Crappy DSE Maths Paper II (2023 B-side)

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Abstract

This is a paper created by a no-lifer. Its sole purpose is to help the reader escape from the responsibilities of real life.

Contents

0 Syllabus and Rules

All of the questions in this paper are based on 2023 DSE Mathematics Paper II [1].

It is advised that all the tactics used to solve the problems are in B-side syllabus.

In Syllabus:

- All of DSE Mathematics Compulsory syllabus
- Intersecting chord/tangent/secant theorem
- Double angle formulas
- sin/cos values of special angles
- Intersection of two circles / Intersection of tangent and circles
- Shoelace formula
- Formulas for four centres of triangle
- AM-GM inequality
- Linear homogeneous recurrence relations
- Dihedral angle formula
- Principle of inclusion-exclusion

Out of Syllabus:

- Cubic formula
- Derivatives and Integrals

1 Problems

Section A

1. If $\dfrac{3a+4b}{6a+7b}=\dfrac{5a+2b}{4a+9b}$ and $a \neq b$, then $a=$

A. $\dfrac{4b-3}{b^2+2}$.

B. $\dfrac{7}{13}b$.

C. $-\dfrac{2}{11}b$.

D. $-\dfrac{11}{9}b$.

2. $\dfrac{2x}{6x-7}-\dfrac{2x+5}{7+6x}=$

A. $\dfrac{5-28x}{36x^2-49}$.

B. $\dfrac{5+28x}{36x^2-49}$.

C. $\dfrac{35+2x}{36x^2-49}$.

D. $\dfrac{35-2x}{36x^2-49}$.

3. $\dfrac{16^{2n+1}27^{n-5}}{4^{n+17}}=$

A. 12^{n-5} .

B. 12^{3n-15} .

C. 24^{n-5} .

D. 24^{3n-15} .

4. $4x^2-16x^4+9y^2-81y^4-12xy+72x^2y^2=$

A. $(2x-3y)^2(2x+3y+1)(1-2x-3y)$.

B. $(2x-3y)^2(2x-3y+1)(1+2x-3y)$.

C. $(2x+3y)^2(2x+3y+1)(1-2x-3y)$.

D. $(2x+3y)^2(2x-3y+1)(1+2x-3y)$.

5. If m, n and c are positive constants such that

$$(mx+5)(x-n)+2m-1 \equiv (n-4)(x+1)x-(2n-3)(x+c)$$

, then $c=$

A. 2 .

B. 3 .

C. 6 .

D. 9 .

6. The number of integers satisfying the inequality $3x - 8 < \dfrac{2x+7}{2} \leq 3(2x+3)$ is

A. 3 .

B. 4 .

C. 5 .

D. 6 .

7. If $0.01645 < x < 0.01654$, which of the following must be true?

A. $x = 0.017$ (correct to 2 significant figures)

B. $x = 0.0164$ (correct to 3 significant figures)

C. $x = 0.016$ (correct to 3 decimal places)

D. $x = 0.0165$ (correct to 4 decimal places)

8. If $f(x) = 3x^2 - 5x - 8$, then $f(3m+2) + f(3m-2) =$

A. $26m^2 - 15m + 8$

B. $26m^2 - 15m - 16$

C. $54m^2 - 30m + 8$

D. $54m^2 - 30m - 16$

9. Let $h(x) = 4kx^3 - 10x^2 + 8$, where k is a real constant. When $h(x)$ is divided by $2x - k$, the remainder is 26. Find the remainder when $h(x)$ is divided by $2x + k$.

A. -13

B. -55

C. 16

D. 54

10. Which of the following statements about the graph of $y = (5-x)(x+3) - 7$ is true?

A. The graph opens upwards.

B. The x-intercepts of the graph are -4 and 2 .

C. The vertex of the graph is $(1, 9)$.

D. The y-intercept of the graph is -7.

11. Marcy sells a vase and a bag for $\$S$ each. She gains $x\%$ on the vase and loses $x\%$ on the bag. After the two transactions, Marcy loses \$40 in total. If the profit of selling the vase is \$80, find S.

