

FY2045 Quantum Mechanics I

Fall 2023

Henning Goa Hugdal

Week 1

Welcome!

Practical information

Lectures

Mondays 10-12 in H3

Thursdays 12-14 in KJL5

Summaries posted on webpage.

Exercises

Fridays 10-12 in R5

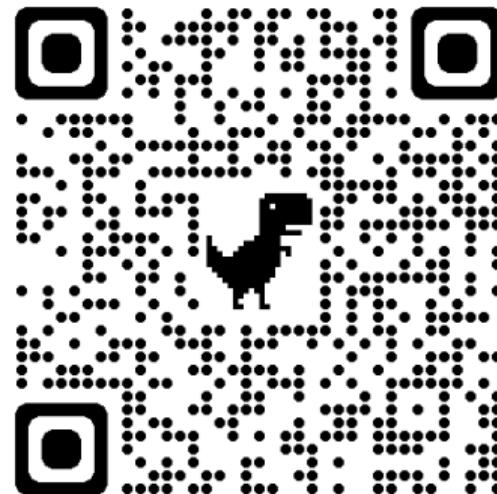
Mandatory exercise(s)! More information on this later.

Office hours

Tuesdays 14:00 – 15:00 in E5-149

Course homepage

hghugdal.github.io/FY2045



Practical information

Learning materials

Lecture notes by Øverbø

Kvantemekanikk by Hemmer

Introduction to Quantum Mechanics by Griffiths and Schroeter

(*Quantum Mechanics* by Bransden and Joachain)



Lecture notes, presentations, and exercises will be posted every week.

Reference group: 3-4 volunteers

- talk to me during the break or send an email

This course

- Continue studying quantum mechanics on the foundations from *TFY4215/FY1006 Introduction to Quantum Physics*
- Develop necessary mathematical tools and present a more general and abstract formulation of quantum mechanics
- Focus on methods and calculations, not “understanding”/interpretation

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“Those who are not shocked when they first come across quantum theory cannot possibly have understood it.”
— Niels Bohr

“I think I can safely say that nobody understands quantum mechanics.”
— Richard Feynman

Topics

- Short introduction and recap
- QM postulates
- Dirac bra/ket notation
- General formulation of QM
- Algebraic methods – harmonic oscillator
- Spin, addition of angular momentum
- Identical particles, Pauli principle and degenerate matter
- Perturbation theory
- Variational method

This list is tentative

Exercise session on **Friday August 25th** will be used for recap of relevant topics from *Introduction to Quantum Mechanics*

Questions?

Short introduction and recap

Why quantum mechanics?

Several experiments led to a crisis of classical physics, e.g.

- Blackbody radiation
- Photoelectric effect
- Compton effect

To explain these results, one had to assume that light can behave like particles with energy and momentum

$$E = hf \quad (h: \text{Planck's constant}, f: \text{frequency})$$

$$p = \frac{hf}{c} \quad (c : \text{speed of light}).$$

Light has both wave-like and particle-like properties!

Can matter have wave-like properties?

Diffraction of electrons

For photons

$$\lambda = \frac{c}{\nu} = \frac{hc}{E} = \frac{h}{p},$$

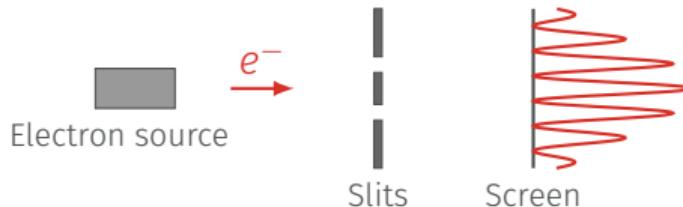
using $E = pc$.

de Broglie postulated that also **particles have a wavelength** given by the same relation.

Gives rise to diffraction patterns.

