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FY2045 Problem set 1 fall 2023

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Problem 1

The normalized eigenfunctions of the harmonic oscillator are given by

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\frac{1}{2}\xi^2} , \quad (1)$$

where $H_n(x)$ are the so-called Hermite polynomials and $\xi = \sqrt{\frac{m\omega}{\hbar}}x$.

a) Calculate the expectation values $\langle x^2 \rangle$ and $\langle p^2 \rangle$ in the ground state $\psi_0(x)$ of the harmonic oscillator. $H_0(x) = 1$.

b) In class we derived the relation

$$\frac{d}{dt}\langle A \rangle = \frac{i}{\hbar}\langle [\hat{H}, \hat{A}] \rangle + \left\langle \frac{\partial \hat{A}}{\partial t} \right\rangle . \quad (2)$$

Choose $\hat{A} = \hat{x}\hat{p}$ and calculate the right-hand side of Eq. (2).

c) In a stationary state, the left-hand side of Eq. (2) vanishes. The resulting equation is an example of the virial theorem. Show that for the harmonic oscillator this implies $\langle T \rangle = \langle V \rangle$, where T is the kinetic energy. Show that the result in ***a***) are in agreement with the virial theorem.

Problem 2

A particle in the harmonic oscillator potential is prepared in the initial state given by

$$\Psi(x,0) \quad = \quad A\left[\psi_0(x) + \psi_1(x)\right] \ , \tag{3}$$

a) Normalize the initial wavefunction. $H_1(x) = 2x$.

b) Find the state $\Psi(x,t)$ and the propability distribution $|\Psi(x,t)|^2$.

c) Find the expectation value $\langle x \rangle$. Use Ehrenfest's theorem to find $\langle p \rangle$.

Problem 3

Consider the delta-function potential

$$V(x) = \beta\delta(x) \ , \tag{4}$$

see Fig. 1.

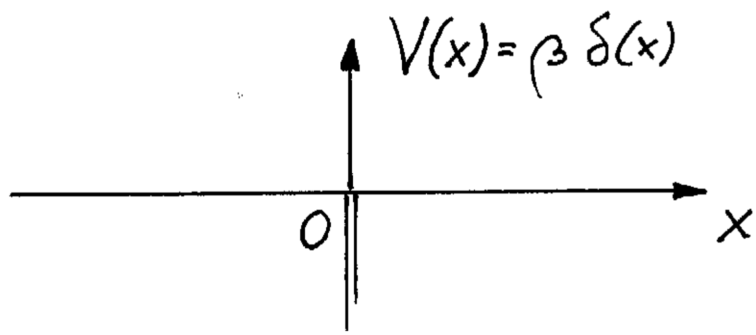

Figure 1: Delta-function potential.

a) For $x \neq 0$, the wavefunction can be written as

$$\psi(x) \quad = \quad Ae^{-kx} + Be^{kx} \ , \tag{5}$$

where A and B are coefficients and the energy is $E = -\frac{\hbar^2 k^2}{2m} < 0$. Explain why k must be real if we consider bound states and why the wavefunction can be written as

$$\psi(x) \quad = \quad Ae^{-k|x|} \ , \qquad k > 0 \ . \tag{6}$$

b) Determine the constant A. Integrate the Schrödinger equation over a small interval $(-\epsilon, \epsilon)$ around $x = 0$ and find β as a function of k.