FY2045 Quantum Mechanics I

Fall 2023

Henning Goa Hugdal

Week 1

Welcome!

Practical information

Lectures

Mondays 10-12 in H3

Thursdays 12-14 in KJL5

Summaries posted on webpage.

Exercises

Fridays 10-12 in R5

Mandatory exercise(s)! More information on this later.

Office hours

Tuesdays 14:00 – 15:00 in E5-149

Course homepage

`hghugdal.github.io/FY2045`

Practical information

Learning materials

Lecture notes by Øverbø

Kvantemekanikk by Hemmer

Introduction to Quantum Mechanics by Griffiths and Schroeter

(*Quantum Mechanics* by Bransden and Joachain)

Lecture notes, presentations, and exercises will be posted every week.

Reference group: 3-4 volunteers

— talk to me during the break or send an email

This course

- Continue studying quantum mechanics on the foundations from *TFY4215/FY1006 Introduction to Quantum Physics*
- Develop necessary mathematical tools and present a more general and abstract formulation of quantum mechanics
- Focus on methods and calculations, not "understanding"/interpretation

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"Those who are not shocked when they first come across quantum theory cannot possibly have understood it."
— Niels Bohr

"I think I can safely say that nobody understands quantum mechanics."
— Richard Feynman

Topics

- Short introduction and recap
- QM postulates
- Dirac bra/ket notation
- General formulation of QM
- Algebraic methods — harmonic oscillator
- Spin, addition of angular momentum
- Identical particles, Pauli principle and degenerate matter
- Perturbation theory
- Variational method

This list is tentative

Exercise session on **Friday August 25th** will be used for recap of relevant topics from *Introduction to Quantum Mechanics*

Questions?

Short introduction and recap

Why quantum mechanics?

Several experiments led to a crisis of classical physics, e.g.

- Blackbody radiation
- Photoelectric effect
- Compton effect

To explain these results, one had to assume that light can behave like particles with energy and momentum

$$E = hf \qquad (h\text{: Planck's constant, } f\text{: frequency})$$

$$p = \frac{hf}{c} \qquad (c\text{ : speed of light}).$$

Light has both wave-like and particle-like properties!

Can matter have wave-like properties?

Diffraction of electrons

For photons

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{h}{p},$$

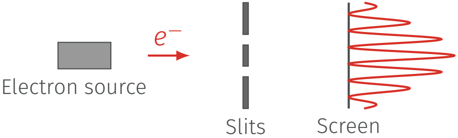
using $E = pc$.

de Broglie postulated that also **particles have a wavelength** given by the same relation.

Gives rise to diffraction patterns.

Double-slit experiment

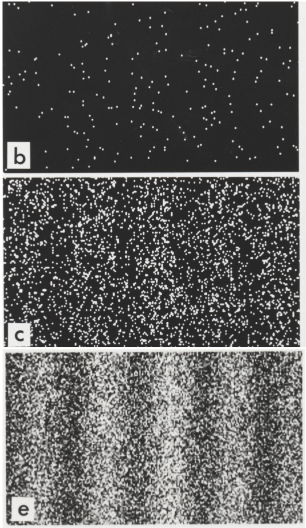
If matter behaves like a wave, we should get interference patterns!



"The most beautiful experiment in physics"[a]

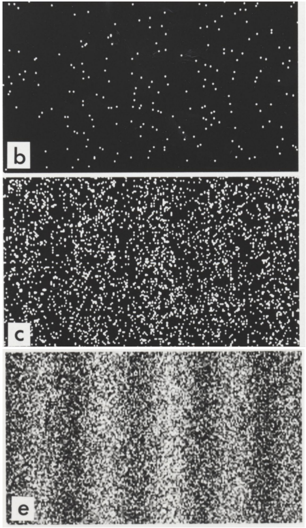
[a] R. P. Crease, *Physics World* **15**, (9) 19 (2022)

Electrons → Neutrons → Atoms → Molecules



Adapted from figure by Dr. Tonomura

Electrons → Neutrons → Atoms → Molecules



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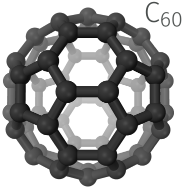
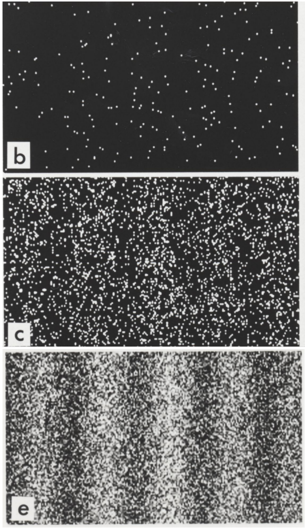


