GENERATION, CLASSIFICATION AND ANALYSIS OF SWITCHED-MODE DC-TO-DC CONVERTERS BY THE USE OF CONVERTER CELLS

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ABSTRACT

A method is presented which combines generation, classification and analysis of DC-to-DC PWM converters. Fundamental blocks known as converter cells can be used to generate a plethora of converters leading to a number of useful new converter topologies. A classification of basic converters is proposed in terms of converter-cell generated families. The converter cells considered may be seen as three-terminal devices for which terminal characteristics may be easily found. DC and AC small-signal models for each converter cell can be derived obviating the need to rederive these models for each of the converters of a particular family.

1. INTRODUCTION

Over the past decade a number of different PWM DC-to-DC converter topologies have appeared in the literature [1-6]. The methods used in deriving these converters do not, however, indicate any unifying connection between them. Whilst it would seem clear that there exists a basic set of converters from which others may be derived, a satisfactory enumeration of this basic set is yet to be determined. The present classification of basic converter topologies [7,8] does not account for the existence of several converters, for example, the well known Sepic converter [2,3]. In this paper it will be seen that, in fact, the Sepic along with its dual/bilateral inversion counterpart and four other converters introduced in [6,9,10,11] form the members of one particular family of basic converters. Thus a unifying connection between seemingly unrelated converters is established. This relationship was not pointed out in [6] (the first and only published record of these converters) and may have been first pointed out in [9] and [10].

Furthermore the "down" and "up" converters when first introduced by Landsman [12] were seen to be simply "topological transformations" of the buck and boost converters. We will see in a later section that these converters are the two other members of the family of converters of which the Cuk converter is a member, as was first pointed out in [9] and subsequently in [10,11,13,14].

Through the concept of a "canonical switching cell" Landsman was able to indicate the simple relationship between the buck, the boost and the buck-boost converters. Subsequently, a somewhat similar approach was taken by Rao [15] in the generation of converters. However, for the generation of *basic* converters Rao's approach suffers in that he considers a switching cell that includes a transformer and moreover he considers only one switching cell, as did Landsman.

An analytical approach to the generation of converters with specified properties or attributes is given by Erickson [5]. The two classes considered in [5] are (1) the class of single-inductor two-topology converters and (2) the class of converters featuring non-pulsating port currents. Converters in these two categories, not considered as distinct according to [5], are listed here because they have different electrical properties. Also, converter classes with conversion ratios which are not bilinear functions of D and D' are considered here as well.

For the generation of converter topologies the approach taken in this paper is as follows; (1) given a converter we may identify a fundamental block hereafter referred to as the "converter cell", (2) with this cell one can generate other converters. This approach is similar to that of Landsman [12] and Rao [15]. However, in contrast to previous work many different converter cells are considered from which different families of converters are derived. As a consequence of considering a large number of converter cells a more adequate classification of basic converter topologies than previously presented is proposed.

However, with the task of having to identify different converter cells comes the requirement of having to impose a certain structure on the converter cell. This is done also in an effort to present a more systematic basic converter topology generation scheme. Thus, to this end, only three-terminal converter cells devoid of transformers are considered. This contrasts Rao's work where the cell he considers is of a four terminal configuration which included a transformer. Whilst imposing a three-terminal structure on the converter cell seems restrictive at first, it is in fact quite efficacious, since for each three-terminal converter cell

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a family of three or six distinct members (depending on the symmetry of the cell) can be derived.

Further in contrast to previous work, viewing the converter cell as a three-terminal device has considerable analytical power. Once the DC and AC small-signal model of a converter cell is derived from its terminal characteristics, the need to perform the analysis of each converter belonging to that particular converter-cell family is obviated. The advantages and limitations of this approach to analysing DC-to-DC converters are explained in Section 4.

In the next section a formal definition of the term "converter cell" is given along with its relationship with basic converter structure. A number of different converter cells are subsequently used in the generation of converter-cell families. A classification of basic converter topologies is then proposed based on classification of converter cells. In Section 3 some selected converter properties and applications are examined. In particular some additions to the sets of two classes of converters previously considered by Erickson are made.

2. GENERATION AND CLASSIFICATION OF CONVERTER-CELL FAMILIES

2.1 Basic Converter Structure

In this paper only two-switch interval (for continuous inductor current conduction) and three-switch interval (for discontinuous conduction) basic PWM converter topologies are considered. Moreover the general structure of DC-to-DC converters is assumed to be as shown in Fig. 1 where we see that a converter may be represented as consisting of three main parts:

1. the input voltage source
2. the converter cell
3. the output voltage sink, which consists of the parallel combination of the load resistance and output capacitor.

The converter cell is defined as the network remaining when the input source and output sink are removed. Alternatively, the converter cell may be defined as a topological combination of reactive elements (L's and C's) and switches arranged such that when an input voltage source and output voltage sink are connected the duty cycle has control of the output voltage. As we are interested in only basic converters, only converters devoid of transformers are considered. Thus the generic converter of Fig. 1 appears as an non-isolated converter with the further constraint of a common input/output line which prevents the load from floating.

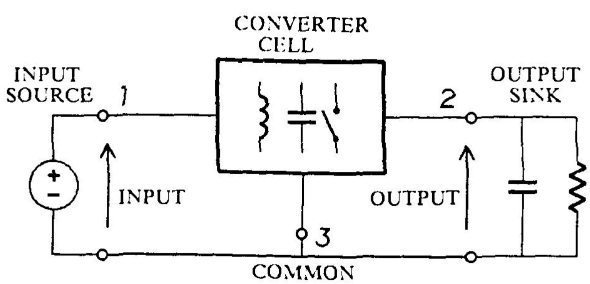


Fig. 1. *General structure of basic DC-to-DC converters.*

A converter cell is now seen as a three-terminal device which can be connected in six different possible ways to the input source and output sink to generate different converters while preserving the general structure shown in Fig. 1. If we denote, arbitrarily, the three terminals of the cell as terminals 1 to 3 as shown in Fig. 2, the six ways of connection, designated as configurations 1 to 6, are listed in Table 1. Thus it is seen that the converter, as shown in Fig. 1, is connected in the configuration corresponding to configuration 5 in Table 1. Note that configurations 5 and 6 are bilateral inversions of each other, as are configurations 1 and 2 and also 3 and 4. Bilateral inversions are simply generated by the inversion of input source and output sink connections of a converter.

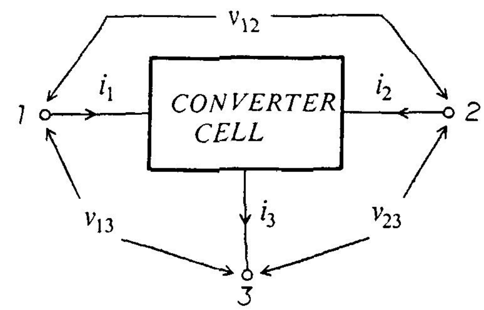


Fig. 2. *A three-terminal converter cell with designated terminal voltages and currents.*

CONFIGURATION NUMBER	1	2	3	4	5	6
COMMON	1	1	2	2	3	3
INPUT	2	3	1	3	1	2
OUTPUT	3	2	3	1	2	1

Table 1. *The six ways of configuring a three-terminal converter cell to the input source and output sink. The table entries represent the terminal number of the converter cell.*

By using the terms input source and output sink we have tacitly assumed the property of unidirectional power flow in the converter from left to right. This is necessary, for the converter, in general, will feature different conversion properties when processing power in different directions, as will be seen later. Take for example the battery charger/discharger application. Power may flow in different directions at different times. Therefore a converter, for example, in configuration 5 which is processing power from left to right during some interval in time will need to be viewed as operating in configuration 6 when the direction of power flow reverses.

