



KMP



PREFIX SUFFIX

- **PREFIX**

- Pattern = ABCABE

- A
- AB
- ABC
- ABCA
- ABCAB
- ABCABE

- **SUFFIX**

- Pattern = ABCABE

- E
- BE
- ABE
- CABE
- BCABE
- ABCABE



PI[] OF "ABCABE"

I	SUBSTRING	PI[I]
0	A	0
1	AB	0
2	ABC	0
3	ABCA	1
4	ABCAB	2
5	ABCABE	0



PI[] OF "ABCABDABCABEABC"

I	SUBSTRING	PI[I]
0	A	0
1	AB	0
2	ABC	0
3	ABCA	1
4	ABCAB	2
5	ABCABD	0
6	ABCABDA	1
7	ABCABDAB	2
8	ABCABDABC	3
9	ABCABDABCA	4
10	ABCABDABCAB	5
11	ABCABDABCABE	0
12	ABCABDABCABEA	1
13	ABCABDABCABEAB	2
14	ABCABDABCABEABC	3



abcdabcef

i	0	1	2	3	4	5	6	7	8
Pattern [i]	a	b	c	d	a	b	c	e	f
NEXT[i]	-1	0	0						

a **b**

a **b**



abcdabcef

i	0	1	2	3	4	5	6	7	8
Pattern [i]	a	b	c	d	a	b	c	e	f
NEXT[i]	-1	0	0	0					

a **b** **c**

a **b** **c**

a **b** **c**



abcdabcef

i	0	1	2	3	4	5	6	7	8
Pattern [i]	a	b	c	d	a	b	c	e	f
NEXT[i]	-1	0	0	0	0				

a b c d

a b c d

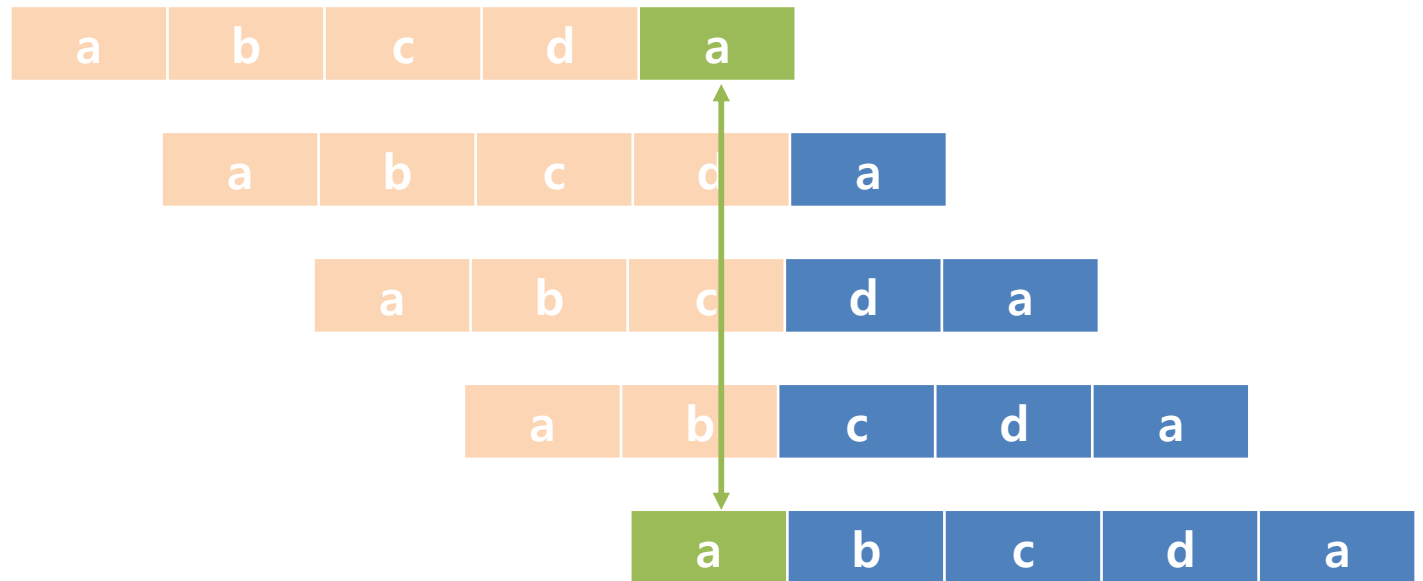
a b c d

a b c d



abcdabcef

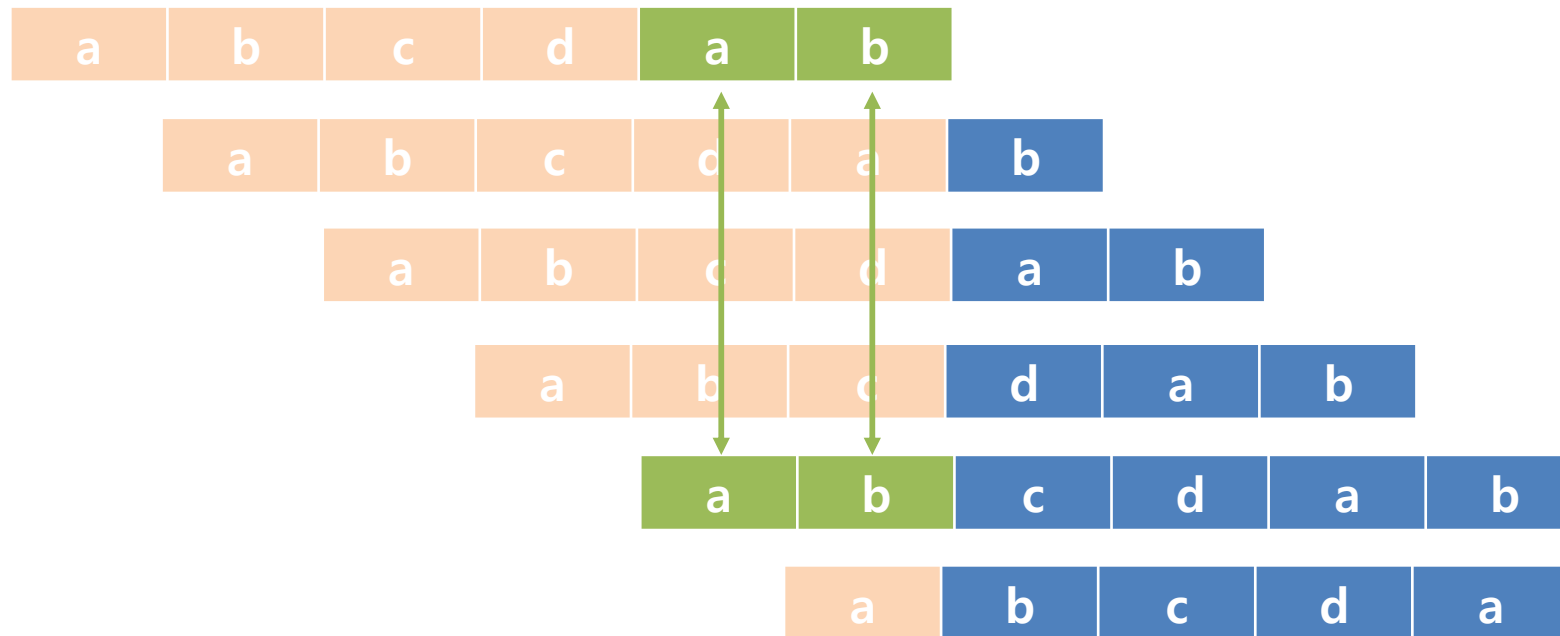
i	0	1	2	3	4	5	6	7	8
Pattern [i]	a	b	c	d	a	b	c	e	f
NEXT[i]	-1	0	0	0	0	1			