Double-slit experiment

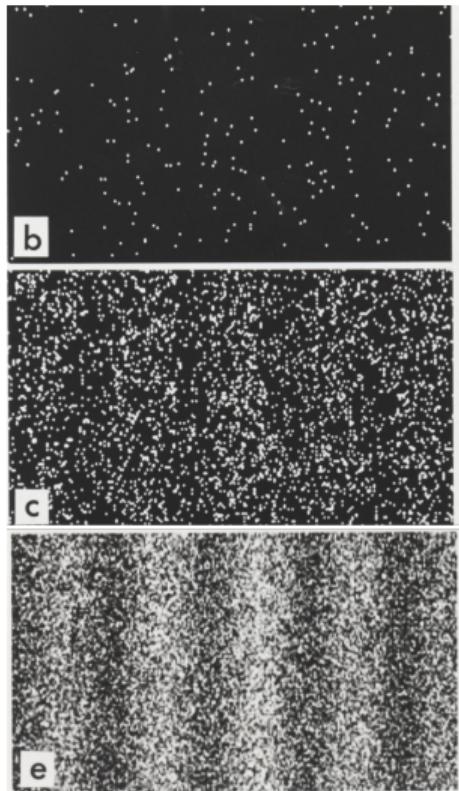
If matter behaves like a wave, we should get interference patterns!



"The most beautiful experiment in physics"^a

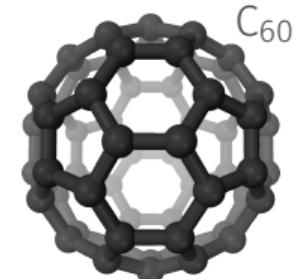
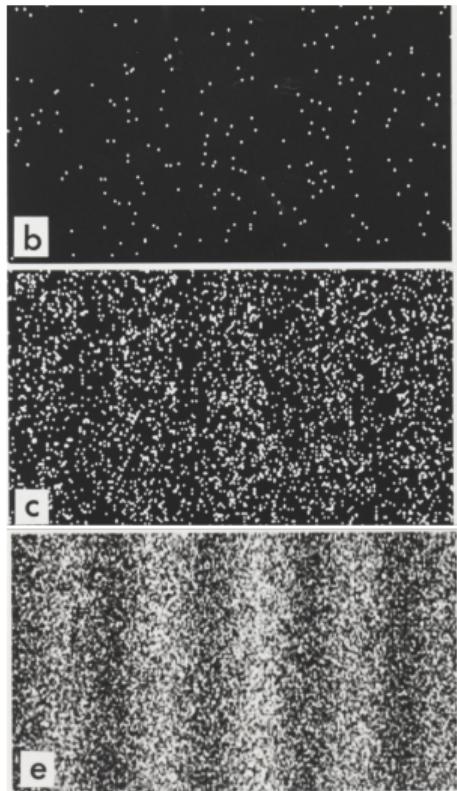
^aR. P. Crease, *Physics World* 15, (9) 19 (2022)