2.2 Converter-Cell Generated Families - A Classification Scheme

It is evident from the six possible connections that, given a particular converter cell, a family of six converters may be derived. If however, for any of these configurations the converter cell is symmetric the number of distinct members of the family reduces to three. It is thus clear that a number of distinct converters may be generated from different converter cells. A classification scheme for these converters based on classification of converter cells is now proposed. These converter cells are classified according to their order, which indicates the number of storage elements used, and according to the number of single pole/single throw switches used. In this paper only four classes of converter cells are considered and although higher order converter cells can be considered their usefulness becomes questionable. The four categories considered are:

1. 1st. order - 2 switch: converter cell A (1 family)
2. 1st. order - 4 switch: converter cell B (1 family)
3. 3rd. order - 2 switch: converter cells C to G (5 families)
4. 3rd. order - 4 switch: converter cells H to N (7 families)

These converter cells are shown in Table 2 along with their corresponding derived converters. No formal synthesis procedure is given for the derivation of these cells and they are simply obtained from known converters. However, several new converter topologies are obtained by connecting these cells in the six different possible ways as explained earlier. These new converters include the following; D1 to D4, E1 to E4, F1 to F4, H1 to H4, I1 to I4, J1 to J4, K1 and K2, L1 and L2, M1 and M2, and also N1 to N4.

A few words about the format of Table 2 are now in order. In Table 2, for each cell considered, the family of converters which is generated from the different configurations, (as enumerated in Table 1), is given. An active switch assignment is also given along with the corresponding voltage conversion ratio (M). For example, from Table 2 we can see that for converter B1 of converter-cell B family, two sets of active switch assignments can be made. First, switches S2 and S4 need to be the active switches for duty ratios (referred to these switches) of less than a half. The resulting voltage gain is $M = D/(2D - 1)$. A second active switch assignment is given with S1 and S3 as the active switches for operation with duty ratios (referred to these switches) of also less than a half. The resulting voltage gain is $M = D'/(1 - 2D)$.

It can be seen from the family of converter-cell A that the buck, boost, and buck-boost belong to the same family. Also, the Cuk converter is seen to be one of the three members of converter-cell C family. Hence the classification of basic converter topologies given in [7,8] as Buck, Boost, Buck-boost and Cuk is not complete.

The classification and choice of converter cells given above is not arbitrary and is now explained. It is seen in this classification that three-switch cells are not considered because they correspond to three-switched intervals in continuous conduction mode as is the case of the converters considered in [16] and [17]. Three-switched interval mode of operation is not specifically considered in this paper although the four-switch converter cell families can provide this mode of operation. For example, in converter B5 of converter-cell B family if S1 is turned off while S3 is on the inductor current will idle through S2 and S3. After this idling period S3 is opened giving rise to three-switched intervals in continuous conduction mode of operation. The motivation for eliminating cells with three switches has thus been explained. Next we consider the order of the cell. It is seen that the storage element in the first order cells is an inductor and not a capacitor. This is explained by requiring all ports of the converter cells considered to be *voltage ports* rather than *current ports*. This requirement is imposed since at the input a voltage source will be connected and at the output we wish to derive a voltage. Those ports which can be considered current ports will be redundant as current ports because the inductor of the source will combine with the inductor of the switch(es). For example, in cell A if port 3-1 is considered as a current port the inductor of the cell will be redundant. Therefore, since current ports are not considered here, a first-order cell with a capacitor as shown in Fig. 3 is not considered. It should be noted that no converters are lost or unaccounted for because of this restriction because converter-cell C automatically will include these cases. The choice of the first-order two-switch cell and also the first-order four-switch cell is now entirely explained.

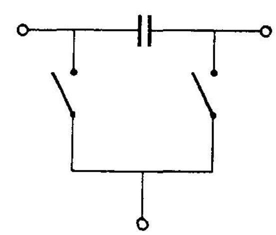


Fig. 3. *A first-order two-switch converter cell with current ports.*

It is seen that second-order two-switch cells are not considered either. This is easily explained by the choice of port characterisation. To obtain a second-order cell from the first-order cell we would require a capacitor to be connected in parallel with any port which would be redundant since all ports are required to be voltage ports. This eliminates the choice of second-order two-switch cells. Since second-order three-switch cells are eliminated the next higher category considered is the third-order two-switch cell, of which all possible configurations with voltage port requirements give rise to converter cells C to G. Except for converter-cell N, the third-order four-switch cells are arrived at by cascading lower order cells. All possible cascade connections are given by converter cells H to M.

Other converter cells which might have been included in one of the four categories considered have not been done so as they were not considered as

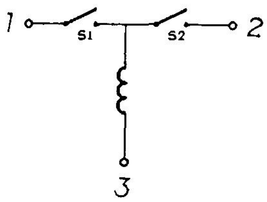
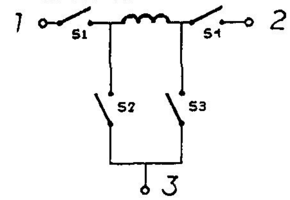
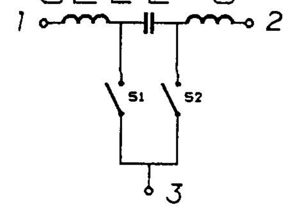
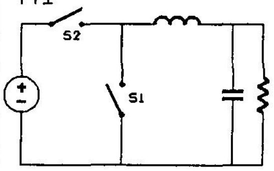
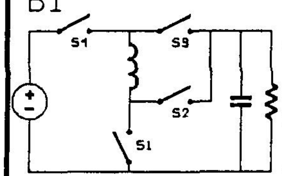
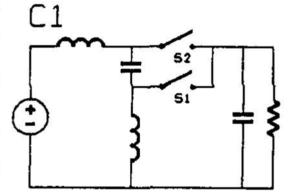
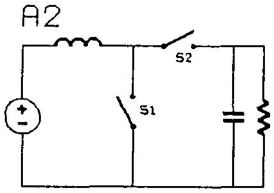
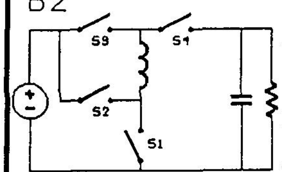
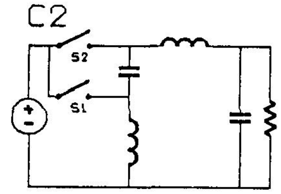
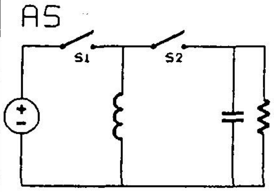
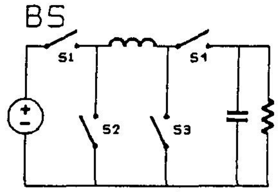
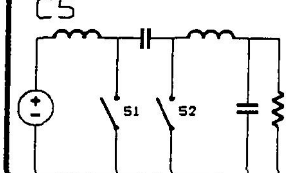
CONFIG.	CELL A	CELL B	CELL C
			
1	A1  S2 : $M = D$	B1  1) S2,S4(D < 0.5) $M = \frac{D}{2D-1}$ 2) S1,S3(D < 0.5) $M = \frac{D'}{1-2D}$	C1  S2 : $M = D$
2	A2  S1 : $M = \frac{1}{D'}$	B2  1) S2,S4(D > 0.5) $M = \frac{2D-1}{D}$ 2) S1,S3(D > 0.5) $M = \frac{1-2D}{D'}$	C2  S1 : $M = \frac{1}{D'}$
3	*THE SAME AS 1 ABOVE*	*THE SAME AS 1 ABOVE*	*THE SAME AS 1 ABOVE*
4	*THE SAME AS 2 ABOVE*	*THE SAME AS 2 ABOVE*	*THE SAME AS 2 ABOVE*
5	A5  S1 : $M = -\frac{D}{D'}$	B5  S1, S3 : $M = \frac{D}{D'}$	C5  S1 : $M = -\frac{D}{D'}$
6	*THE SAME AS 5 ABOVE*	*THE SAME AS 5 ABOVE*	*THE SAME AS 5 ABOVE*

Table 2. *The converter-cell generated families of converters. See the text for an explanation of the active switch/conversion gain (M) entries in the table.*

