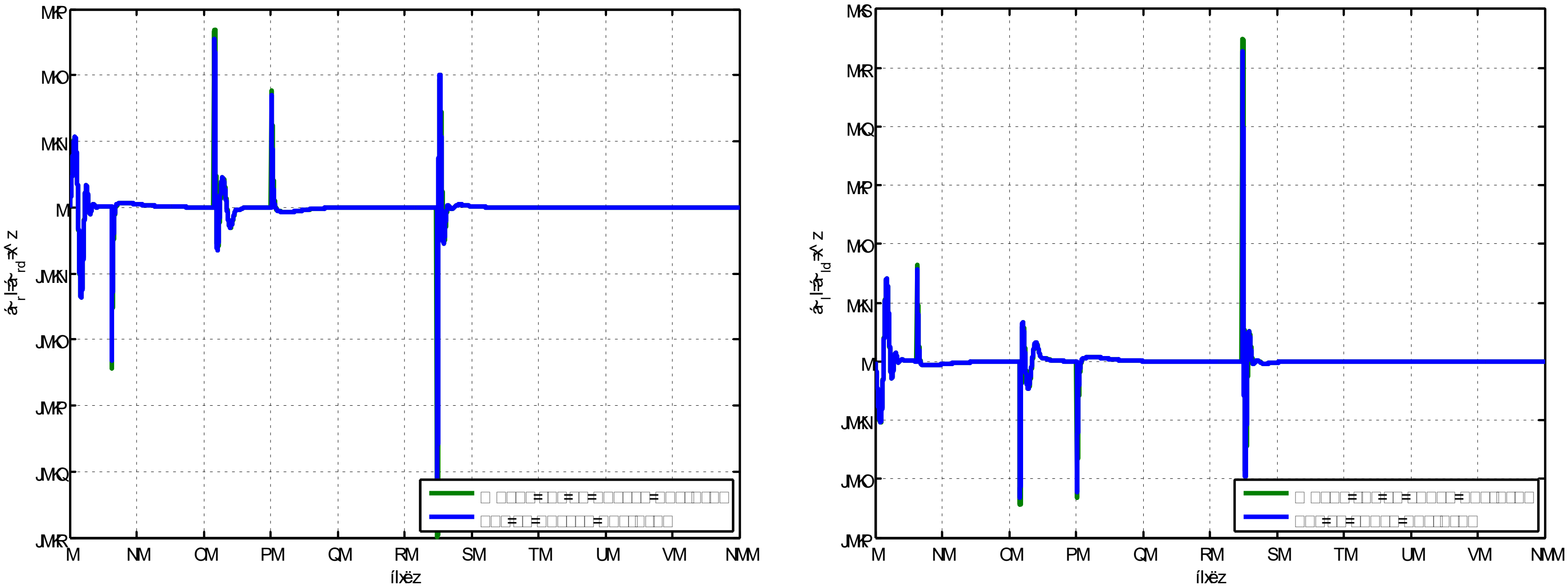


Fig.16

Right and left motor currents:



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Conclusions:

- A study of the behavior of the closed nonlinear system shows that the selected synthetic approaches to system positional control of wheeled mobile robot suitable for achieving the desired objectives of management.

- Based on the results it can be argued that the purpose of the thesis - synthesis of managing mobile robot is successfully executed, the results obtained are appropriate and validated by dynamic simulation management system.

Thanks!