By Benjah-bmm27

Electrons → Neutrons → Atoms → **Molecules**



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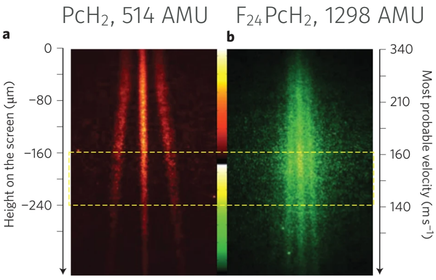
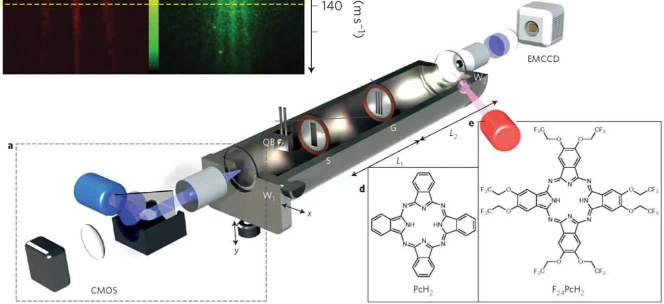
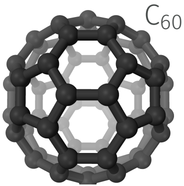




Adapted from Juffmann *et al.*, *Nature Nanotechnology* **7**, 297–300 (2012)



By Benjah-bmm27

So, what do photons and electrons really behave like — waves or particles?

They really behave ”like **neither**.”[1]

[1] *The Feynman Lectures on Physics*, Feynman, Leighton and Sands, Vol. III, ch. 1-1. **Recommended read!**

A particle of mass m moving in one dimension in a potential $V(x)$ is described by a wavefunction $\Psi(x,t)$ that is a solution of the **Schrödinger equation**

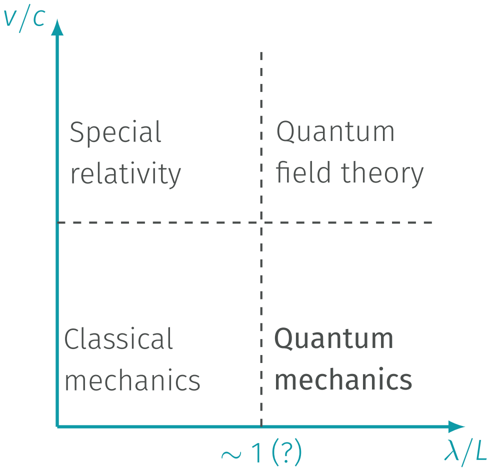
$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi,$$

a linear partial differential equation. $|\Psi|^2$ interpreted as a probability density.

Planck's reduced constant

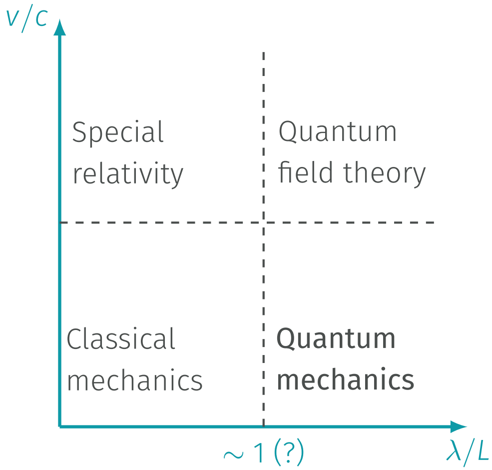
$$\hbar = \frac{h}{2\pi} = 1.054\,571\,817 \times 10^{-34}\,\mathrm{J\,s}.$$

When do we need Quantum mechanics?



- L : characteristic size of system
- $\lambda = h/p$: de Broglie wavelength for object with momentum p
- v : speed of object
- c : speed of light

When do we need Quantum mechanics?



Are we "quantum" when biking?

$$\lambda_H \sim 3 \times 10^{-37}\ \mathrm{m}$$

No, definitely not!

