Crappy DSE Maths Paper II (2023 B-side)

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January 12, 2024

Abstract

This is a paper created by a no-lifer. Its sole purpose is to help the reader escape from the responsibilities of real life.

Contents

1 Problems

Section A

1. If $\dfrac{3a+4b}{6a+7b}=\dfrac{5a+2b}{4a+9b}$ and $a \neq b$, then $a=$

A. $\dfrac{4b-3}{b^2+2}$.

B. $\dfrac{7}{13}b$.

C. $-\dfrac{2}{11}b$.

D. $-\dfrac{11}{9}b$.

2. $\dfrac{2x}{6x-7}-\dfrac{2x+5}{7+6x}=$

A. $\dfrac{5-28x}{36x^2-49}$.

B. $\dfrac{5+28x}{36x^2-49}$.

C. $\dfrac{35+2x}{36x^2-49}$.

D. $\dfrac{35-2x}{36x^2-49}$.

3. $\dfrac{16^{2n+1}27^{n-5}}{4^{n+17}}=$

A. 12^{n-5} .

B. 12^{3n-15} .

C. 24^{n-5} .

D. 24^{3n-15} .

4. $4x^2-16x^4+9y^2-81y^4-12xy+72x^2y^2=$

A. $(2x-3y)^2(2x+3y+1)(1-2x-3y)$.

B. $(2x-3y)^2(2x-3y+1)(1+2x-3y)$.

C. $(2x+3y)^2(2x+3y+1)(1-2x-3y)$.

D. $(2x+3y)^2(2x-3y+1)(1+2x-3y)$.

5. If m, n and c are positive constants such that

$$(mx+5)(x-n)+2m-1 \equiv (n-4)(x+1)x-(2n-3)(x+c)$$

, then $c=$

A. 2 .

B. 3 .

C. 6 .

D. 9 .

6. The number of integers satisfying the inequality $3x - 8 < \dfrac{2x+7}{2} \leq 3(2x+3)$ is

A. 2 .

B. 3 .

C. 4 .

D. 5 .

7. If $0.01645 < x < 0.01654$, which of the following must be true?

A. $x = 0.017$ (correct to 2 significant figures)

B. $x = 0.0164$ (correct to 3 significant figures)

C. $x = 0.016$ (correct to 3 decimal places)

D. $x = 0.0165$ (correct to 4 decimal places)

8. If $f(x) = 3x^2 - 5x - 8$, then $f(3m+2) + f(3m-2) =$

A. $26m^2 - 15m + 8$

B. $26m^2 - 15m - 16$

C. $54m^2 - 30m + 8$

D. $54m^2 - 30m - 16$

9. Let $h(x) = 4kx^3 - 10x^2 + 8$, where k is a real constant. When $h(x)$ is divided by $2x - k$, the remainder is 26. Find the remainder when $h(x)$ is divided by $2x + k$.

A. -13

B. -55

C. 16

D. 54

10. Which of the following statements about the graph of $y = (5-x)(x+3) - 7$ is true?

A. The graph opens upwards.

B. The x-intercepts of the graph are -4 and 2 .

C. The vertex of the graph is $(1, 9)$.

D. The y-intercept of the graph is -7.

11. Marcy sells a vase and a bag for $\$S$ each. She gains $x\%$ on the vase and loses $x\%$ on the bag. After the two transactions, Marcy loses \$40 in total. If the profit of selling the vase is \$80, find S.

A. 480

B. 450

C. 320

D. 240

12. The actual area of a park is 0.5625 km^2 . If the area of the park on a map is 625cm^2 , then the scale of the map is

A. $1:90$

B. $1:3\,000$

C. $1:75\,000$

D. $1:9\,000\,000$

13. It is given that z partly varies directly as x^2 and partly varies inversely as the cube root of y . When $x = 6$ and $y = 27$, $z = 7$. When $x = 15$ and $y = 125$, $z = -41$. When $x = 21$ and $y = 729$, $z =$

A. -93

B. -45

C. 103

D. 125

14. Let a_n be the nth term of a sequence. If $a_3 = 7$, $a_9 = 1393$ and $a_{n+2} = 2a_{n+1} + a_n$ for any positive integer n , then $a_6 =$

A. 99

B. 143

C. 198

D. 237

15. A right pyramid has a height of h cm and a square base of side s cm. Its volume is 11200 cm^3 and its total surface area is 3920 cm^2 . If $s > h$, find s .