A. 480

B. 450

C. 320

D. 240

12. The actual area of a park is 0.5625 km^2 . If the area of the park on a map is 625cm^2 , then the scale of the map is

A. $1 : 90$

B. $1 : 3\,000$

C. $1 : 75\,000$

D. $1 : 9\,000\,000$

13. It is given that z partly varies directly as x^2 and partly varies inversely as the cube root of y . When $x = 6$ and $y = 27$, $z = 7$. When $x = 15$ and $y = 125$, $z = -41$. When $x = 21$ and $y = 729$, $z =$

A. -93

B. -45

C. 103

D. 125

14. Let a_n be the nth term of a sequence. If $a_3 = 7$, $a_9 = 1393$ and $a_{n+2} = 2a_{n+1} + a_n$ for any positive integer n , then $a_6 =$

A. 99

B. 143

C. 198

D. 237

15. A right pyramid has a height of h cm and a square base of side s cm. Its volume is 11200 cm^3 and its total surface area is 3920 cm^2 . If $s > h$, find s .

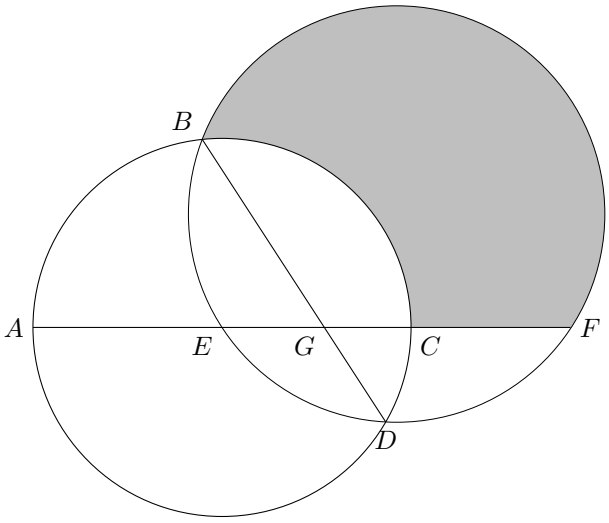
A. 15

B. 20

C. 40

D. 42

16. In the figure, E is the centre of the circle $ABCD$, and $BEDF$ is another circle. It is given that C and E lie on AF . Let G be the point of intersection of AF and BD . If $BG = 15$ cm , $DG = 8$ cm and $\angle BGE = 60^\circ$, find the area of the shaded region correct to the nearest cm^2.



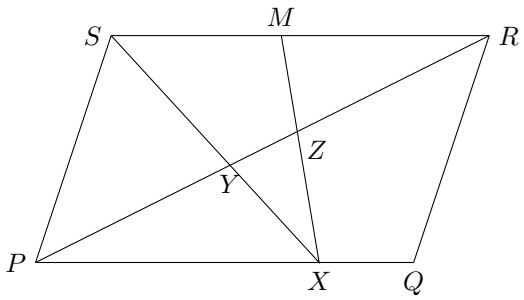
A. 320 cm^2

B. 341 cm^2

C. 353 cm^2

D. 399 cm^2

17. In the figure, $PQRS$ is a parallelogram. Let X be a point lying on PQ , and let M be the mid-point of SR . Let PR and SX intersect at Y , and PR and MX intersect at Z . If the area of quadrilateral $SYZM$ and the area of quadrilateral $QRZX$ are 648 cm^2 and 1040 cm^2 respectively, then the area of $\triangle SPY$ is



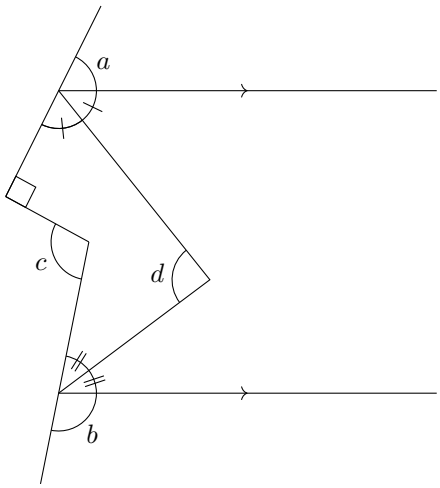
A. 672 cm^2

B. 720 cm^2

C. 848 cm^2

D. 936 cm^2

18. According to the figure, which of the following must be true?



I. $a + b + c = 270°$

II. $a + b + d = 180°$

III. $2c - d = 360°$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

19. It is given that $ABCD$ is a parallelogram. Denote the point of intersection of AC and BD by E. If $\angle ABE = \angle CBE$, then which of the following must be true?