Postulates of Quantum Mechanics

A: The operator postulate

To each physical observable quantity F there corresponds in quantum-mechanical theory a linear operator $\hat{F}$.

$$x \rightarrow \hat{x} = x,$$

$$p \rightarrow \hat{p} = -i\hbar\frac{d}{dx}$$

Observable must be real $\rightarrow$ $\hat{F}$ must be **hermitian**, $\hat{F} = \hat{F}^{\dagger}$.

Ø2.2, Ø7.1, H2.1

B: The wavefunction postulate

The state of a system is described, as completely as possible, by the wave function $\Psi(q_n, t)$. The time development of the wave function (and hence of the state) is determined by the Schrödinger equation,

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi,$$

where $\hat{H}$ is the Hamiltonian of the system.

C: The expectation value postulate

When a large number of measurements of an observable F is made on a system which is prepared in a state $\Psi(q_1, q_2, \cdots, q_n, t)$ (before each measurement), the average $\bar{F}$ of the measured values will approach the theoretical expectation value, which is postulated to be

$$\langle F \rangle = \int \Psi^* \hat{F} \Psi \, d\tau,$$

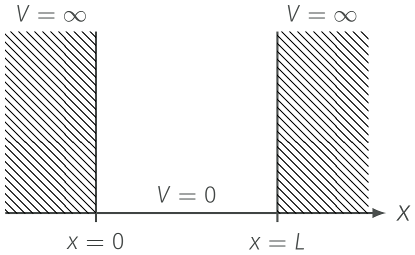
where $d\tau = dq_1 dq_2 \cdots dq_n$ and where the integration goes over the whole range of each of the variables.

D: The measurement postulate

(i) The only possible result of a precise measurement of an observable F is **one of the eigenvalues** f_n of the corresponding linear operator $\hat{F}$.

(ii) Immediately after the measurement of the eigenvalue f_n, the system is in an eigenstate of $\hat{F}$, namely, the eigenstate ψ_n corresponding to the measured eigenvalue f_n.

Example — Infinite square well



$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x),$$

$$V(x) = \begin{cases} 0, & \text{if } 0 < x < L, \\ \infty, & \text{otherwise.} \end{cases}$$

Ø2.1, H3.2, G2.2

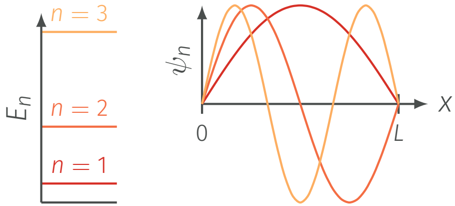
Solution

$$\Psi_n = \psi_n e^{-iE_n t/\hbar},$$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2},$$

$$\psi_n = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L},$$

with $n = 1, 2, 3 \ldots$



Time-dependence of expectation values

$$\frac{d}{dt}\langle F\rangle = \frac{i}{\hbar}\left\langle\left[\hat{H},\hat{F}\right]\right\rangle + \left\langle\frac{\partial\hat{F}}{\partial t}\right\rangle$$

Ehrenfest's theorem

For system with Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$, we have

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m},$$
$$\frac{d\langle p\rangle}{dt} = -\left\langle\frac{dV}{dx}\right\rangle$$

Measurement of a degenerate eigenvalue

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Non-degenerate case

$$\hat{F}\psi_n = f_n\psi_n$$

has only one solution ψ_n for eigenvalue f_n.

If a measurement of observable F gives f_n, the system is in state ψ_n immediately after the measurement.

Measurement of a degenerate eigenvalue

Degenerate case

$$\hat{F}\psi_{ni} = f_n\psi_{ni}, \quad i = 1, 2, \ldots g_n.$$

Complete set, expand general state as

$$\Psi = \sum_n \sum_{i=1}^{g_n} c_{ni}\psi_{ni},$$

resulting in probability of measuring f_n

$$P_n = \sum_{i=1}^{g_n} |c_{ni}|^2.$$

Define function with eigenvalue f_n,

$$\Psi_n = \sum_{i=1}^{g_n} c_{ni}\psi_{ni}.$$

Immediately after a measurement of eigenvalue f_n, the system is in the normalized state

$$\frac{\Psi_n}{||\Psi_n||},$$

with $||\Psi_n||$ the norm of Ψ_n.

∅7.2