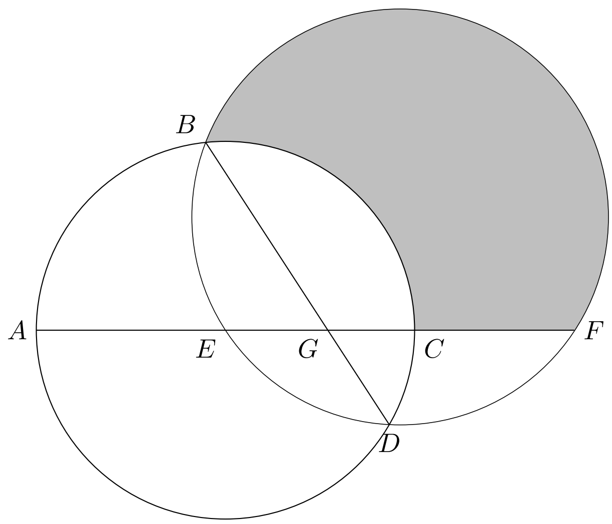
A. 15

B. 20

C. 40

D. 42

16. In the figure, E is the centre of the circle $ABCD$, and $BEDF$ is another circle. It is given that C and E lie on AF . Let G be the point of intersection of AF and BD . If $BG = 15$ cm , $DG = 8$ cm and $\angle BGE = 60°$, find the area of the shaded region correct to the nearest cm^2.



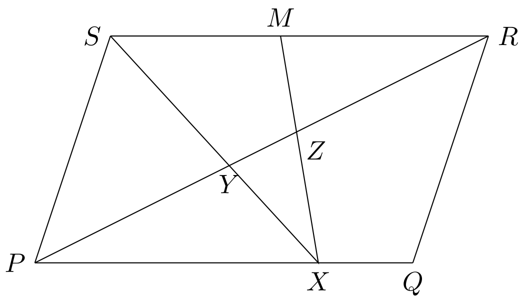
A. 320 cm^2

B. 341 cm^2

C. 353 cm^2

D. 399 cm^2

17. In the figure, $PQRS$ is a parallelogram. Let X be a point lying on PQ , and let M be the mid-point of SR . Let PR and SX intersect at Y , and PR and MX intersect at Z . If the area of quadrilateral $SYZM$ and the area of quadrilateral $QRZX$ are 648 cm^2 and 1040 cm^2 respectively, then the area of $\triangle SPY$ is