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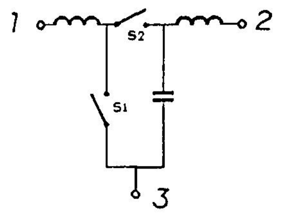
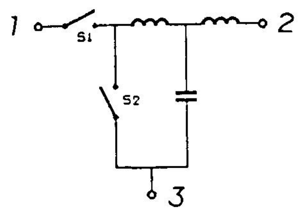
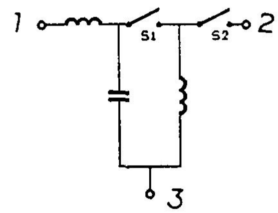
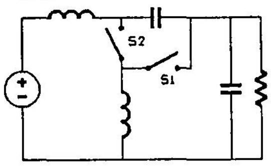
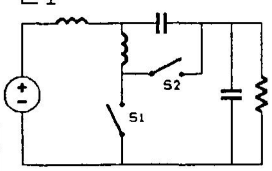
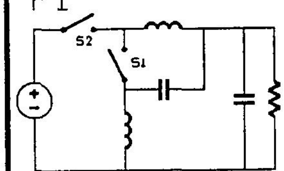
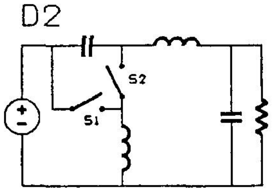
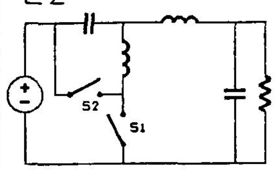
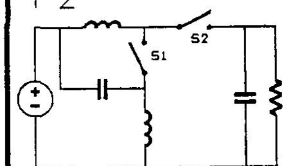
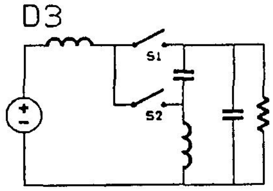
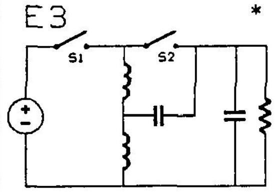
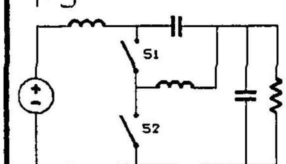
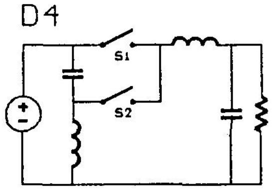
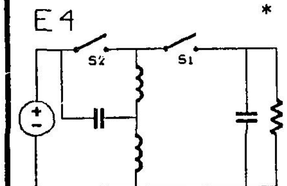
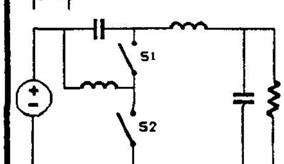
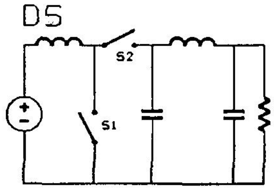
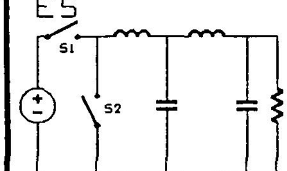
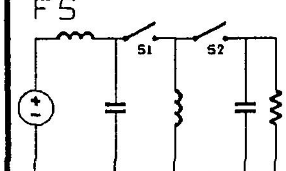
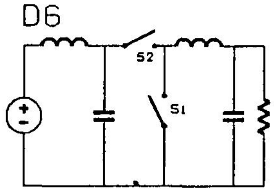
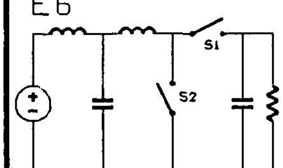
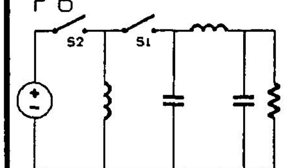
			
1	D1  S2 : $M = -\frac{D}{D'}$	E1  S1 : $M = \frac{1}{D'}$	F1 *  S2 : $M = D$
2	D2  S1 : $M = -\frac{D}{D'}$	E2  S2 : $M = D$	F2 *  S1 : $M = \frac{1}{D'}$
3	D3  S2 : $M = \frac{1}{D'}$	E3 *  S1 : $M = -\frac{D}{D'}$	F3  S1 : $M = D$
4	D4  S1 : $M = D$	E4 *  S2 : $M = -\frac{D}{D'}$	F4  S2 : $M = \frac{1}{D'}$
5	D5  S1 : $M = \frac{1}{D'}$	E5  S1 : $M = D$	F5  S1 : $M = -\frac{D}{D'}$
6	D6  S2 : $M = D$	E6  S2 : $M = \frac{1}{D'}$	F6  S2 : $M = -\frac{D}{D'}$

Table 2 (cont'd).

* *The LC branch connected from source to ground or from sink to ground is redundant in these converters.*

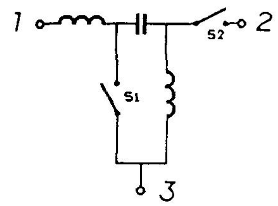
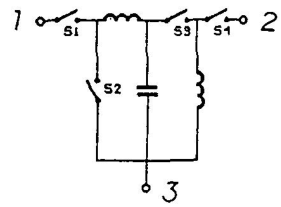
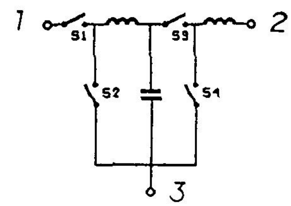
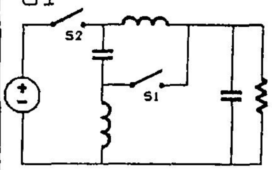
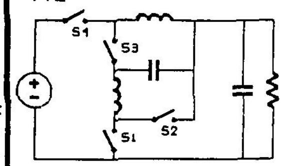
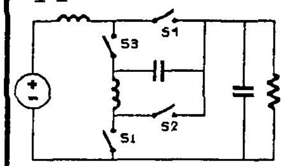
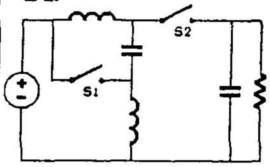
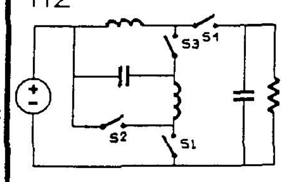
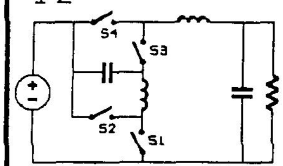
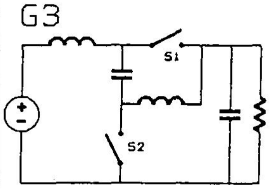
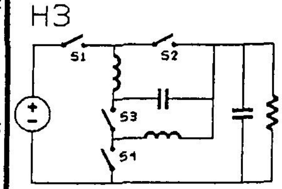
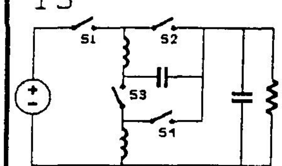
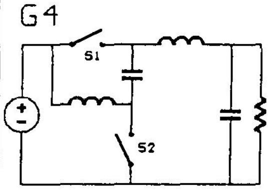
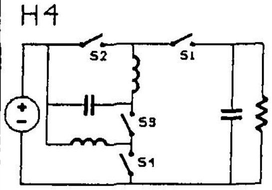
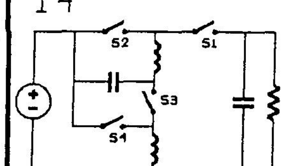
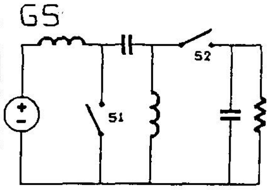
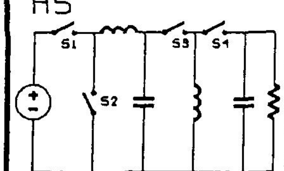
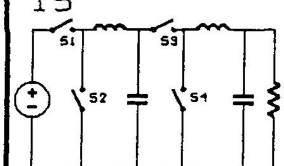
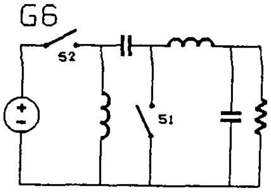
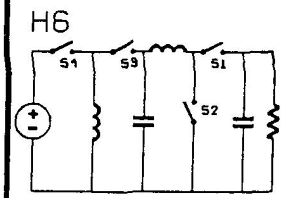
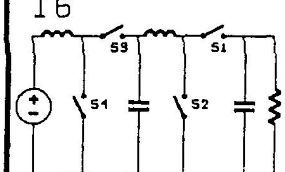
CONFIG.	CELL G	CELL H	CELL I
			