abcdabcef

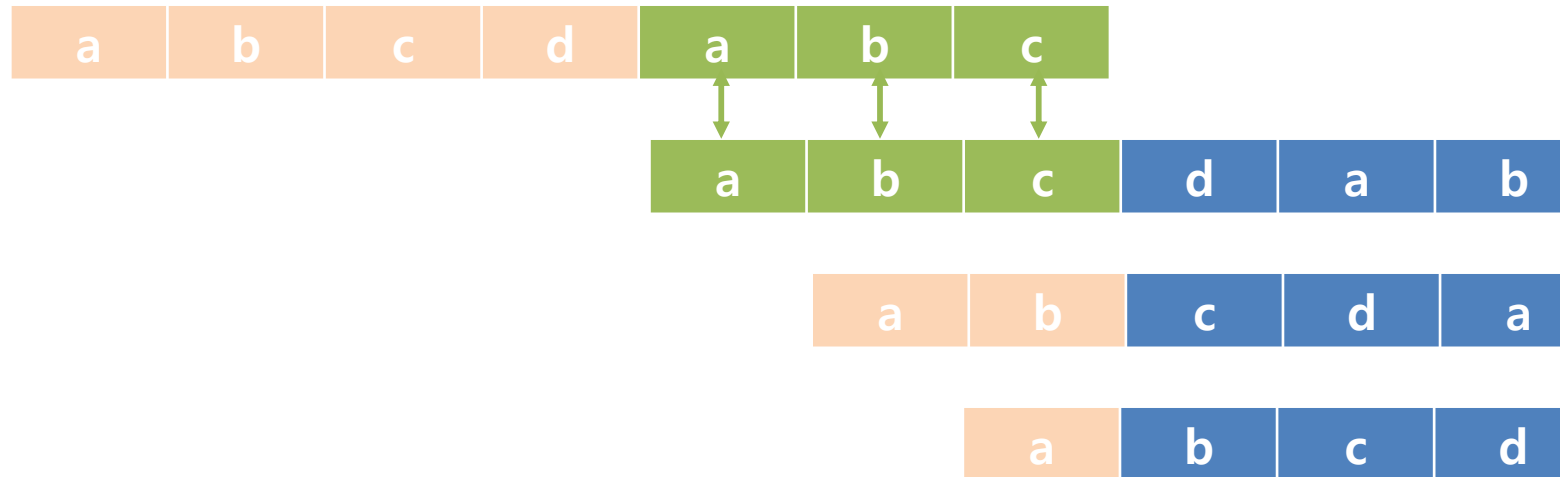
i	0	1	2	3	4	5	6	7	8
Pattern [i]	a	b	c	d	a	b	c	e	f
NEXT[i]	-1	0	0	0	0	1	2		





abcdabcef

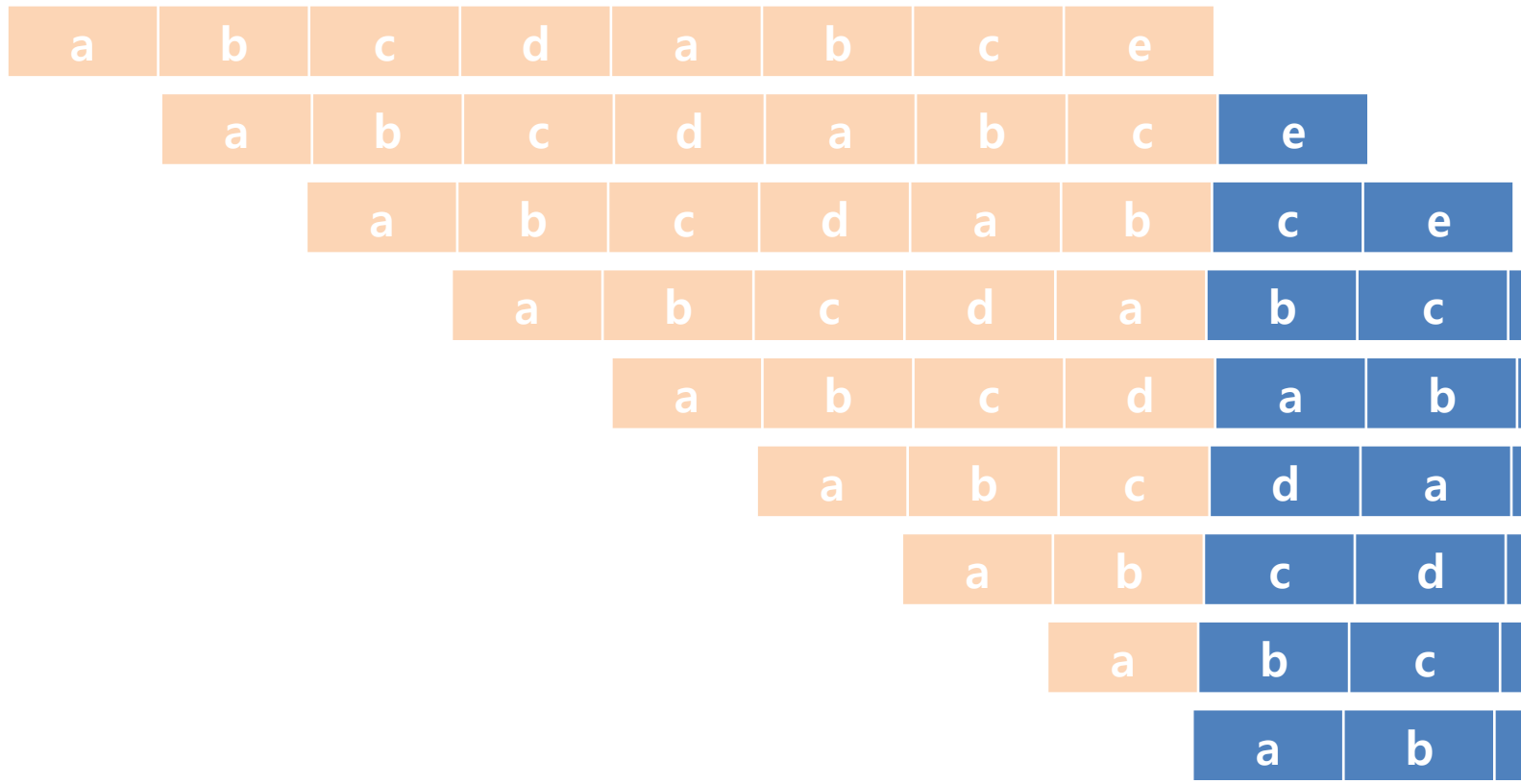
i	0	1	2	3	4	5	6	7	8
Pattern [i]	a	b	c	d	a	b	c	e	f
NEXT[i]	-1	0	0	0	0	1	2	3	





