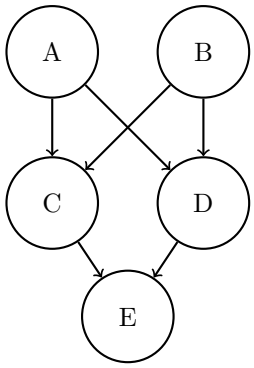
CS 188 Summer 2023 Final Review Bayes Nets Solutions

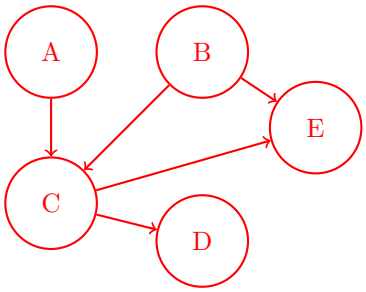
Q1. Bayes Nets and Joint Distributions

(a) Write down the joint probability distribution associated with the following Bayes Net. Express the answer as a product of terms representing individual conditional probabilities tables associated with this Bayes Net:



$P(A)P(B)P(C|A,B)P(D|A,B)P(E|C,D)$

(b) Draw the Bayes net associated with the following joint distribution:
$P(A) \cdot P(B) \cdot P(C|A,B) \cdot P(D|C) \cdot P(E|B,C)$



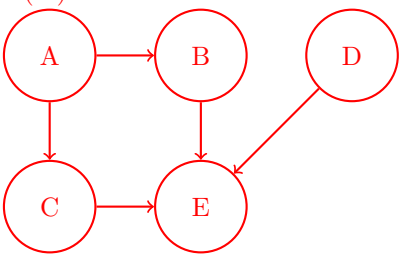
(c) Do the following products of factors correspond to a valid joint distribution over the variables A, B, C, D? (Circle TRUEFALSE.)

(i)	TRUE	FALSE	$P(A) \cdot P(B) \cdot P(C\|A) \cdot P(C\|B) \cdot P(D\|C)$
(ii)	TRUE	FALSE	$P(A) \cdot P(B\|A) \cdot P(C) \cdot P(D\|B,C)$
(iii)	TRUE	FALSE	$P(A) \cdot P(B\|A) \cdot P(C) \cdot P(C\|A) \cdot P(D)$
(iv)	TRUE	FALSE	$P(A\|B) \cdot P(B\|C) \cdot P(C\|D) \cdot P(D\|A)$

(d) What factor can be multiplied with the following factors to form a valid joint distribution? (Write "none" if the given set of factors can't be turned into a joint by the inclusion of exactly one more factor.)

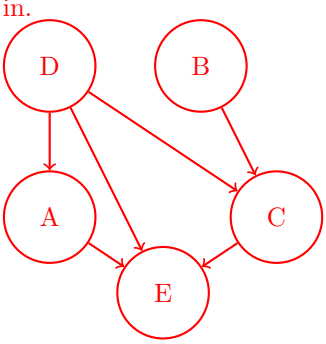
(i) $P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(E|B,C,D)$

P(D) is missing. D could also be conditioned on A,B, and/or C without creating a cycle (e.g. $P(D|A, B, C)$). Here is an example bayes net that would represent the distribution after adding in $P(D)$:

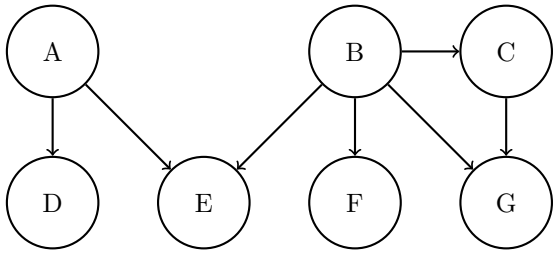


(ii) $P(D) \cdot P(B) \cdot P(C|D, B) \cdot P(E|C, D, A)$

P(A) is missing to form a valid joint distributions. A could also be conditioned on B, C, and/or D (e.g. $P(A|B, C, D)$. Here is a bayes net that would represent the distribution is $P(A|D)$ was added in.



(e) Answer the next questions based off of the Bayes Net below:
All variables have domains of {-1, 0, 1}



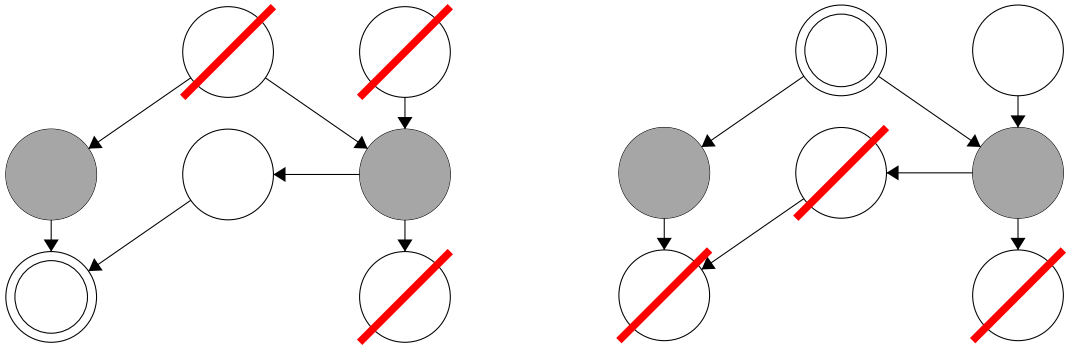
(i) Before eliminating any variables or including any evidence, how many entries does the factor at G have?
The factor is $P(G|B,C)$, so that gives $3^3 = 27$ entries.