1	G1  1) S2 ($D < 0.5$): $M = \frac{D}{2D - 1}$ 2) S1 ($D < 0.5$): $M = \frac{D'}{1 - 2D}$	H1  1) S2,S4 : $M = \frac{D}{D + D'^2}$ 2) S1,S4 : $M = \frac{1}{1 + D'}$	I1  1) S1,S3 : $M = \frac{1}{1 - D^2}$ 2) S1,S4 : $M = \frac{1}{1 - DD'}$
2	G2  1) S1 ($D > 0.5$): $M = \frac{1 - 2D}{D'}$ 2) S2 ($D > 0.5$): $M = \frac{2D - 1}{D}$	H2  1) S1,S3 : $M = \frac{D' + D^2}{D'}$ 2) S2,S3 : $M = 1 + D$	I2  1) S2,S4 : $M = 1 - D'^2$ 2) S1,S4 : $M = 1 - DD'$
3	G3  1) S1 ($D < 0.5$): $M = \frac{D}{2D - 1}$ 2) S2 ($D < 0.5$): $M = \frac{D'}{1 - 2D}$	H3  1) S1,S3 : $M = \frac{D^2}{D' + D^2}$ 2) S1,S4 : $M = \frac{D'}{1 + D'}$	I3  1) S1,S3 : $M = \frac{D^2}{D^2 - 1}$ 2) S1,S4 : $M = \frac{DD'}{DD' - 1}$
4	G4  1) S2 ($D > 0.5$): $M = \frac{1 - 2D}{D'}$ 2) S1 ($D > 0.5$): $M = \frac{2D - 1}{D}$	H4  1) S2,S4 : $M = \frac{D + D'^2}{D'^2}$ 2) S1,S4 : $M = \frac{1 + D'}{D'}$	I4  1) S2,S4 : $M = \frac{D'^2 - 1}{D'^2}$ 2) S1,S4 : $M = \frac{DD' - 1}{DD'}$
5	G5  S1 : $M = \frac{D}{D'}$	H5  1) S1,S3 : $M = -\frac{D^2}{D'}$ 2) S1,S4 : $M = -D'$	I5  1) S1,S3 : $M = D^2$ 2) S1,S4 : $M = DD'$
6	G6  S2 : $M = \frac{D}{D'}$	H6  1) S2,S4 : $M = -\frac{D}{D'^2}$ 2) S1,S4 : $M = -\frac{1}{D'}$	I6  1) S2,S4 : $M = \frac{1}{D'^2}$ 2) S1,S4 : $M = \frac{1}{DD'}$

Table 2 (cont'd).

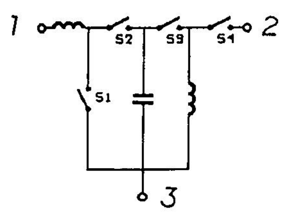
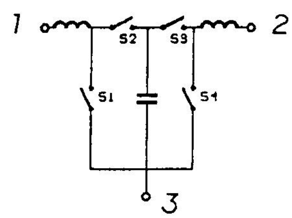
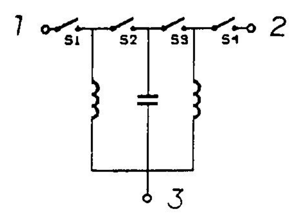
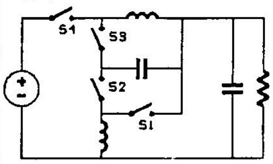
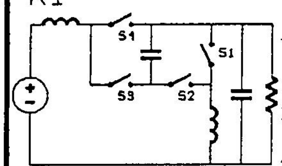
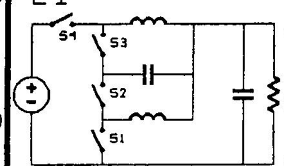
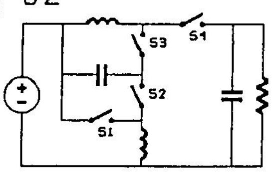
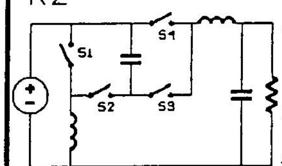
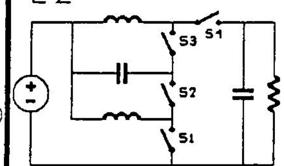
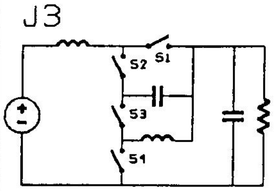
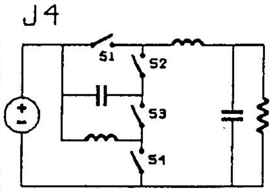
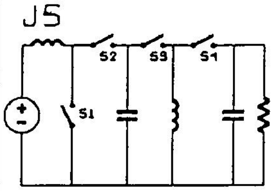
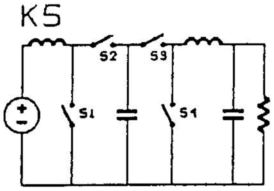
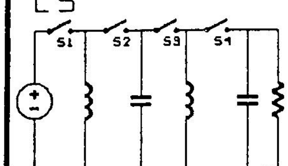
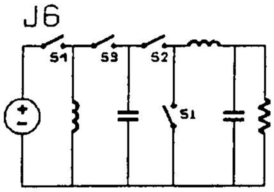
CONFIG.	CELL J 	CELL K 	CELL L 
1	J1  S2,S4 : $M = \dfrac{D^2}{D' + D^2}$	K1  1) S1,S3(D < 0.5) $M = \dfrac{D'}{1 - 2D}$ 2) S2,S4(D < 0.5) $M = \dfrac{D}{2D - 1}$	L1  1) S2,S4(D < 0.5) $M = \dfrac{D^2}{2D - 1}$ 2) S1,S3(D < 0.5) $M = \dfrac{D'^2}{1 - 2D}$
2	J2  S1,S3 : $M = \dfrac{D + D'^2}{D'^2}$	K2  1) S1,S3(D > 0.5) $M = \dfrac{1 - 2D}{D'}$ 2) S2,S4(D > 0.5) $M = \dfrac{2D - 1}{D}$	L2  1) S1,S3(D > 0.5) $M = \dfrac{1 - 2D}{D'^2}$ 2) S2,S4(D > 0.5) $M = \dfrac{2D - 1}{D^2}$
3	J3  S1,S3 : $M = \dfrac{D}{D + D'^2}$	*THE SAME AS 1 ABOVE*	*THE SAME AS 1 ABOVE*
4	J4  S2,S4 : $M = \dfrac{D' + D^2}{D'}$	*THE SAME AS 2 ABOVE*	*THE SAME AS 2 ABOVE*
5	J5  S1,S3 : $M = -\dfrac{D}{D'^2}$	K5  S1,S3 : $M = \dfrac{D}{D'}$	L5  S1,S3 : $M = \left(\dfrac{D}{D'}\right)^2$
6	J6  S2,S4 : $M = -\dfrac{D^2}{D'}$	*THE SAME AS 5 ABOVE*	*THE SAME AS 5 ABOVE*

Table 2 (cont'd).