I. $\angle BAE + \angle CDE = \angle BCE$

II. $AD^2 = 2AE^2$

III. $AC^2 + BD^2 = 2(AB^2 + BC^2)$

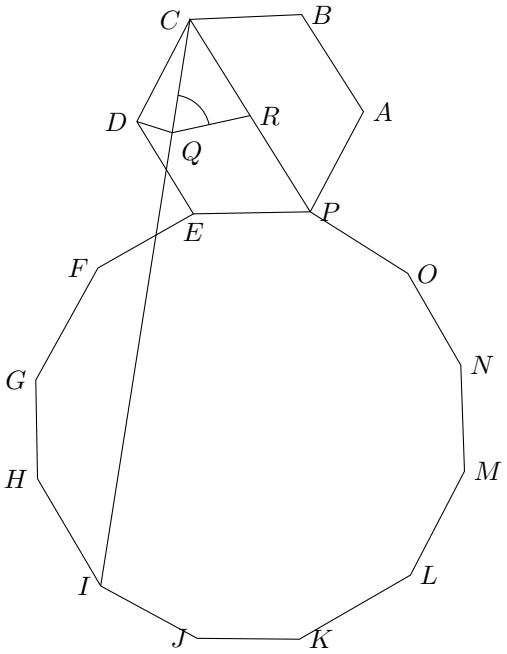
A. II only

B. III only

C. II and III only

D. I, II and III

20. The figure shows the regular hexagon $ABCDEP$ and the regular dodecagon $EFGHIJKLMNOP$. Q is a point on CI such that $DQ \perp CI$, and R is the mid-point of CP . Find $\angle CQR$.



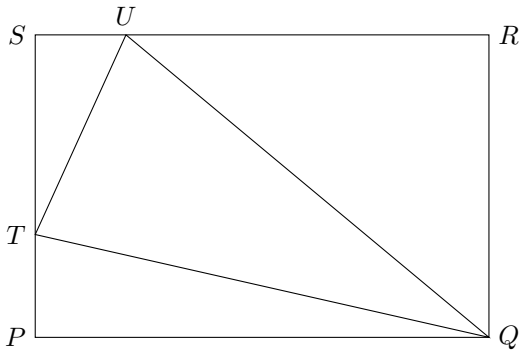
A. $60°$

B. $72°$

C. $75°$

D. $78°$

21. In the figure, $PQRS$ is a rectangle. Let U and T be points lying on SR and SP respectively such that $\angle UTQ = 90°$, $\angle TUS = \angle TUQ$ and $\angle TQP = \angle TQU$. Which of the following must be true?



I. $TU^2 = SU \cdot UQ$

II. $\triangle UST \sim \triangle QRU$

III. $ST = TP$

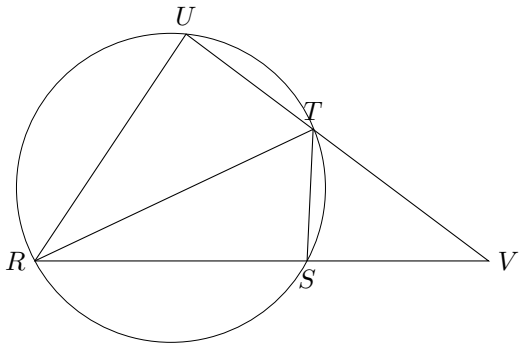
A. I only

B. II only

C. I and III only

D. I, II and III

22. In the figure, RT is the diameter of the circle $RSTU$, and $ST = TU$. RS produced and UT produced meet at point V . If $RT = 1547$ cm and $TV = 845$ cm, then $RV =$



