A Formula sheet

Schrödinger equation (time-dependent and time-independent)

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$$
$$\hat{H}|\psi\rangle = E|\psi\rangle$$

Thermodynamics

$$dW = PdV$$

Eigenvalues and eigenvectors

$$\det(A-\lambda I) = 0$$
$$(A-\lambda I)\mathbf{v} = 0$$

Some properties of the Dirac delta function and Heaviside step function

$$\int dx\ f(x)\delta(x-a) = f(a)$$
$$\frac{1}{2\pi}\int dx\ e^{i(k-k_0)x} = \delta(k-k_0)$$
$$\Theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x \leq 0. \end{cases}$$
$$\frac{d}{dx}\Theta(x) = \delta(x)$$
$$\int_{-\infty}^{\infty} dx\ \left[\frac{d}{dx}\delta(x)\right] f(x) = -\int_{-\infty}^{\infty} dx\ \delta(x)\left[\frac{d}{dx}f(x)\right]$$

Various physical constants

$$\hbar = 1.054\,571\,817\times 10^{-34}\,\mathrm{J\,s} = 6.582\,119\,569\times 10^{-16}\,\mathrm{eV\,s}$$
$$m_e = 9.109\,383\,701\,5\times 10^{-31}\,\mathrm{kg}$$
$$e = 1.602\,176\,634\times 10^{-19}\,\mathrm{C}$$
$$c = 299\,792\,458\,\mathrm{m\,s^{-1}} \approx 3\times 10^{8}\,\mathrm{m\,s^{-1}}$$
$$\mu_0 = \frac{1}{\epsilon_0 c^2} = \frac{4\pi\alpha}{e^2}\frac{\hbar}{c} = 1.256\,637\,062\,12\times 10^{-6}\,\mathrm{N\,A^{-2}}$$
$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$
$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{e^2 m_e} = 5.29\times 10^{-11}\,\mathrm{m}$$

$$\mu_B = \frac{\hbar e}{2m_e} = 9.274\,010\,078\,3 \times 10^{-24}\,\mathrm{J\,T^{-1}} = 5.788\,381\,806\,0 \times 10^{-5}\,\mathrm{eV\,T^{-1}},$$

Commutators and anticommutators

$$\begin{aligned}
[A, B] &\equiv AB - BA \\
[AB, C] &= [A, C]B + A[B, C] \\
[A + B, C] &= [A, C] + [B, C] \\
\{A, B\} &\equiv AB + BA \\
[\hat{x}, \hat{p}_x] &= i\hbar \\
[\hat{S}_x, \hat{S}_y] &= i\hbar \hat{S}_z
\end{aligned}$$

Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Taylor expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{d}{dx} \right)^n f(x) \bigg|_{x=a} (x-a)^n$$

Some potentially useful integrals

$$\begin{aligned}
\int_{-\infty}^{\infty} dx\ e^{-a(x+b)^2} &= \sqrt{\frac{\pi}{a}} \\
\int_{-\infty}^{\infty} dx\ e^{-ax^2+bx+c} &= \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}+c} \\
\int_{-\infty}^{\infty} dx\ x^{2n} e^{-ax^2} &= \left(-\frac{\partial}{\partial a} \right)^n \int_{-\infty}^{\infty} dx\ e^{-ax^2}
\end{aligned}$$

Cylindrical coordinates

$$\begin{aligned}
x = r\cos\phi, \quad y &= r\sin\phi, \quad z = z \\
\boldsymbol{\nabla} f &= \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \\
\boldsymbol{\nabla} \cdot \mathbf{A} &= \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\
\nabla^2 f &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \\
\int d\mathbf{r} &= \int dz\ d\phi\ dr\ r
\end{aligned}$$

Spherical coordinates

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$

$$\boldsymbol{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\boldsymbol{\nabla}\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$$\int d\mathbf{r} = \int d\phi\ d\theta\ dr\ \ \sin\theta r^2$$