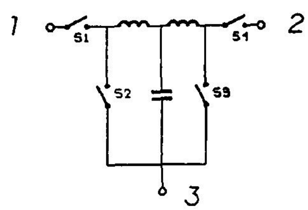
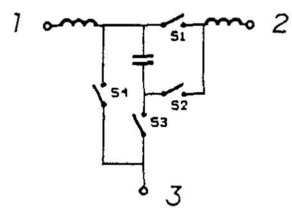
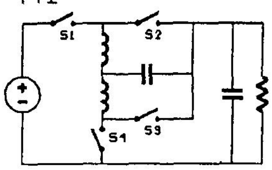
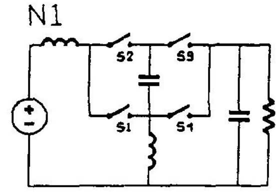
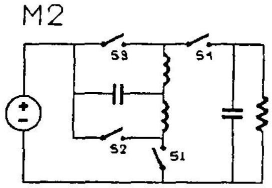
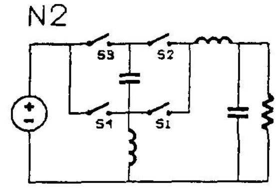
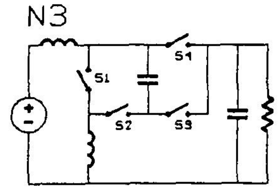
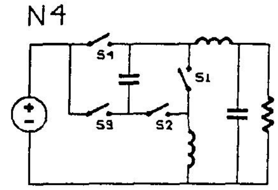
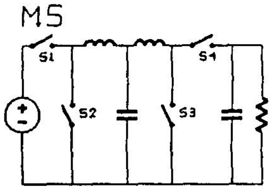
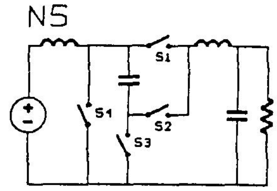
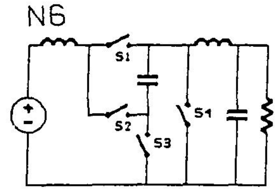
CONFIG.	CELL M	CELL N
		
1	M1  1) S1,S3(D < 0.5) $M = \frac{D}{2D - 1}$ 2) S2,S4(D < 0.5) $M = \frac{D'}{1 - 2D}$	N1  S1,S3 (D < 0.5): $M = \frac{D}{D'}$
2	M2  1) S2,S4(D > 0.5) $M = \frac{2D - 1}{D}$ 2) S1,S3(D > 0.5) $M = \frac{1 - 2D}{D'}$	N2  S2,S4 (D > 0.5): $M = \frac{D}{D'}$
3	*THE SAME AS 1 ABOVE*	N3  S2,S4 (D > 0.5): $M = \frac{2D - 1}{D}$
4	*THE SAME AS 2 ABOVE*	N4  S1,S3 (D < 0.5): $M = \frac{D'}{1 - 2D}$
5	M5  S1,S3 : $M = \frac{D}{D'}$	N5  S2,S4 (D > 0.5): $M = \frac{1 - 2D}{D'}$
6	*THE SAME AS 5 ABOVE*	N6  S1,S3 (D < 0.5): $M = \frac{D}{2D - 1}$

Table 2 (cont'd).

basic. Fig. 4 shows three such converter cells. The series LC branch in the cells of Fig. 4a and 4b do not alter the basic behaviour of the cell without these extra elements. It is interesting to note in passing, however, that by implementing suitable coupling between the two inductors of the cell, zero current ripple in terminal 3 (which may represent the input or output ripple current of a converter) may be achieved, to a first order. A more detailed examination of these converter cells in a buck configuration can be found in [18].

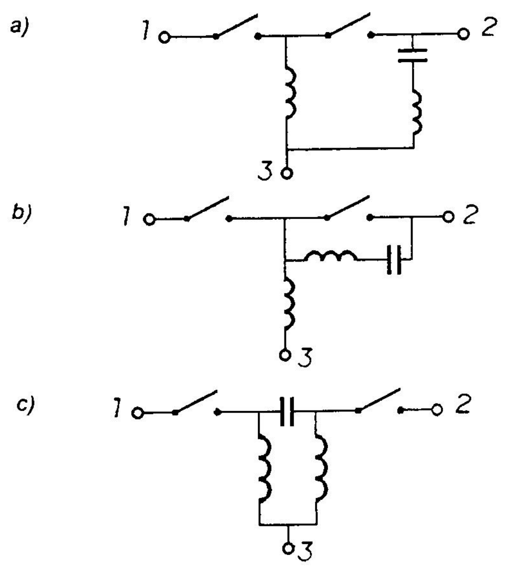


Fig. 4. *These converter cells are examples of cells which have not been considered as basic and, as such, are not included in the categorisation of basic converters.*

3. SELECTED CONVERTER PROPERTIES AND APPLICATIONS

Having now generated a plethora of converters from fourteen different converter cells let us now look at some converter properties and applications.

3.1 The Class of Two-Topology Single-Inductor Converters

Members of converter-cell families A and B belong to the class of two-topology single-inductor converters. Tapped inductors can be used in family B to reduce the number of switches required. The converter of Fig. 5 may be derived from converter B5 in this way. These converters have the same *M* but are electrically distinct. A simple permutation of the cell in Fig. 5 will generate the Watkins-Johnson converter and its inverse as considered in [5].

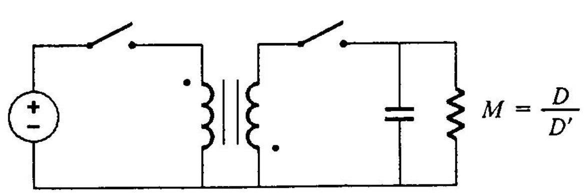


Fig. 5. *A member of the class of single-inductor, two-topology converters.*

3.2 The Class of Converters Featuring Non-pulsating Port Currents

Erickson [5] has listed seven converters as being the distinct members of this class. Thirteen other converters which belong to this class, not considered as distinct in [5], are listed here because they have different electrical properties. For example, converters C1 and D6, considered identical by Erickson [28], have different average currents in one of the inductors.

Let us now examine a port current property of a converter cell. With reference to Fig. 2 we see that $\tilde{i}_1 + \tilde{i}_2 = \tilde{i}_3$, where $\tilde{i}_1$, $\tilde{i}_2$ and $\tilde{i}_3$ denote the instantaneous current levels flowing in terminals 1, 2 and 3, respectively. Therefore if the input and output currents $\tilde{i}_1$ and $\tilde{i}_2$, respectively, are non-pulsating, so $\tilde{i}_3$ will also be non-pulsating. Therefore, if one converter of a particular family features non-pulsating port currents then all converters of this family will feature the same property. It is seen then, that converters derived from converter cells C, D, K and N all feature non-pulsating port currents and it is from these converter cells that the thirteen extra members of this class may be derived.

3.3 Converter Applications

From the plethora of converters generated from the fourteen converter-cells considered we find some new and peculiar and useful DC conversion ratios emerge. For example, converters featuring DC conversion ratios that allow an output voltage of either polarity to be achieved may prove to be useful in DC-to-AC applications. Converters with the following conversion ratios allow either polarity of output voltage: $(2D - 1)/D$, $(1 - 2D)/D'$, $(2D - 1)/D^2$ and $(1 - 2D)/D'^2$ where $D' \equiv 1 - D$. These conversion ratios are featured by converters in the following converter-cell families: B,G,K,L,M and N. Note also that these conversion ratios achieve a large voltage step-down without requiring a very small duty ratio (and without a transformer). In fact for duty ratios very close to $D = 0.5$ extremely large step-downs may be achieved and also at these values of duty ratios operation at very high frequencies is feasible. In conjunction, these properties are desirable for some applications [19].

Whilst all switches so far have been considered as ideal bidirectional switches, it is more convenient and cheaper in actual implementations (and if the application permits) to use unidirectional switches. This leads to the task of active switch assignment. From an appreciation of the function of an active switch one can make this assignment. For every pair of switches in a converter there is an associated inductor. An active switch provides a path for an increasing current in this inductor. This clearly must be the case as an active switch can turn off this increasing current, whereas, an inactive switch could not do so. A procedure for active switch assignment is given in the following. Initially the active switch(es) can be arbitrarily assigned. A steady-state analysis is then performed on the converter from which positive inductor current directions and capacitor voltage polarities can be indicated. Given the direction of inductor current flow, an increasing current in an inductor implies a certain voltage polarity across the inductor. For the active switch assignment under consideration to be satisfactory the voltage polarities across the inductor(s) (found from the steady-state analysis) during the interval DT_s (when the active switch is on) must be consistent with the implied voltage direction of an increasing current in the inductor during this interval of the switching cycle. If there is a conflict an alternative switch assignment is made and the procedure is repeated until a satisfactory assignment is achieved.