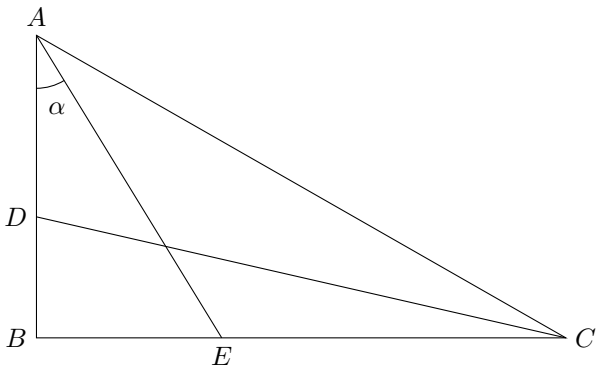
A. 2028 cm

B. 2096 cm

C. 2147 cm

D. 2192 cm

23. In the figure, $\triangle ABC$ is a right-angled triangle with $\angle ABC = 90°$. D and E are points lying on AB and BC respectively such that AE bisects $\angle BAC$ and CD bisects $\angle ACB$. Find $\dfrac{CE}{AD}$.



A. $\dfrac{1 + \tan\alpha}{1 - \tan\alpha}$

B. $\dfrac{\tan\alpha(1 + \tan^2\alpha)}{1 - \tan^2\alpha}$

C. $\dfrac{2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha}$

D. $\dfrac{\sin\alpha(\cos\alpha + \sin\alpha)}{\cos\alpha(\cos\alpha - \sin\alpha)}$

24. The rectangular coordinates of the point P are $(-1,\, 2+\sqrt{3})$. P is rotated clockwise about the origin through $45°$ and then reflected with respect to the x-axis. Find the y-coordinate of its image.

A. $\dfrac{-\sqrt{2}-\sqrt{10}}{2}$

B. $\dfrac{\sqrt{6}+2\sqrt{2}}{4}$

C. $\dfrac{1-\sqrt{5}}{4}$

D. $\dfrac{-\sqrt{6}-3\sqrt{2}}{2}$

25. If a and b are integer constants such that the straight lines $(a+7)x+5y+9a-21=0$ and $(b+3)x-6y+2b=0$ are perpendicular to each other, and the y-coordinate of their intersection is 9, then the x-coordinate of their intersection is

A. -4 .

B. 4 .

C. 6 .

D. 9 .

26. The equations of the straight lines l and L are $3x+4y-10=0$ and $7x+24y-35=0$ respectively, and they intersect at the point E. l cuts the y-axis at the point A while L cuts the x-axis at the point B. Let P be a moving point in the rectangular coordinate plane such that the perpendicular distance from P to l is equal to the perpendicular distance from P to L. Denote the locus of P by Γ. Which of the following are true?

I. The straight line $66x+44y-185=0$ lies on Γ.

II. $AE=BE$.

III. Γ passes through the mid-point of AB.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

27. The equations of the circles C_1 and C_2 are $x^2+y^2+8x+2y-128=0$ and $3x^2+3y^2+66x-12y-756=0$ respectively. Let G_1 and G_2 be the centres of C_1 and C_2 respectively. Let A and B be the intersections of C_1 and C_2 respectively. Which of the following must be true?

I. $AB=G_1G_2$.

II. The origin lies outside $\triangle G_1G_2A$.

III. The area of $\triangle G_1G_2B$ is 29.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

28. A box contains four cards numbered 1, 2, 3 and 4 respectively while another box contains five cards numbered 5, 6, 7, 8 and 9 respectively. If two numbers are drawn without replacement from each box, find the probability that the sum of the four numbers drawn is divisible by 4.

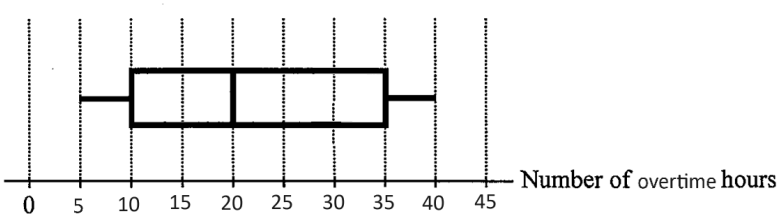
A. $\frac{7}{30}$

B. $\frac{1}{4}$

C. $\frac{4}{15}$

D. $\frac{17}{60}$

29. The box-and-whisker diagram below shows the distribution of the numbers of overtime hours of some engineers in a week. Find the interquartile range of the distribution.