abcdabcef

i	0	1	2	3	4	5	6	7	8
Pattern [i]	a	b	c	d	a	b	c	e	f
NEXT[i]	-1	0	0	0	0	1	2	3	0





KMP Algorithm π Table Example

ab

	i	j							
i	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0						

$\pi[0] = -1;$

$\pi[1] = 0;$

$i = 0, j = 1$

If(Pattern[i] != Pattern[j])

//.....

$\pi[j+1] = \dots\dots\dots;$

$j = j + 1$



KMP Algorithm π Table Example

abb

i

j

<i>i</i>	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0	0					



KMP Algorithm π Table Example

abb**a**

	i			j					
<i>i</i>	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0	0	1				

If(Pattern[i]==Pattern[j])

$\pi[j+1] = \pi[j] + 1;$

$i++; j++;$



KMP Algorithm π Table Example

abbab

		i			j				
<i>i</i>	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0	0	1	2			

If(Pattern[i]==Pattern[j])

$\pi[j+1] = \pi[j] + 1;$

$i++; j++;$



KMP Algorithm π Table Example

abbabd

i



j

i	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0	0	1	2			



KMP Algorithm π Table Example

abbabda

i

j

<i>i</i>	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0	0	1	2	0	1	



KMP Algorithm π Table Example

abbabdaa

i



j

<i>i</i>	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0	0	1	2	0	1	



KMP Algorithm π Table Example



abbabdaa

i

j

i	0	1	2	3	4	5	6	7	8
Pattern	a	b	b	a	b	d	a	a	b
πi	-1	0	0	0	1	2	0	1	1



ABACABABAC

i	0	1	2	3	4	5	6	7	8	9
Pattern [i]	A	B	A	C	A	B	A	B	A	C
PI[i]	-1	0	1	0	1	2	3	2	3	4

A B A C A B A B A C



KMP Algorithm π Table Example

i	0	1	2	3	4	5	6	7	8	9	10
P_i	a	b	a	a	b	a	b	a	a	b	c
π_i	-1	0	0	1	1	2	3	0	1	1	2

i	0	1	2	3	4	5	6	7	8	9
P_i	a	b	a	b	a	b	a	b	c	a
π_i	-1	0	0	1	2	3	4	5	6	0



KMP Algorithm

T ABC ABCDAB ABCDABCDABDE
 P ABCDABD

i	0	1	2	3	4	5	6
P_i	A	B	C	D	A	B	D
π_i	-1	0	0	0	0	1	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
A	B	C	D																			