Space does not permit a more detailed discussion of active switch assignment to be given here. However in [11] examples of the procedure for active switch assignment are given which include the case when the state variable polarities change when the duty cycle varies over its full range; a property of converters derived from converter-cells B,G and K to N. As previously stated, active switch assignments for all the converters of Table 2 are given there. Semiconductor switch implementations for these converters can be found in [11].

4. ANALYSIS

As we have seen, a family of converters may be derived from one converter cell. Thus there exists a general relationship between the terminal voltages and currents between the different members of a family. In particular, the DC voltage ratios of different family members are simply related. Moreover each converter cell can be seen as a three-terminal device and, as such, DC and AC small-signal averaged models for each converter cell can be derived obviating the need to rederive these models for each of the converters of a particular family.

One should take special note of how this approach is analogous to ordinary transistor circuit analysis. A converter cell and a transistor are both non-linear three-terminal devices. Operation of these devices for small excursions about a DC operating point allows one to derive small-signal models, for example, the hybrid-pi model of a transistor. Once these models have been derived the non-linear device itself, be it the converter cell or a transistor, in a particular circuit may be replaced point-by-point by its linearised model. This allows ordinary linear circuit analysis techniques to be used to analyse the circuit within which the non-linear device was imbedded, be it a converter or a transistor circuit.

With this analogy with ordinary transistor circuits we can see the analogy between the configurations of a converter cell in a family corresponding to the different configurations of a transistor in a circuit e.g. common base or common emitter configurations.

In the following the general relationship between the DC voltage conversion ratios of different family members will be derived (for the particular case of continuous inductor current conduction). Next, as an example of the "transistor circuit analysis approach" proposed here, the DC and AC small-signal models of the converter cell of the family of first-order two-switch converters (converter-cell A) will be derived. These models will then be used to analyse one of the converters generated by this cell, viz., the boost converter.

4.1 General DC Conversion Ratio Relationships Between Converter-Cell Family Members

In the following we will see how the DC voltage conversion ratios of different members of a family are related. With reference to Fig. 2 let us denote the voltages between terminals 1 and 2, 2 and 3, and also 1 and 3 of the converter cell as v_{12}, v_{23} and v_{13}, respectively. Now, if we consider the converter cell in configuration 5 we have, as indicated in Table 1, terminal 3 as the common terminal, the input source being connected across terminals 1 and 2, and the output is taken across terminals 2 and 3. Let us now denote the voltage gain in continuous conduction mode from the input to the output as some function, f, of the duty cycle, D. Thus we have,

$$\left.\frac{\textit{output voltage}}{\textit{input voltage}}\right|_{\textit{config. } 5} = \frac{v_{23}}{v_{13}} = f(D) \quad (1)$$

So given (1) we may now derive the converter DC voltage gain relationship of any of the other configurations. For example, let us find the DC gain of a converter in configuration 4. Thus we would like to find

$$\left.\frac{\textit{output voltage}}{\textit{input voltage}}\right|_{\textit{config. } 4} = \frac{v_{12}}{v_{32}} = g(D) \quad (2)$$

where g is some function of D. Given (1) we can find how $g(D)$ is related to $f(D)$:

$$g(D) = \frac{f(D) - 1}{f(D)} \quad (3)$$

A summary of the DC voltage gains (M) for all the six different configurations is given in Table 3 in terms of the voltage gain of the converter in configuration 5, i.e. in terms of $f(D)$. Note that the voltage gain entries given in Table 3 hold if the active switch position in the converter cell is invariant for the different configurations. In practice we find that the active switch position changes. This happens, in particular, for family members which are bilateral inversions of each other. If the active switch position changes the relevant entry in Table 3 for the conversion ratio also changes by the simple replacement of $1 - D$ for D .

CONFIG. NUMBER	1	2	3	4	5	6
M	$\frac{1}{1-f(D)}$	$1-f(D)$	$\frac{f(D)}{f(D)-1}$	$\frac{f(D)-1}{f(D)}$	$f(D)$	$\frac{1}{f(D)}$

Table 3. *Relationship of the DC voltage conversion ratios (M) for the different members of a family.*

4.2 DC and AC Small-Signal Converter-Cell Models

The DC and AC small-signal models of any converter cell can be derived and used in analysing the converters comprising the members of the family generated by the cell. As an example, in the following the DC and AC small-signal models of converter-cell A will be derived for both continuous and discontinuous inductor current operation. These models will then be used to derive the DC operating conditions and the control-to-output transfer function for the specific case of the boost converter.

4.2.1 Model of Converter-Cell A in Continuous Conduction Mode

Converter-cell A shown in Table 2 is again shown in Fig. 6a where the *average* terminal currents i_1, i_2 and i_3 and *average* terminal voltages v_{12}, v_{13} and v_{23} are indicated. The instantaneous currents $\tilde{i}_1, \tilde{i}_2$ and $\tilde{i}_3$ are shown in Fig. 6b for the case of continuous conduction mode. In the analysis that follows, S1 is designated to be the active switch with duty ratio d. If the active switch assignment is changed to S2 as in the case of family members 1,4 and 6 of converter-cell A, then in all the results obtained D changes to D' and $\hat{d}$ to $-\hat{d}$.

Since the average voltage across L is zero, the average terminal voltages v_{13} and v_{23} are equal to the average terminal voltages v_{1a} and v_{2a} of the switches, respectively. Consequently, the simplified circuit shown in Fig. 6c need only be considered. From Fig. 6c we have the following equations for the average terminal voltages and currents

$$i_1 = d i_3 \tag{4a}$$

$$v_{12} = \frac{v_{32}}{d} \tag{4b}$$

Small-signal perturbation of these equations gives

$$\hat{i}_1 = I_3 \hat{d} + D \hat{i}_3 \tag{5a}$$

$$\hat{v}_{12} = \frac{1}{D}\hat{v}_{32} - \frac{V_{32}}{D^2}\hat{d} \tag{5b}$$

Equations (5a) and (5b) correspond to the equivalent circuit model of Fig. 6d which when combined with the inductor gives the equivalent circuit of converter-cell A as shown in Fig. 7. The buck, boost and buck-boost converters, which are members of converter-cell A family, can be analysed simply by replacing cell A with its equivalent circuit model. An example of the boost converter shown in Fig. 8 is now given.

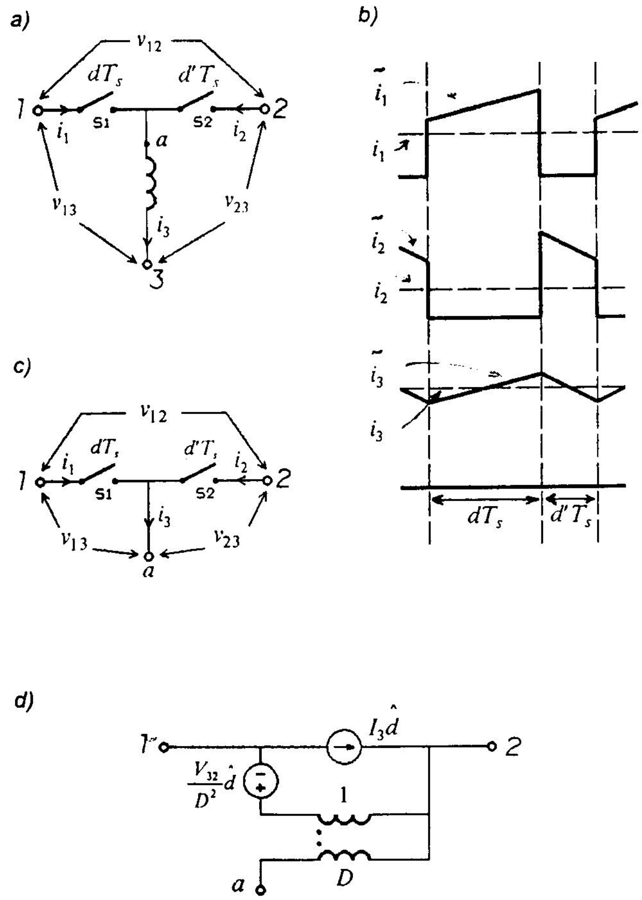


Fig. 6. *a) Converter-cell A showing designated terminal voltages and currents and switch on-times for continuous conduction.*
b) Typical instantaneous terminal currents, $\tilde{i}_1, \tilde{i}_2$ and $\tilde{i}_3$, *and average terminal currents,* i_1, i_2 and i_3, *for converter-cell A in continuous inductor current operation.*
c) Simplified circuit of converter-cell A for which an equivalent circuit model will be found.
d) Equivalent circuit model for (c).