A. 10

B. 20

C. 25

D. 35

30. In a company, the salary of a part-time employee is \$6075 while the salary of a full-time employee is \$8075. Originally, the mean salary of all employees is \$7500. After 8 part-time employees become full-time employees, the mean salary of all employees is increased by \$200. Find the original number of part-time employees.

A. 15

B. 23

C. 25

D. 33

Section B

31. $\text{B00B1E5FEE1900D}_{16} =$

A. $721013 \times 16^{10} + 942062 \times 16^5 + 102414$

B. $45067 \times 16^{11} + 124414 \times 16^5 + 102413$

C. $16^{14} + 2846 \times 16^8 + 392929 \times 16^4 + 36877$

D. $11 \times 16^{14} + 2846 \times 16^9 + 392929 \times 16^4 + 36877$

32. The L.C.M. of $a^4b^2 + ab^5$, $2a^4b + 4a^3b^2 + 2a^2b^3$ and $3a^3b - 3a^2b^2 + 3ab^3$ is

A. $ab(a+b)(a^2 - ab + b^2)$

B. $3ab^2(a-b)^2(a^2 + ab + b^2)$

C. $6a^2b^2(a+b)^2(a^2-ab+b^2)$

D. $6a^2b^2(a+b)(a^2-ab+b^2)$

33. It is given that $\log_8 y$ is a linear function of $\log_{32} x$. The intercepts on the vertical axis and on the horizontal axis of the graph of the linear function are 8 and 3 respectively. Which of the following must be true?

A. $x^8y^3 = 8^{24}$

B. $x^8y^3 = 16^{30}$

C. $x^3y^5 = 32^{15}$

D. $x^8y^5 = 64^{20}$

34. Define $z_1 = \dfrac{3}{1+ki}$ and $z_2 = \dfrac{5}{i-2m}$, where k and m are real numbers. Find the maximum value of the imaginary part of z_1z_2 .

A. $\dfrac{15}{8}$

B. $\dfrac{45}{8}$

C. $\dfrac{15}{4}$

D. $\dfrac{45}{4}$

35. Let $f(x) = 5x^2 - 30mx + 12m^2$, where m is a real constant. Which of the following statements about the graph of $y = -\dfrac{1}{3}f(2x+7m)$ must be true?

I. The y-coordinate of the vertex of the graph is $11m^2$.

II. The equation of the axis of symmetry of the graph is $x - 5m = 0$.

III. The area of the triangle formed by the x-intercepts and the vertex of the graph is less than $15m^3$.

A. I only

B. II only

C. I and III only

D. II and III only

36. Let $S(n)$ be the sum of the first nth term of an arithmetic sequence. If $S(16) = 1112$ and $S(34) - S(19) = 4095$, find the least value of k such that $S(k) > 6 \times 10^7$.

A. 2686

B. 2687

C. 3304

D. 3305

37. Consider the following system of inequalities:

$$\begin{cases} 3x - 7y + 13 \geq 0 \\ 6x + 5y + 7 \geq 0 \\ 2x - 5y - 31 \leq 0 \\ 7x + 3y - 47 \leq 0 \end{cases}$$

Let D be the region which represents the solution of the above system of inequalities. Find the constant k such the minimum value of $8x + 6y + k$ is 13 , where (x, y) is a point lying in D.

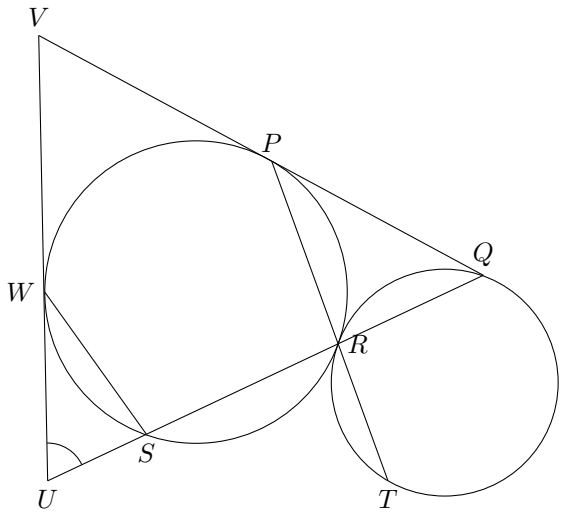
A. -10

B. 6

C. 20

D. 23

38. In the figure, PRS and QTR are circles that are externally tangent to each other at R. It is given that PRT and QRS are straight lines, and PQ is the common tangent to the two circles. QS is produced to U and QP is produced to V such that UV is tangent to circle PRS at W. If $UV = UQ$ and $\angle WSR = 96°$, then $\angle WUS =$



