

Hannah Gibson
LC 330 ~~HW~~

HW
9/18/16

1) a. $\sum_{i=2}^{30} \left(\frac{1}{8}\right)^i = \left(\frac{1}{8}\right)^2 + \left(\frac{1}{8}\right)^3 + \dots + \left(\frac{1}{8}\right)^{30}$ Finite sum

~~0.0022~~

b. $\sum_{i=0}^{\infty} \frac{9}{7^i} = 9 \sum_{i=0}^{\infty} \left(\frac{1}{7}\right)^i$ Infinite sum
Geometric

~~9~~ $9 \left(\frac{1}{1 - \frac{1}{7}}\right) = 10.5$

c. $\sum_{i=1}^N (6i^3 + 3i - 9) = 6 \sum_{i=1}^N i^3 + 3 \sum_{i=1}^N i - 9 \sum_{i=1}^N 1$

$= 6 \left(\frac{N^2(N+1)^2}{4} \right) + 3 \left(\frac{N(N+1)(2N+1)}{6} \right) + 9N$

$= \frac{6(N^2(N^2+2N+1))}{4} + \frac{1}{2}(N^2+N)(2N+1) + 9N$

$= \frac{3}{2}(N^4 + 2N^3 + N^2) + \frac{1}{2}(2N^3 + N^2 + 2N^2 + N) + 9N$

$= \frac{3}{2}N^4 + \frac{5N^3}{2} + 3N^2 + \frac{19N}{2}$

d.

$$\sum_{i=8}^{250} \frac{1}{i}$$

RomMe
Sum

$$\approx 3.621$$

e.

~~$$\sum_{i=4}^{\infty} \frac{i}{32^i}$$~~

~~$$\sum_{i=4}^{\infty} \frac{i}{32^i}$$~~

~~$$= \sum_{i=1}^{\infty} i \left(\frac{1}{32}\right)^i$$~~

~~$$\sum_{i=1}^{\infty} i \left(\frac{1}{32}\right)^i$$~~

this is derivative of $\sum_{i=0}^{\infty} \left(\frac{1}{32}\right)^{i+1}$

~~$$\sum_{i=1}^{\infty} i \left(\frac{1}{32}\right)^i = \sum_{i=1}^{\infty} i \left(\frac{1}{32}\right)^i - \sum_{i=0}^{\infty} i \left(\frac{1}{32}\right)^i$$~~

$$\sum_{i=1}^{\infty} i \left(\frac{1}{32}\right)^i = \sum_{i=0}^{\infty} i \left(\frac{1}{32}\right)^i - \sum_{i=0}^{\infty} i \left(\frac{1}{32}\right)^i$$

$$= \frac{\frac{1}{32}}{\left(1 - \left(\frac{1}{32}\right)^2\right)} - 0 \approx 0.0313$$

$$2) a. x^{21} \cdot x^{22} \cdot x^{23} \cdots x^{77}$$

$$= \prod_{k=21}^{77} x^k$$

$$\log_x \left(\prod_{k=21}^{77} x^k \right) = \sum_{k=21}^{77} \log_x (x^k) = \sum_{k=21}^{77} k = 2418$$

$$\boxed{x^{21} \cdot x^{23} \cdots x^{77} = x^{2418}}$$

$$b. \log_{24}(76 \cdot 76 \cdot 76) = \frac{\log_{10}(76 \cdot 76 \cdot 76)}{\log_{10}(24)} \approx 24.09$$

$$c. \boxed{32^{\log_{32} 84} = 84}$$

$$d. \log_{49}((7x)^y) = \boxed{y \cdot \log_{49}(7x)}$$

$$e. \sum_{i=1}^{2N} \log_{54}(i) = \log_{54} \left(\prod_{i=1}^{2N} i \right) \neq \log_{54}(1 \cdot 2 \cdots 3 \cdots)$$

$$3) \binom{8}{3} = \frac{8!}{8! (8-3)!} = \frac{8!}{8! \cdot 5!} = \boxed{56}$$

6. ~~Even~~ Even when 2 odd, 4
 even or 3 even

~~99~~
~~3~~

Even $\rightarrow n \neq 2, 2, 0$
 Odd $\rightarrow n \neq 2, 2, 1$

$\{1, 2, \dots, 99\}$ of 2 = $\{1, 0, \dots, 0\}$
 $\left[\begin{smallmatrix} 2 \text{ odd}, 4 \text{ even} \end{smallmatrix} \right] \left[\begin{smallmatrix} 3 \text{ even} \end{smallmatrix} \right]$

$$\binom{5}{2} \cdot \binom{49}{1} + \binom{49}{3} = 49^2 \cdot 25 + 49 \cdot 47 \cdot 8$$

$$= \boxed{78449}$$