Tensors

We now have a fully developed autodifferentiation system built around scalars. This system is correct, but you saw during training that it is inefficient. Every scalar number requires building an object, and each operation requires storing a graph of all the values that we have previously created. Training requires repeating the above operations, and running models, such as a linear model, requires a `for` loop over each of the terms in the network.

This module introduces and implements a *tensor* object that will solve these problems. Tensors group together many repeated operations to save Python overhead and to pass off grouped operations to faster implementations.

Guides

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For this module we have implemented the skeleton `tensor.py` file for you. This is similar in spirit to `scalar.py` from the last assignment. Before starting, it is worth reading through this file to have a sense of what a Tensor does. Each of the following tasks asks you to implement the methods this file relies on:

- `tensor_data.py` : Indexing, strides, and storage
- `tensor_ops.py` : Higher-order tensor operations
- `tensor_functions.py` : Autodifferentiation-ready functions

Tasks 2.1: Tensor Data - Indexing

The MiniTorch library implements the core tensor backend as `minitorch.TensorData`. This class handles indexing, storage, transposition, and low-level details such as strides. You will first implement these core functions before turning to the user-facing class `minitorch.Tensor`.

> **Todo**
>
> Complete the following functions in `minitorch/tensor_data.py`, and pass tests marked as `task2_1`.

`minitorch.index_to_position(index: Index, strides: Strides) -> int`

Converts a multidimensional tensor `index` into a single-dimensional position in storage based on strides.

Parameters:

- **index** – index tuple of ints
- **strides** – tensor strides

Returns:

- `int` – Position in storage

`minitorch.to_index(ordinal: int, shape: Shape, out_index: OutIndex) -> None`

Convert an `ordinal` to an index in the `shape`. Should ensure that enumerating position 0 ... size of a tensor produces every index exactly once. It may not be the inverse of `index_to_position`.

Parameters:

- **ordinal** (`int`) – ordinal position to convert.
- **shape** – tensor shape.
- **out_index** – return index corresponding to position.

`minitorch.tensor_data.TensorData.permute(*order: int) -> TensorData`

Permute the dimensions of the tensor.

Parameters:

- ***order** (`int`) – a permutation of the dimensions

Returns:

- `TensorData` – New `TensorData` with the same storage and a new dimension order.

Tasks 2.2: Tensor Broadcasting

> **Todo**
>
> Complete following functions in `minitorch/tensor_data.py` and pass tests marked as `task2_2`.

`minitorch.shape_broadcast(shape1: UserShape, shape2: UserShape) -> UserShape`

Broadcast two shapes to create a new union shape.

Parameters:

- **shape1** – first shape
- **shape2** – second shape

Returns:

- `UserShape` – broadcasted shape

Raises:

- `IndexingError` – if cannot broadcast

`minitorch.broadcast_index(big_index: Index, big_shape: Shape, shape: Shape, out_index: OutIndex) -> None`

Convert a `big_index` into `big_shape` to a smaller `out_index` into `shape` following broadcasting rules. In this case it may be larger or with more dimensions than the `shape` given. Additional dimensions may need to be mapped to 0 or removed.

Parameters:

- **big_index** – multidimensional index of bigger tensor
- **big_shape** – tensor shape of bigger tensor
- **shape** – tensor shape of smaller tensor
- **out_index** – multidimensional index of smaller tensor

Returns:

- `None` – None

Tasks 2.3: Tensor Operations

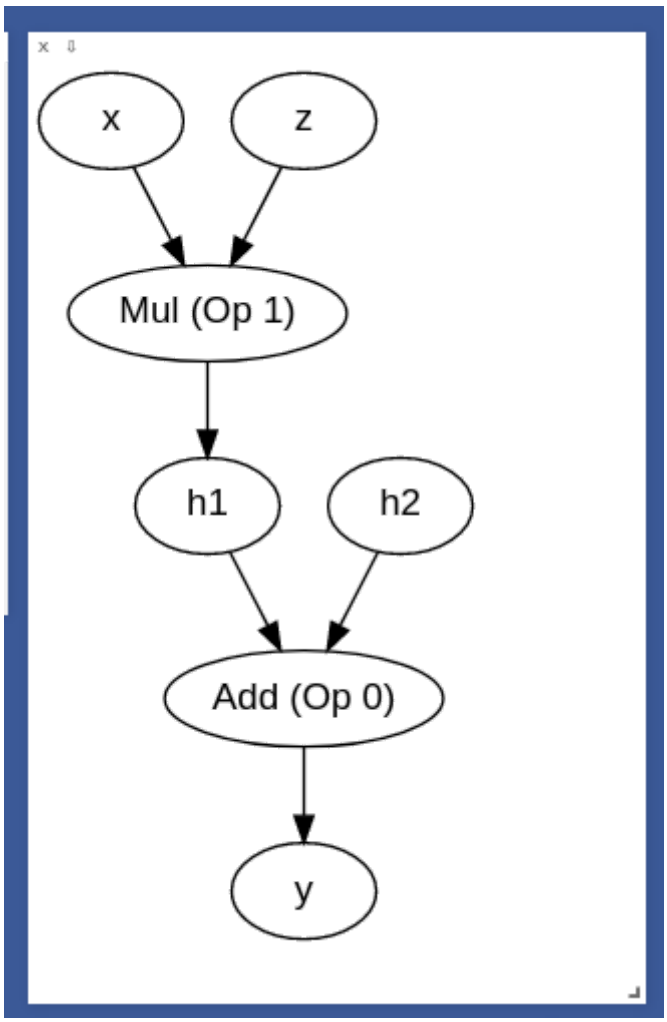
Tensor operations apply high-level, higher-order operations to all elements in a tensor simultaneously. In particular, you can map, zip, and reduce tensor data objects together. On top of this foundation, we can build up a `Function` class for Tensor, similar to what we did for the

ScalarFunction. In this task, you will first implement generic tensor operations and then use them to implement `forward` for specific operations.

We have built a debugging tool for you to observe the workings of your expressions to see how the graph is built. You can alter the expression at in `Streamlit` to view the graph

```
y = x * z + 10.0
```

```
>>> python project/show_expression.py
```



Todo

Add functions in `minitorch/tensor_ops.py` and `minitorch/tensor_functions.py` for each of the following, and pass tests marked as `task2_3`.

```
minitorch.tensor_ops.tensor_map(fn: Callable[[float], float]) -> Callable[[Storage, Shape, Strides, Storage, Shape, Strides], None]
```

Low-level implementation of tensor map between tensors with *possibly different strides*.

Simple version:

- Fill in the `out` array by applying `fn` to each value of `in_storage` assuming `out_shape` and `in_shape` are the same size.

Broadcasted version:

- Fill in the `out` array by applying `fn` to each value of `in_storage` assuming `out_shape` and `in_shape` broadcast. (`in_shape` must be smaller than `out_shape`).

Parameters:

- **fn** (`Callable[[float], float]`) – function from float-to-float to apply

Returns:

- `Callable[[Storage, Shape, Strides, Storage, Shape, Strides], None]` – Tensor map function.

`minitorch.tensor_ops.tensor_zip(fn: Callable[[float, float], float]) -> Callable[[Storage, Shape, Strides, Storage, Shape, Strides, Storage, Shape, Strides], None]`

Low-level implementation of tensor zip between tensors with *possibly different strides*.

Simple version:

- Fill in the `out` array by applying `fn` to each value of `a_storage` and `b_storage` assuming `out_shape` and `a_shape` are the same size.

Broadcasted version:

- Fill in the `out` array by applying `fn` to each value of `a_storage` and `b_storage` assuming `a_shape` and `b_shape` broadcast to `out_shape`.