A. $54°$

B. $56°$

C. $62°$

D. $68°$

39. Find the range of values of c such that the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ and the straight line $cx - 4y - 21c + 20 = 0$ intersect.

A. $c \leq \dfrac{3}{5}$ or $c \geq \dfrac{20}{21}$

B. $c \leq \dfrac{13}{21}$ or $c \geq 3$

C. $\dfrac{3}{5} \leq c \leq \dfrac{20}{21}$

D. $\dfrac{13}{21} \leq c \leq 3$

40. In tetrahedron $ABCD$, $BA : BC : BD = 13 : 7 : 15$ and $AC : AD : CD = 9 : 7 : 10$. Let θ be the angle between $\triangle ABD$ and $\triangle CBD$. If $AD = 2BC$, then $\cos\theta =$

A. $-\dfrac{39}{70}$.

B. $-\dfrac{79}{196}$.

C. $-\dfrac{53}{126}$.

D. $-\dfrac{39}{40}$.

41. The equations of three sides of a triangle are $20x - 21y + 33a = 0$, $40x + 9y + 151a = 0$ and $y = a$, where a is a constant. If the x-coordinates of the in-centre of the triangle is 43, then $a =$

A. -51 .

B. -15 .

C. 8 .

D. 34 .

42. There are five couples in a shop. If all of these 10 people form a queue such that none of them stand next to their partners, how many different queues can be formed?

A. 1 263 360

B. 1 244 600

C. 2 365 440

D. 3 624 960

43. Josh is playing a game. In each turn, he throws 3 six-sided fair dice at the same time, and he wins the turn only when the sum obtained is 12. If Josh plays 30 turns, find the probability that he wins more than 4 turns in total, correct to 4 decimal places.

A. 0.1615

B. 0.2628

C. 0.3188

D. 0.5757

44. Peter finds that he got the same marks in two tests. In the first test, the mean and the standard deviation of the marks are 87 and 6 respectively. In the second test, the mean and the standard deviation of the marks are 80 and 4 respectively. If Peter's standard score in the second test is 2 more than his standard score in the first test, then his marks in both tests is

A. 84 .

B. 88 .

C. 90 .

D. 94 .

45. It is given that n is an integer. Let u_1, v_1 and w_1 be the standard deviation, the mean and the range of the group of the numbers $\{15 - 7n,\ 53 - 7n,\ 56 - 7n,\ 59 - 7n,\ 97 - 7n\}$ respectively while u_2, v_2 and w_2 be the standard deviation, the mean and the range of the group of the numbers $\{15 - 8n,\ 53 - 2n,\ 56 - 14n,\ 59 - 7n,\ 97 - 9n\}$ respectively. Which of the following must be true?

I. $v_1 > v_2$

II. $w_1 < w_2$

III. $u_1 < u_2$

A. I only

B. II only

C. I and III only

D. II and III only

2 Solutions

.

1. D
2. D
3. B
4. A
5. B
6. A (from D)
7. D
8. C
9. B
10. C
11. A
12. B (from C)
13. A
14. A
15. C (from A)
16. B (from D)
17. A
18. B (from B)
19. C (from C)
20. C (from A)
21. C (from C)
22. A
23. D (from D)
24. D
25. B
26. C (from B)
27. A (from C)
28. C
29. C (from C)
30. B
31. D (from B)
32. C
33. D (from A)
34. A
35. C
36. D (from B)
37. D (from D)
38. D
39. D
40. B
41. B (from C)
42. A
43. B (from C)
44. C
45. A (from A)

References

[1] HKEAA, "2023 dse mathematics compuslory part paper 2." [Online]. Available: https://dse.life/static/pp/m0/eng/dse/2023/p2.pdf