(ii) Now we observe $e = 1$ and want to query $P(D|e = 1)$, and you get to pick the first variable to be eliminated.

- Which choice would create the **largest** factor f_1?
  Eliminating B first would give the largest f_1:, $f_1(A, F, G, C, e) = \sum_{B=b} P(b)P(e|A, b)P(F|b)P(G|b, C)P(C|b)$. This factor has 3^4 entries.
- Which choice would create the **smallest** factor f_1?
  eliminating F first would give smallest factors of 3 entries: $f_1(B) = \sum_f P(f|B)$. Eliminating D is not correct because D is the query variable.

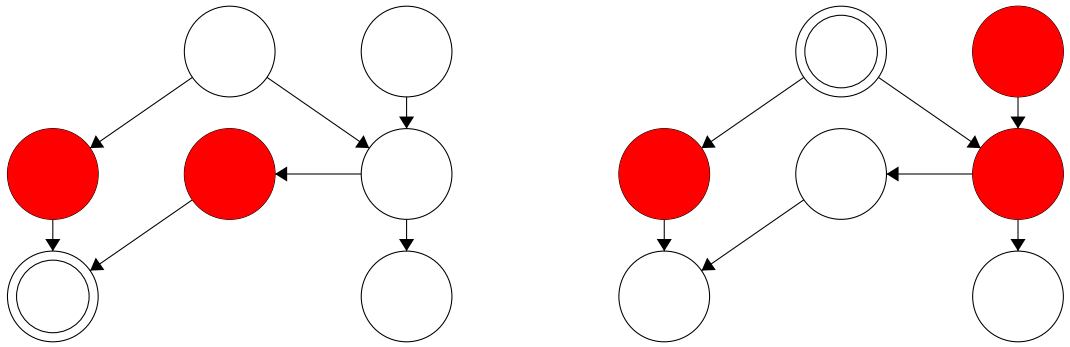
Q2. Moral Graphs

(a) For each of the following queries, we want to preprocess the Bayes net before performing variable elimination. Query variables are double-circled and evidence variables are shaded. Cross off all the variables that we can ignore in performing the query. If no variables can be ignored in one of the Bayes nets, write "None" under that Bayes net.



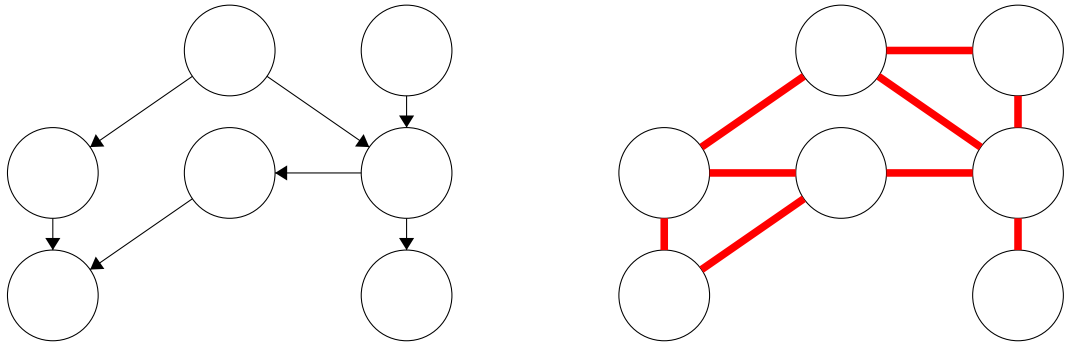
Let B be a Bayes net with a set of variables V. The **Markov blanket** of a variable $v \in V$ is the smallest set of variables $S \subset V$ such that for any variables $v' \in V$ such that $v \neq v'$ and $v' \notin S$, $v \perp\!\!\!\perp v' | S$. Less formally, v is independent from the entire Bayes net given all the variables in S.

(b) In each of the following Bayes nets, shade in the Markov blanket of the double-circled variable.



The **moral graph** of a Bayes net is an **undirected** graph with the same vertices as the Bayes net (i.e. one vertex corresponding to each variable) such that each variable has an edge connecting it to every variable in its Markov blanket.

(c) Add edges to the graph on the right so that it is the moral graph of the Bayes net on the left.



(d) The following is a query in a moral graph for a larger Bayes net (the Bayes net is not shown). Cross off all the variables that we can ignore in performing the query.