Number of characters matched so far $K = 3$
 p movement $k - \pi[K] = 3 - \pi[3] = 3 - 0 = 3$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
			A																			

Number of characters matched so far $K = 0$

p movement $k - \pi [K] = 0 - \pi[0] = 0 - (-1) = 1$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
				A	B	C	D	A	B	D												

Number of characters matched so far $K = 6$

$p \text{ movement } k - \pi [K] = 6 - \pi [6] = 6 - 2 = 4$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
								A	B	C												

Number of characters matched so far $K = 2$
 p movement $k - \pi [K] = 2 - \pi [2] = 2 - 0 = 2$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
										A												

Number of characters matched so far $K = 0$

p movement $k - \pi [K] = 0 - \pi[0] = 0 - (-1) = 1$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
											A	B	C	D	A	B	D					

Number of characters matched so far $K = 6$

p movement $k - \pi [K] = 6 - \pi [6] = 6 - 2 = 4$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
															A	B	C	D	A	B	D	

Number of characters matched so far $K = 7$
 p movement $k - \pi [K] =$



KMP algorithm

- $Pi[i] \quad O(M)$
- Pattern Matching $O(N+M)$



KMP Algorithm π Table Example

i	0	1	2	3	4	5	6	7	8
P_i	a	b	b	a	b	d	a	a	b
π_i									

i	0	1	2	3	4	5	6	7	8	9	10
P_i	a	b	a	a	b	a	b	a	a	b	c
π_i											

i	0	1	2	3	4	5	6	7	8	9
P_i	a	b	a	b	a	b	a	b	c	a
π_i										



KMP Algorithm

T ABC ABCDAB ABCDABCDABDE
 P ABCDABD

i	0	1	2	3	4	5	6
P_i	A	B	C	D	A	B	D
π_i	-1	0	0	0	0	1	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
A	B	C	D																			

Number of characters matched so far $K = 3$
 p movement $k - \pi [K] = 3 - \pi[3] = 3 - 0 = 3$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
			A																			

Number of characters matched so far $K = 0$

$$p \text{ movement } k - \pi [K] = 0 - \pi[0] = 0 - (-1) = 1$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
				A	B	C	D	A	B	D												

Number of characters matched so far $K = 6$

$$p \text{ movement } k - \pi [K] = 6 - \pi [6] = 6 - 2 = 4$$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
								A	B	C												

Number of characters matched so far $K = 2$
 p movement $k - \pi [K] = 2 - \pi [2] = 2 - 0 = 2$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
										A												

Number of characters matched so far $K = 0$
 p movement $k - \pi [K] = 0 - \pi [0] = 0 - (-1) = 1$



KMP Algorithm

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
											A	B	C	D	A	B	D					

Number of characters matched so far $K = 6$

p movement $k - \pi [K] = 6 - \pi [6] = 6 - 2 = 4$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A	B	C		A	B	C	D	A	B		A	B	C	D	A	B	C	D	A	B	D	E
															A	B	C	D	A	B	D	

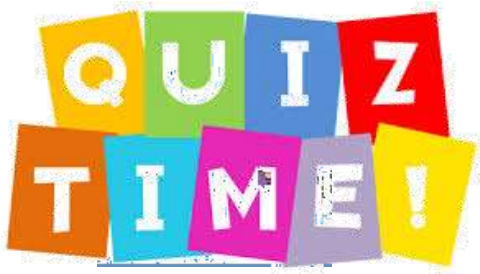
Number of characters matched so far $K = 7$

p movement $k - \pi [K] =$



KMP algorithm

- $Pi[i] \quad O(M)$
- Pattern Matching $O(N+M)$



Exercise 1

- KMP ALGORITHM
- Output
 - In the first line, print how many times the Pattern appears in the entire Text. In the second line, the location of the pattern is displayed one after the other

Input		Output
Text	: ABC ABCDAB ABCDABCDABDE	1
Pattern	: ABCDABD	16