Electrons → Neutrons → Atoms → Molecules



Adapted from figure by Dr. Tonomura

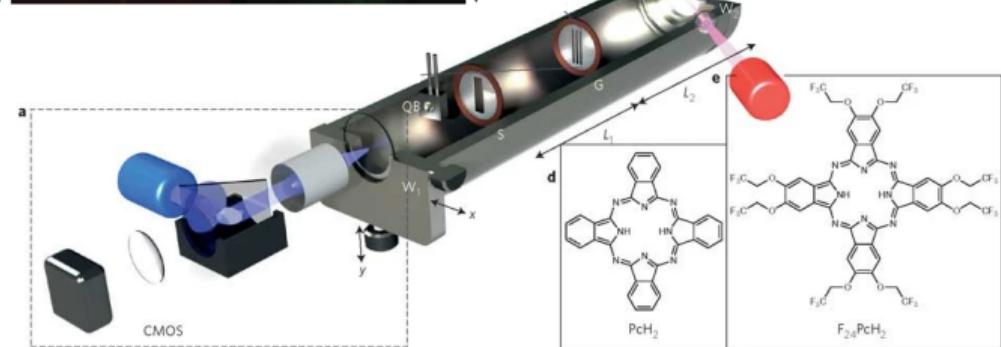
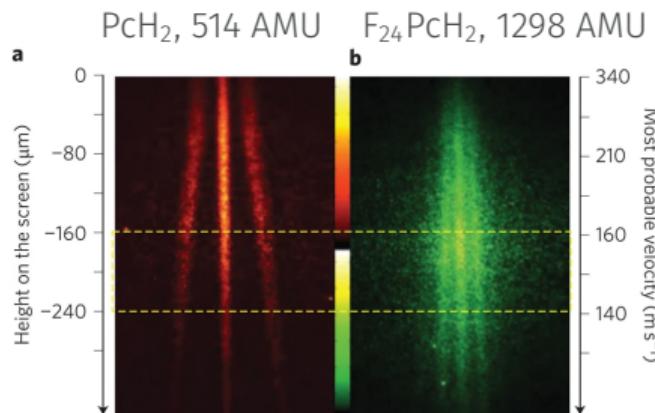
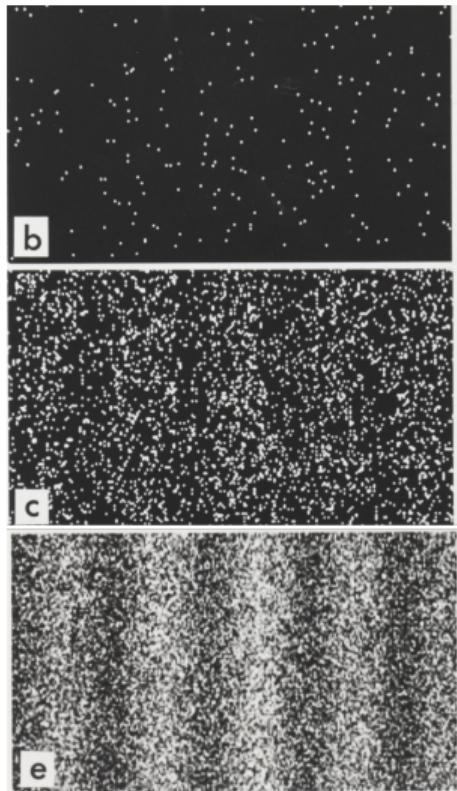
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By Benjah-bmm27

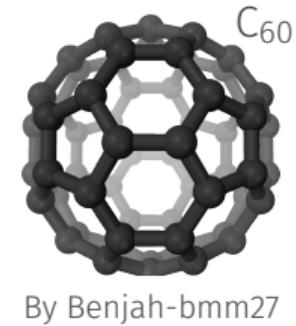
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Electrons → Neutrons → Atoms → Molecules



Adapted from figure by Dr. Tonomura

Adapted from Juffmann *et al.*, *Nature Nanotechnology* 7, 297–300 (2012)



By Benjah-bmm27

So, what do photons and electrons really behave like — waves or particles?

They really behave "like **neither**."¹

¹*The Feynman Lectures on Physics*, Feynman, Leighton and Sands, Vol. III, ch. 1-1. Recommended read!

Theoretical description – Quantum mechanics

A particle of mass m moving in one dimension in a potential $V(x)$ is described by a wavefunction $\Psi(x, t)$ that is a solution of the **Schrödinger equation**

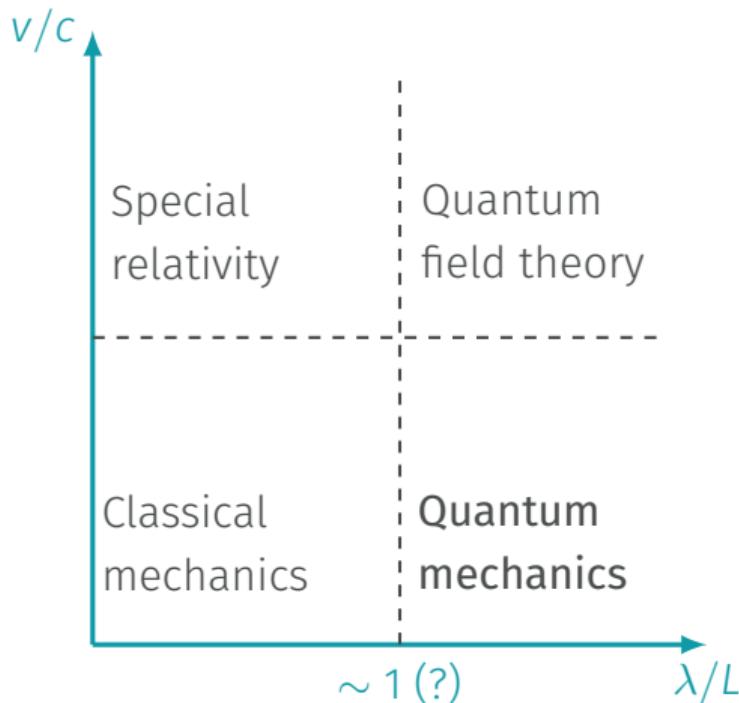
$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi,$$