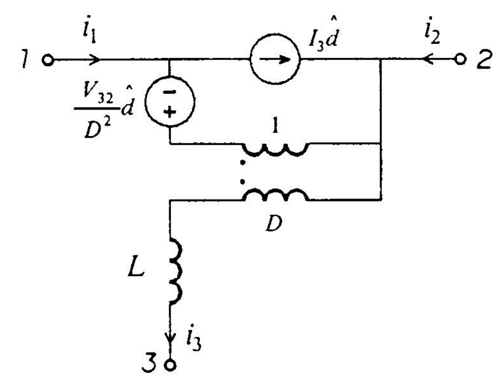


Fig. 7. *DC and AC small-signal averaged model of converter-cell A. This model can be used to derive transfer functions for the buck, the boost and buck-boost converters.*

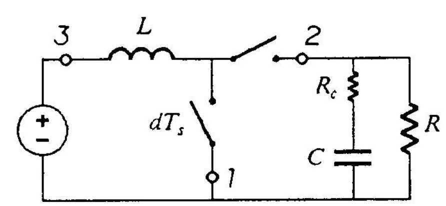


Fig. 8. *The boost converter with converter-cell A terminals identified.*

4.2.1.1 Example - Boost Converter in Continuous Conduction Mode

DC Analysis

Point-by-point replacement of converter-cell A gives the DC equivalent circuit model of the boost converter as shown in Fig. 9. Note that the perturbation sources have been removed for DC analysis. The current

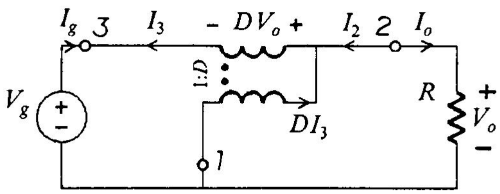


Fig. 9. *DC model of the boost converter in continuous conduction.*

conversion ratio is easily determined to be

$$\frac{I_o}{I_g} = \frac{I_2}{I_3} = 1 - D \equiv D' \tag{6}$$

The voltage conversion ratio can now be approximately written as

$$M \equiv \frac{V_o}{V_g} \cong \frac{1}{1 - D} \equiv \frac{1}{D'} \tag{7}$$

This result is approximate since the power dissipation in the esr of the capacitor and the extra power in the output due to the pulsating current $\tilde{i}_2$ do not appear in this analysis. This is a slight limitation of this analysis which can be easily overcome by noting that the actual conversion ratio can be written as

$$M = \frac{I_g}{I_o}\eta \tag{8}$$

where the efficiency η can be easily estimated from

$$\eta = \frac{P_o}{P_{in}} = \frac{I_o^2 R}{I_o^2 R + \langle(\tilde{i}_2 - I_2)^2\rangle R_c \| R} \tag{9}$$

where the squared rms value of the pulsating component of the current in terminal 2 can be easily estimated from Fig. 6b to be

$$\langle(i_2 - I_2)^2\rangle \cong (i_3 - I_2)^2 D' + I_o^2 D = \frac{D}{D'} I_o^2 \tag{10}$$

where in the last step (6) is used. Combining (8), (9) and (10) gives a more accurate, but still approximate, result which agrees with the result given in [26] :

$$M \cong \frac{1}{D' + D\dfrac{R_c}{R + R_c}} \cong \frac{1}{D'} \tag{11}$$

It is clear therefore that this added refinement is unnecessary and that (7) is fairly accurate. This refinement was given only to show and explain the slight discrepancy of (7) with the result given in [26]. It should be pointed out that the parasitic resistance of the inductor and the voltage drops of the switches can be easily accounted for in this model and the results would agree with the results given in [27].

Neglecting the refinement in the voltage conversion ratio we can derive, from the DC model of Fig. 9, the following quantities which will be needed in the AC analysis:

$$V_{32} = -\frac{D}{1 - D} V_g \tag{12}$$

$$I_3 = -\frac{V_g}{(1 - D)^2 R} \tag{13}$$

AC Small-Signal Analysis

Replacing converter-cell A point-by-point in Fig. 8 by its AC model we get the small-signal equivalent model of the boost converter in continuous conduction mode as shown in Fig. 10. Since we are only going to determine the control-to-output transfer function, $\hat{v}_o/\hat{d}$,

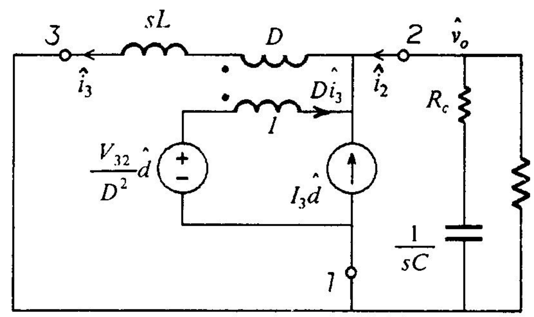


Fig. 10. *AC small-signal model of the boost converter in continuous conduction used in deriving the control-to-output transfer function.*

V_g is replaced by a short circuit. From Fig. 10 the following equations can be written:

$$\hat{v}_o = -\hat{i}_2 R \frac{1 + sR_cC}{1 + s(R + R_c)C} \tag{14}$$

$$\hat{i}_2 = -I_3\hat{d} - D\hat{i}_3 + \hat{i}_3 \tag{15}$$

$$\hat{v}_o + \left(\frac{V_{32}}{D^2}\hat{d} - \hat{v}_o\right)D = sL\hat{i}_3 \tag{16}$$

Equations (12) through (16) can be solved to obtain the control-to-output transfer function:

$$\frac{\hat{v}_o}{\hat{d}} = G_{od}\frac{(1 + s/s_{z1})(1 + s/s_{z2})}{1 + \dfrac{s}{\omega_o Q} + \dfrac{s^2}{\omega_o^2}} \tag{17}$$

where:

$$G_{od} = \frac{V_g}{D'^2} \tag{18a}$$

$$\omega_o = \frac{D'}{\sqrt{LC}}\sqrt{\frac{R}{R + R_c}} \cong \frac{D'}{\sqrt{LC}} \tag{18b}$$

$$Q = \frac{1}{\omega_o(R_cC + L/D'^2R)} \tag{18c}$$

$$s_{z1} = R_cC \qquad s_{z2} = -\frac{D'^2R}{L} \tag{18d - e}$$

If these results are compared to those results that would be obtained by the method of state-space averaging [26], then there would be a slight discrepancy in Q, s_{z2} and ω_o, but not in s_{z1}. These discrepancies can often be neglected as explained in

[27] and will be given here only for completeness. Thus, from state-space averaging we would get for the boost converter

$$s_{z2} = -\frac{D'^2}{L}(R - R_c||R) \cong -\frac{D'^2 R}{L} \tag{19}$$

$$\omega_o = \frac{D'}{\sqrt{LC}}\sqrt{\frac{R + (R_c||R)DD'}{R + R_c}} \cong \frac{D'}{\sqrt{LC}} \tag{20}$$

Hence, we see that (17) and (18) are very adequate for the determination of $\hat{v}_o/\hat{d}$.