Parameters:

- **fn** (`Callable[[float, float], float]`) – function mapping two floats to float to apply

Returns:

- `Callable[[Storage, Shape, Strides, Storage, Shape, Strides, Storage, Shape, Strides], None]` – Tensor zip function.

`minitorch.tensor_ops.tensor_reduce(fn: Callable[[float, float], float]) -> Callable[[Storage, Shape, Strides, Storage, Shape, Strides, int], None]`

Low-level implementation of tensor reduce.

- `out_shape` will be the same as `a_shape` except with `reduce_dim` turned to size `1`

Parameters:

- **fn** (`Callable[[float, float], float]`) – reduction function mapping two floats to float

Returns:

- `Callable[[Storage, Shape, Strides, Storage, Shape, Strides, int], None]` – Tensor reduce function.

minitorch.tensor_functions.Mul.forward(ctx: Context, a: Tensor, b: Tensor) -> Tensor
staticmethod

minitorch.tensor_functions.Sigmoid.forward(ctx: Context, t1: Tensor) -> Tensor
staticmethod

minitorch.tensor_functions.ReLU.forward(ctx: Context, t1: Tensor) -> Tensor
staticmethod

minitorch.tensor_functions.Log.forward(ctx: Context, t1: Tensor) -> Tensor staticmethod

minitorch.tensor_functions.Exp.forward(ctx: Context, t1: Tensor) -> Tensor staticmethod

minitorch.tensor_functions.LT.forward(ctx: Context, a: Tensor, b: Tensor) -> Tensor
staticmethod

minitorch.tensor_functions.EQ.forward(ctx: Context, a: Tensor, b: Tensor) -> Tensor
staticmethod

minitorch.tensor_functions.Permute.forward(ctx: Context, a: Tensor, order: Tensor) -
> Tensor staticmethod

minitorch.tensor_functions.IsClose.forward(ctx: Context, a: Tensor, b: Tensor) ->
Tensor staticmethod

Tasks 2.4: Gradients and Autograd

Similar to `minitorch.Scalar`, `minitorch.Tensor` is a Variable that supports autodifferentiation. In this task, you will implement `backward` functions for tensor operations.

> **Todo**
>
> Complete following functions in `minitorch/tensor_functions.py`, and pass tests marked as `task2_4`.

`minitorch.tensor_functions.Mul.backward(ctx: Context, grad_output: Tensor) -> Tuple[Tensor, Tensor]` staticmethod

`minitorch.tensor_functions.Sigmoid.backward(ctx: Context, grad_output: Tensor) -> Tensor` staticmethod

`minitorch.tensor_functions.ReLU.backward(ctx: Context, grad_output: Tensor) -> Tensor` staticmethod

`minitorch.tensor_functions.Log.backward(ctx: Context, grad_output: Tensor) -> Tensor` staticmethod

`minitorch.tensor_functions.Exp.backward(ctx: Context, grad_output: Tensor) -> Tensor` staticmethod

`minitorch.tensor_functions.LT.backward(ctx: Context, grad_output: Tensor) -> Tuple[Tensor, Tensor]` staticmethod

`minitorch.tensor_functions.EQ.backward(ctx: Context, grad_output: Tensor) -> Tuple[Tensor, Tensor]` staticmethod

`minitorch.tensor_functions.Permute.backward(ctx: Context, grad_output: Tensor) -> Tuple[Tensor, float]` staticmethod

Task 2.5: Training

If your code works you should now be able to move on to the tensor training script in `project/run_tensor.py`. This code runs the same basic training setup as in `module1`, but now utilize your tensor code.

> **Todo**
>
> Implement the missing `forward` functions in `project/run_tensor.py`. They should do exactly the same thing as the corresponding functions in `project/run_scalar.py`, but now use the tensor code base.
>
> - Train a tensor model and add your results for all datasets to the README.
> - Record the time per epoch reported by the trainer. (It is okay if it is slow).