a linear partial differential equation. $|\Psi|^2$ interpreted as a probability density.

Planck's reduced constant

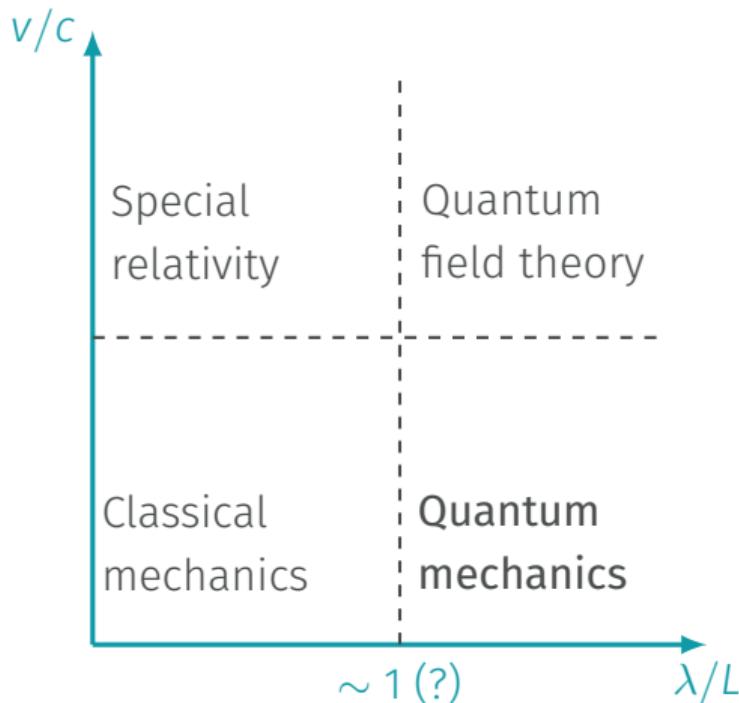
$$\hbar = \frac{h}{2\pi} = 1.054571817 \times 10^{-34} \text{ Js.}$$

When do we need Quantum mechanics?



- L : characteristic size of system
- $\lambda = h/p$: de Broglie wavelength for object with momentum p
- v : speed of object
- c : speed of light

When do we need Quantum mechanics?



Are we “quantum” when biking?

$$\lambda_H \sim 3 \times 10^{-37} \text{ m}$$

No, definitely not!

Postulates of Quantum Mechanics

A: The operator postulate

To each physical observable quantity F there corresponds in quantum-mechanical theory a linear operator \hat{F} .

$$x \rightarrow \hat{x} = x,$$

$$p \rightarrow \hat{p} = -i\hbar \frac{d}{dx}$$

Observable must be real $\rightarrow \hat{F}$ must be hermitian, $\hat{F} = \hat{F}^\dagger$.

Ø2.2, Ø7.1, H2.1

B: The wavefunction postulate

The state of a system is described, as completely as possible, by the wave function $\Psi(q_n, t)$. The time development of the wave function (and hence of the state) is determined by the Schrödinger equation,

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi,$$

where \hat{H} is the Hamiltonian of the system.