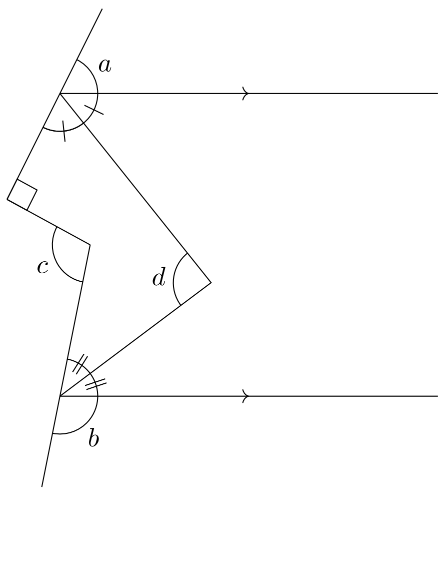
A. 672 cm^2

B. 720 cm^2

C. 848 cm^2

D. 936 cm^2

18. According to the figure, which of the following must be true?



I. $a + b + c = 270°$

II. $a + b + d = 180°$

III. $2c - d = 360°$

A. I only

B. I and II only

C. I and III only

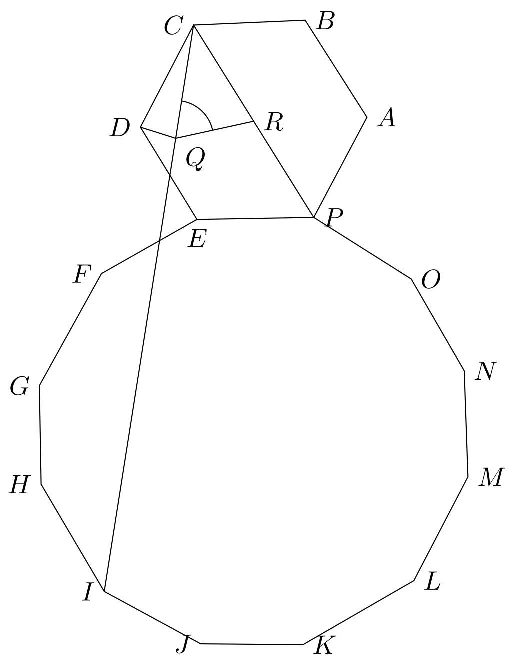
D. I, II and III

19. It is given that $ABCD$ is a parallelogram. Denote the point of intersection of AC and BD by E. If $\angle ABE = \angle CBE$, then which of the following must be true?

I. $\angle BAE + \angle CDE = \angle BCE$
II. $AD^2 = 2AE^2$
III. $AC^2 + BD^2 = 2(AB^2 + BC^2)$

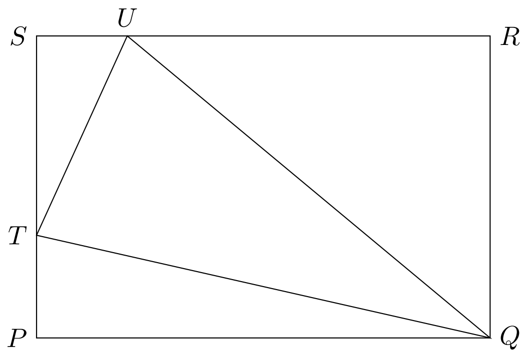
A. II only
B. III only
C. II and III only
D. I, II and III

20. The figure shows the regular hexagon $ABCDEP$ and the regular dodecagon $EFGHIJKLMNOP$. Q is a point on CI such that $DQ \perp CI$, and R is the mid-point of CP . Find $\angle CQR$.



A. 60°
B. 72°
C. 75°
D. 78°

21. In the figure, $PQRS$ is a rectangle. Let U and T be points lying on SR and SP respectively such that $\angle UTQ = 90°$, $\angle TUS = \angle TUQ$ and $\angle TQP = \angle TQU$. Which of the following must be true?



I. $TU^2 = SU \cdot UQ$

II. $\triangle UST \sim \triangle QRU$

III. $ST = TP$

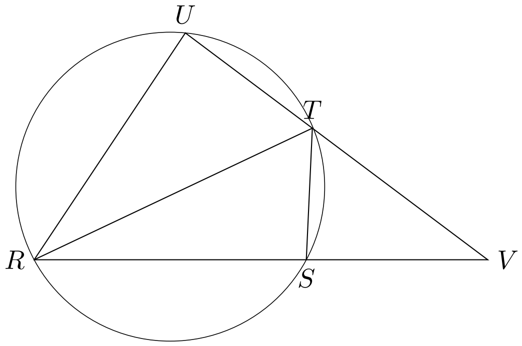
A. I only

B. II only

C. I and III only

D. I, II and III

22. In the figure, RT is the diameter of the circle $RSTU$, and $ST = TU$. RS produced and UT produced meet at point V . If $RT = 1547$ cm and $TV = 845$ cm, then $RV =$



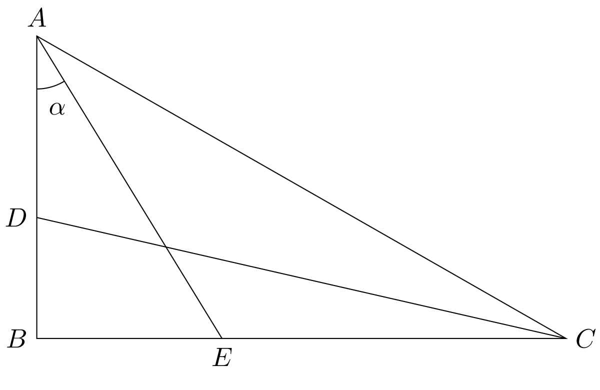
A. 2023 cm

B. 2028 cm

C. 2147 cm

D. 2192 cm

23. In the figure, $\triangle ABC$ is a right-angled triangle with $\angle ABC = 90°$. D and E are points lying on AB and BC respectively such that AE bisects $\angle BAC$ and CD bisects $\angle ACB$. Find $\dfrac{CE}{AD}$.



