EVENT STUDIES IN UNBALANCED PANELS

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A fundamental question in health economics is the effect of a policy on the birthweight of children. Consider a simple situation where there are three periods $t = 0, 1, 2$ and the policy intervention occurred between periods 0 and 1. There are mothers affected by the intervention who gave birth in periods 0 and 1 (referred to as group A hereafter), 0 and 2 (group B), and 1 and 2 (group C). Mothers unaffected by the intervention who gave birth in periods 0 and 1 (group D) and 0 and 2 (group E) are used as control groups. This is an unbalanced panel because not every group is observed in every year. For example, group A did not give birth in period 1. In fact, no group here is observed for all periods but the following analysis extends also to situations where some groups have all years available, that is, if there are mothers who gave birth in every period.

The canonical difference-in-differences (DD) design would compare the average change in birthweight in children of the same mother between mothers who were and who were not affected by the policy. For the effect one period out, one would use only data from groups A and D. For the effect two periods out, one would use only data from groups B and E. In the event study design, these data are pooled in a two-way fixed effects model

$$Y = \beta_1 1\{t = 1\} \times \textit{treated} + \beta_2 1\{t = 2\} \times \textit{treated} + \alpha_j + \tau_t + U, \tag{1}$$

where α_j is a mother or group fixed effect and τ_t is a time fixed effect. Least squares estimates of the coefficients β_1 and β_2 are often interpreted is the same way as one or two period DD estimates.

One would expect this interpretation to not change if "always-treated" units in group C were included because they seem to do nothing when it comes to identifying either of the treatment effects β_1 and β_2. Regression intuition from balanced panels may suggest that group C should simply drop out when two-way fixed effects (TWFE) are included, but this is not true and inclusion of group C generally has an impact on the least squares estimates of β_1 and β_2. As I will outline below, the impact can be positive or negative depending on the underlying assumptions.

To see what least squares actually estimates in the simplest possible setting, consider a TWFE model for the mean of outcome $Y_{j,t}(0)$ in the absence of the intervention,

$$\mathbb{E}Y_{j,t}(0) = \alpha_j + \tau_t,$$

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TABLE 1. $\mathbb{E}Y_{j,t}$ implied by TWFE in an unbalanced panel

	$j=A$	$j=B$	$j=C$	$j=D$	$j=E$
$t=0$	α_A	α_B	$\times$	α_D	α_E
$t=1$	$\alpha_A+\tau_1+\beta_{1,A}$	$\times$	$\alpha_B+\tau_1+\beta_{1,C}$	$\alpha_D+\tau_1$	$\times$
$t=2$	$\times$	$\alpha_B+\tau_2+\beta_{2,B}$	$\alpha_C+\tau_2+\beta_{2,C}$	$\times$	$\alpha_E+\tau_2$

where α_j only varies across groups $j=A,B,C,D,E$ and τ_0 is (without loss of generality) normalized to zero. Denote the period $t=1,2$ causal effect on treated observations $Y_{j,t}$ by

$$\beta_{j,t}=\mathbb{E}[Y_{j,t}-Y_{j,t}(0)], \qquad j=A,B,C.$$

Table 1 summarizes the implied structure of $\mathbb{E}Y_{j,t}$ for all groups and time periods, where "$\times$" indicates that this time-group combination is not observed.

Suppose, for simplicity, that each group A-E has the same number of observations and denote the least squares estimates of β_1 and β_2 in equation (1) by $\hat{\beta}_1$ and $\hat{\beta}_2$. Then tedious but straightforward algebra shows that

$$\mathbb{E}\hat{\beta}_1=\frac{2}{3}\beta_{1,A}+\frac{1}{3}\beta_{1,C}+\frac{1}{3}\beta_{2,B}-\frac{1}{3}\beta_{2,C}$$

and similarly

$$\mathbb{E}\hat{\beta}_2=\frac{2}{3}\beta_{2,B}+\frac{1}{3}\beta_{2,C}+\frac{1}{3}\beta_{1,A}-\frac{1}{3}\beta_{1,C}.$$

In words, if the "always-treated" group C is included in the regression, then not only does that group not drop out but influences the estimates in both periods. Moreover, the least squares estimand in period 1 now includes causal effects from period 2 and vice versa. The causal effect from the other period even enters with a negative weight.

However, from the perspective of the health policy intervention, there may be little reason to believe that the treatment effects are different across groups at any given time. If groups are comparable enough such that treatment effects in groups A and C at time 1 are the same and that treatment effects in groups B and C at time 2 are the same, then including the always-treated group C adds identifying power. If $\beta_{1,A}=\beta_{1,C}=:\beta_1$ and $\beta_{2,B}=\beta_{2,C}=:\beta_2$, the preceding two displays simplify to $\mathbb{E}\hat{\beta}_1=\beta_1$ and $\mathbb{E}\hat{\beta}_2=\beta_2$. In this case, adding group C to the event study adds additional information and therefore efficiency to the estimation of β_1 and β_2 that would otherwise not be available when estimating period-by-period DD effects.