C: The expectation value postulate

When a large number of measurements of an observable F is made on a system which is prepared in a state $\Psi(q_1, q_2, \dots, q_n, t)$ (before each measurement), the average \bar{F} of the measured values will approach the theoretical expectation value, which is postulated to be

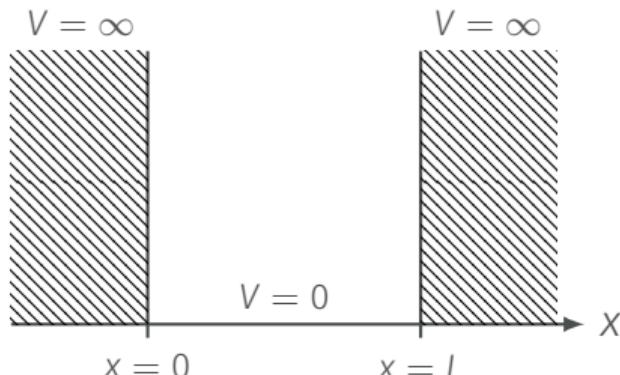
$$\langle F \rangle = \int \Psi^* \hat{F} \Psi d\tau,$$

where $d\tau = dq_1 dq_2 \cdots dq_n$ and where the integration goes over the whole range of each of the variables.

D: The measurement postulate

- (i) The only possible result of a precise measurement of an observable F is one of the eigenvalues f_n of the corresponding linear operator \hat{F} .
- (ii) Immediately after the measurement of the eigenvalue f_n , the system is in an eigenstate of \hat{F} , namely, the eigenstate ψ_n corresponding to the measured eigenvalue f_n .

Example – Infinite square well



$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}) = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x),$$

$$V(x) = \begin{cases} 0, & \text{if } 0 < x < L, \\ \infty, & \text{otherwise.} \end{cases}$$

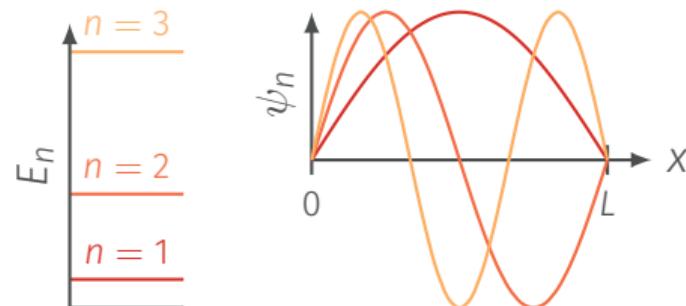
Solution

$$\Psi_n = \psi_n e^{-iE_n t/\hbar},$$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2},$$

$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L},$$

with $n = 1, 2, 3 \dots$



Time-dependence of expectation values

Ehrenfest's theorem

For system with Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$, we have

$$\frac{d}{dt}\langle F \rangle = \frac{i}{\hbar} \left\langle [\hat{H}, \hat{F}] \right\rangle + \left\langle \frac{\partial \hat{F}}{\partial t} \right\rangle$$

$$\frac{d\langle x \rangle}{dt} = \frac{\langle p \rangle}{m},$$

$$\frac{d\langle p \rangle}{dt} = - \left\langle \frac{dV}{dx} \right\rangle$$

Measurement of a degenerate eigenvalue

D: The measurement postulate

(i) The only possible result of a precise measurement of an observable F is **one of the eigenvalues f_n** of the corresponding linear operator \hat{F} .

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Non-degenerate case

$$\hat{F}\psi_n = f_n\psi_n$$

has only one solution ψ_n for eigenvalue f_n .

If a measurement of observable F gives f_n , the system is in state ψ_n immediately after the measurement.

Measurement of a degenerate eigenvalue

Degenerate case

$$\hat{F}\psi_{ni} = f_n\psi_{ni}, \quad i = 1, 2, \dots, g_n.$$

Complete set, expand general state as

$$\Psi = \sum_n \sum_{i=1}^{g_n} c_{ni} \psi_{ni},$$

resulting in probability of measuring f_n

$$P_n = \sum_{i=1}^{g_n} |c_{ni}|^2.$$

Define function with eigenvalue f_n ,

$$\Psi_n = \sum_{i=1}^{g_n} c_{ni} \psi_{ni}.$$

Immediately after a measurement of eigenvalue f_n , the system is in the normalized state

$$\frac{\Psi_n}{\|\Psi_n\|},$$

with $\|\Psi_n\|$ the norm of Ψ_n .