A. $\dfrac{1+\tan\alpha}{1-\tan\alpha}$

B. $\dfrac{\tan\alpha(1+\tan^2\alpha)}{1-\tan^2\alpha}$

C. $\dfrac{2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha}$

D. $\dfrac{\sin\alpha(\cos\alpha+\sin\alpha)}{\cos\alpha(\cos\alpha-\sin\alpha)}$

24. The rectangular coordinates of the point P are $(-1,\,2+\sqrt{3})$. P is rotated clockwise about the origin through $45°$ and then reflected with respect to the x-axis. Find the y-coordinate of its image.

A. $\dfrac{-\sqrt{2}-\sqrt{10}}{2}$

B. $\dfrac{\sqrt{6}+2\sqrt{2}}{4}$

C. $\dfrac{1-\sqrt{5}}{4}$

D. $\dfrac{-\sqrt{6}-3\sqrt{2}}{2}$

25. If a and b are integer constants such that the straight lines $(a+7)x+5y+9a-21=0$ and $(b+3)x-6y+2b=0$ are perpendicular to each other, and the y-coordinate of their intersection is 9, then the x-coordinate of their intersection is

A. -4 .

B. 4 .

C. 6 .

D. 9 .

26. The equations of the straight lines l and L are $3x+4y-10=0$ and $7x+24y-35=0$ respectively, and they intersect at the point E. l cuts the y-axis at the point A while L cuts the x-axis at the point B. Let P be a moving point in the rectangular coordinate plane such that the perpendicular distance from P to l is equal to the perpendicular distance from P to L. Denote the locus of P by Γ. Which of the following are true?

I. The straight line $66x+44y-185=0$ lies on Γ.

II. $AE=BE$.

III. Γ passes through the mid-point of AB.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

27. The equations of the circles C_1 and C_2 are $x^2+y^2+8x+2y-128=0$ and $3x^2+3y^2+66x-12y-756=0$ respectively. Let G_1 and G_2 be the centres of C_1 and C_2 respectively. Let A and B be the intersections of C_1 and C_2 respectively. Which of the following must be true?

I. $AB=C_1C_2$.

II. The origin lies outside $\triangle C_1C_2A$.

III. The area of $\triangle C_1C_2A$ is 29.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

28. A box contains four cards numbered 1, 2, 3 and 4 respectively while another box contains five cards numbered 5, 6, 7, 8 and 9 respectively. If two numbers are drawn without replacement from each box, find the probability that the sum of the four numbers drawn is divisible by 4.

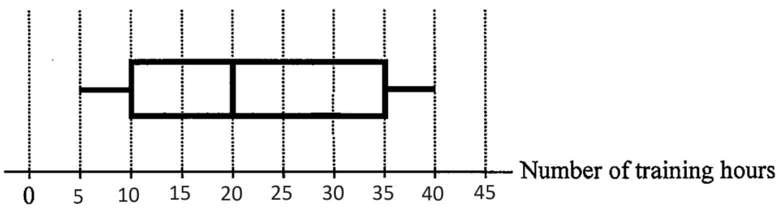
A. $\frac{7}{30}$

B. $\frac{1}{4}$

C. $\frac{4}{15}$

D. $\frac{17}{60}$

29. The box-and-whisker diagram below shows the distribution of the numbers of overtime hours of some engineers in a week. Find the interquartile range of the distribution.



A. 10

B. 20

C. 25

D. 35

30. In a company, the salary of a part-time employee is \$6075 while the salary of a full-time employee is \$8075. Originally, the mean salary of all employees is \$7500. After 8 part-time employees become full-time employees, the mean salary of all employees is increased by \$200. Find the original number of part-time employees.

A. 15

B. 23

C. 25

D. 33

Section B

31. $101110101011011_2 =$

A. 15

B. 23

C. 25

D. 33

2 Solutions

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1. D
2. D
3. B
4. A
5. B
6. B (from D)
7. D
8. C
9. B
10. C
11. A
12. B (from C)
13. A
14. A
15. C (from A)
16. B (from D)
17. A
18. C (from B)
19. C (from C)
20. C (from A)
21. C (from C)
22. B (from B)
23. D (from D)
24. D
25. B
26. C (from B)
27. A (from C)
28. C
29. C (from C)
30. B

References