4.2.2 Model of Converter-Cell A in Discontinuous Conduction Mode

Fig. 11a shows converter-cell A with the designated terminal voltages and currents and switch on-times for the cell operating in discontinuous conduction. Typical terminal current waveforms, for this mode, are shown in Fig. 11b. Note there exists a subinterval, d_3T_s, when both switches are off. From the instantaneous waveforms we can find the average currents. Thus

$$i_1 = \frac{d^2 v_{13} T_s}{2L} \tag{21a}$$

$$i_2 = -\frac{d_2{}^2 v_{23} T_s}{2L} \tag{21b}$$

where d_2, defined in Fig. 11, will next be determined. From volt-second balance considerations on the inductor we find

$$d_2 = -\frac{d v_{13}}{v_{23}} \tag{22}$$

Thus (21b) becomes

$$i_2 = -\frac{d^2 v_{13}^2 T_s}{2L v_{23}} \tag{23}$$

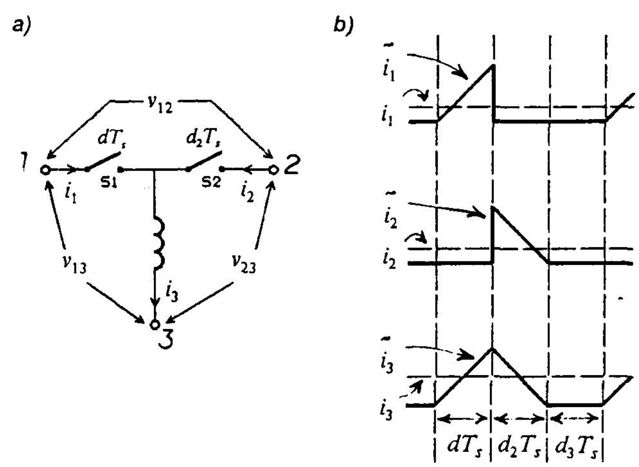


Fig. 11. *a) Converter-cell A showing designated terminal voltages and currents and switch on-times for discontinuous conduction. During the d_3T_s sub-interval both switches are open.*
b) Typical instantaneous terminal currents, $\tilde{i}_1$, $\tilde{i}_2$ and $\tilde{i}_3$, and average terminal currents, i_1, i_2 and i_3, for converter-cell A in discontinuous inductor current operation.

Following a similar procedure as before, we find from (21a) and (23) the DC averaged model is given by

$$I_1 = \frac{D^2 V_{13} T_s}{2L} \tag{24a}$$

and

$$I_2 = -\frac{I_1 V_{13}}{V_{23}} \tag{24b}$$

To find the AC small-signal averaged model we perturb and linearise (21a) and (23) which gives

$$\hat{i}_1 = \frac{D^2 T_s}{2L}\hat{v}_{13} + \frac{2I_1}{D}\hat{d} \tag{25a}$$

$$\hat{i}_2 = -\frac{2I_1}{V_{23}}\hat{v}_{13} - \frac{2I_1 V_{13}}{D V_{23}}\hat{d} + \frac{D^2 T_s V_{13}^2}{2L V_{23}^2}\hat{v}_{23} \tag{25b}$$

From (24) and (25) a DC and AC small-signal averaged circuit model can be drawn. This model is given in Fig. 12. One should note that in this model R_{13} is a DC and AC resistance whereas r_{23} is an AC-only resistance.

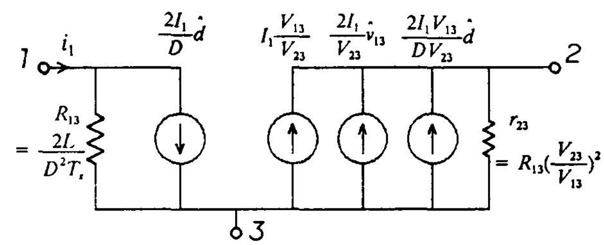


Fig. 12. *DC to AC small-signal averaged model for converter-cell A operating in discontinuous conduction.*

4.2.2.1 Example - Boost Converter in Discontinuous Conduction

In a similar fashion to that for continuous conduction, a DC and AC analysis will now be performed on the boost converter shown in Fig. 8 for discontinuous conduction. However, now for convenience, the esr of the output capacitor, R_c, will be neglected.

DC Analysis

Replacing converter-cell A in Fig. 8 point-by-point by its DC model in discontinuous conduction leads to the model of Fig. 13. From this model we see that

$$V_o = \frac{I_1 V_{13}}{V_{23}} R \tag{26}$$

Also from Fig. 13 we see,

$$\begin{aligned} V_{13} &= -V_g \\ V_{23} &= V_o - V_g \\ I_1 &= -\frac{V_g}{R_{13}} = -\frac{D^2 T_s}{2L} V_g \end{aligned} \tag{27}$$

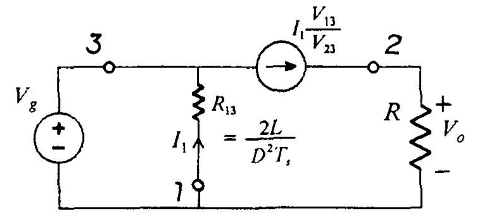


Fig. 13. *DC model of the boost converter in discontinuous conduction.*

Substituting (27) into (26) leads to

$$D = \sqrt{KM(M - 1)} \tag{28}$$

where $M \equiv \frac{V_o}{V_g}$ and $K \equiv \frac{2L}{RT_s}$.

Solving the quadratic in M in (28) gives the DC voltage conversion ratio:

$$M = \frac{1 + \sqrt{1 + 4D^2/K}}{2} \tag{29}$$

AC Small-Signal Analysis

The AC model of the boost converter in discontinuous conduction used to derive the control-to-output transfer function is shown in Fig. 14 and is given by substituting point-by-point the converter-cell model of Fig. 12 into the boost converter of Fig. 8.

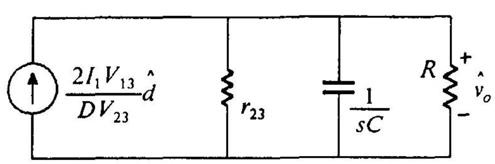


Fig. 14. *AC small-signal model of the boost converter in discontinuous conduction used in deriving the control-to-output transfer function.*

AC resistance r_{23}, as specified in the model of Fig. 12, is given by

$$r_{23} = \left(\frac{M - 1}{M}\right)R \tag{30}$$

where (27) and (28) have been used in deriving (30).

The circuit model of Fig. 14 gives the required result:

$$\frac{\hat{v}_o}{\hat{d}} = \frac{2V_o}{2M - 1}\sqrt{\frac{M - 1}{KM}}\,\frac{1}{1 + s\left(\frac{M - 1}{2M - 1}\right)RC} \tag{31}$$

5. CONCLUSION

From consideration of the structure of basic converter topologies one can identify three fundamental component blocks: (1) the input source, (2) the converter cell, and, (3) the output sink. Whilst the input source and output sink are invariant in structure, the converter cell, in contrast, may be either variant or invariant in structure. It is also variant in configuration to the input source and output sink. The converter-cell structure considered is of a three-terminal arrangement. The variations in configuration, of a particular converter cell, between input source and output sink, lead to the generation of a family of converters derived from this cell. Depending on the symmetry of the cell, three or six different converters may be generated. Different converter-cell structures will generate different converter families. Fourteen distinct basic converter cells have been considered in this paper. A classification of basic converter topologies has been proposed based on classification of converter cells which are categorised according to their order and switch number. The fourteen cells considered have been categorised into four different classes. A number of new basic converter topologies have been generated. From these basic topologies, isolated versions may be derived. For an illustration of how isolation may be introduced into basic topologies see [20] and [21]. By increasing the order and/or the switch number a greater number of converter cells may be considered. However, the usefulness of the resulting converters would be dubious.

Not only can a converter cell be used to generate converters and also be the basis for classification of converter topologies but it can also be used in the analysis of the converters it generates. DC and AC small-signal models of a converter cell can be derived by treating the cell as a three-terminal device. These models may be used in analysing the converters of the related converter-cell family. This approach to converter analysis is similar to transistor circuit analysis. Both continuous and discontinuous inductor conduction modes of operation of converter cells can be handled by the proposed method. As an example, the DC and AC small-signal models of converter-cell A were derived in continuous and discontinuous inductor current modes of operation. These models were subsequently used to analyse one of the converters generated by this cell, namely the boost converter. Whilst the results of this analysis are not new , the methodology is. The simple analysis approach presented here does not require any matrix manipulation and furthermore, since linearised circuit models result, use of standard computer circuit analysis programs such as SPICE may be made.

The approach presented here need not be limited in application to only PWM converters. This approach in identifying a three-terminal structure can also, for example, be applied, for the purpose of generation and analysis of converters [22], to the recently introduced quasi-resonant classes of converters [23,24,25].

Acknowledgement

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