Lecture notes FY2045 Quantum Mechanics I — 1 Short introduction and recap

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1 *Why do we need quantum mechanics?*

Several experiments in the late 1800s and early 1900s produced results which could not be explained by classical physics. This lead to a crisis! A few examples are

- Black-body radiation
- Photoelectric effect
- Compton effect

To explain these results one had to assume that light can behave like particles — *photons*, coming in quanta of energy

$$E = hf, \tag{1}$$

where h is Planck's constant and f is the light's frequency, with momentum

$$p = \frac{hf}{c}, \tag{2}$$

where c is the speed of light. **Light has both wave-like and particle-like properties!**

Given this realization, it was perhaps natural to ask: **Can matter have wave-like properties?** It turned the answer is yes.

Diffraction of electrons For photons, we have

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{h}{p},$$

where we have used $E = hf = pc$. de Broglie postulated that also particles have a wavelength given by

$$\lambda = \frac{h}{p}. \tag{3}$$

This gives rise to diffraction patterns, as was confirmed experimentally.

Double-slit experiment If matter behaves like a wave, we should also get interference patterns, e.g. in a double-slit experiment. This was confirmed experimentally: even when sending one electron at a time, one gets an interference pattern.

Note: When the electron travels through the slits it is "wave-like", giving rise to interference. When it hits the screen, it is "particle-like", giving rise to a point on the screen.

If one tries to observe the electron at one of the slits to see which way it goes, the interference pattern disappears![1]

[1] For a great read on the subtleties of double-slit experiment, see Feynman *et al.* (1963) Vol. III, Ch. 1 — Quantum behavior.

2 Theoretical description — Quantum mechanics

In the mid 1920s the full theory of quantum mechanics was developed, with several alternative formulations, which we will discuss more later. One formulation is Schrödingers formulation — wave mechanics:

A particle with mass m moving in 1D in a potential $V(x)$ is described by a wavefunction $\Psi(x,t)$ that is a solution of the Schrödinger equation (SE)

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\Psi. \tag{4}$$

$|\Psi|^2$ is interpreted as a probability density.

$\hbar = h/(2\pi)$ is Planck's *reduced* constant, which is what we will most often use, and often refer to simply as Planck's constant.

3 When do we need quantum mechanics?

Roughly speaking, one can say that we need quantum mechanics when (1) the characteristic length scale L of the system is comparable to or smaller than the de Broglie wavelength λ [eq. (3)] of the object and (2) the speed v of the object is much lower than the speed of light c, see fig. 1.

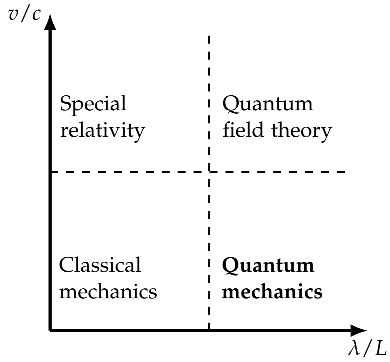

Figure 1: The figure illustrates what the relevant physical theory is depending on the ratios λ/L and v/c, where λ is the de Broglie wavelength, L is the characteristic system size, v is the speed of the object, and c is the speed of light.

References

R. Feynman, R. Leighton, and M. Sands. *The Feynman Lectures on Physics* (Addison-Wesley, Boston, 1963). URL `http://www.feynmanlectures.caltech.edu/`.