Homework 7

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Problem 3.3.

$$
\begin{aligned}
\hat{\theta}_{MLE} &= \arg\max_{\theta} \sum_{i=1}^{k} \log \left(\binom{n_i}{m_i} (f(x_i;\theta))^{m_i} (1 - f(x_i;\theta))^{n_i - m_i} \right) \\
&= \arg\max_{\theta} \sum_{i=1}^{k} \log \left((f(x_i;\theta))^{m_i} (1 - f(x_i;\theta))^{n_i - m_i} \right) \\
&= \arg\max_{\theta} \sum_{i=1}^{k} \left[m_i \log(f(x_i;\theta)) + (n_i - m_i) \log(1 - f(x_i;\theta)) \right]
\end{aligned}
$$

Problem 3.4. As we are focusing on one training example, we do not need to take the summation as we did in Problem 3.2, so we can compute the derivatives using the chain rule as follows:

$$
\begin{aligned}
\frac{\partial ll}{\partial w_0} &= \frac{\partial ll}{\partial f} \cdot \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial w_0} \\
&= \left(\frac{m_i}{f(x_i;\theta)} + \frac{(-1)(n_i - m_i)}{1 - f(x_i;\theta)} \right) \cdot \sigma(z)(1-\sigma(z)) \cdot n_{white} \\
&= \left(\frac{m_i}{f(x_i;\theta)} + \frac{m_i - n_i}{1 - f(x_i;\theta)} \right) \cdot \sigma(z)(1-\sigma(z)) \cdot n_{white} \\
\frac{\partial ll}{\partial w_1} &= \frac{\partial ll}{\partial f} \cdot \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial w_1} \\
&= \left(\frac{m_i}{f(x_i;\theta)} + \frac{m_i - n_i}{1 - f(x_i;\theta)} \right) \cdot \sigma(z)(1-\sigma(z)) \cdot n_{black} \\
\frac{\partial ll}{\partial b_0} &= \frac{\partial ll}{\partial f} \cdot \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial b_0} \\
&= \left(\frac{m_i}{f(x_i;\theta)} + \frac{m_i - n_i}{1 - f(x_i;\theta)} \right) \cdot \sigma(z)(1-\sigma(z)) \cdot 1 \\
&= \left(\frac{m_i}{f(x_i;\theta)} + \frac{m_i - n_i}{1 - f(x_i;\theta)} \right) \cdot \sigma(z)(1-\sigma(z))
\end{aligned}
$$

as required.

Problem 6.2.

$$\begin{aligned}
\frac{\partial y}{\partial z} &= \frac{\partial}{\partial z}(g(z)) \\
&= g(z)(1-g(z)) \\
\frac{\partial z}{\partial w_0} &= \frac{\partial}{\partial w_0}(w_0 x_0 + w_1 x_1) \\
&= x_0
\end{aligned}$$

as required.

Problem 6.3. We could express the rate of change of upstream loss L with respect to w_0 using the chain rule as follows:

$$\frac{\partial L}{\partial w_0} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial w_0}$$

Therefore, we could express the update rule for w_0 as follows:

$$w_0 \leftarrow w_0 - \alpha \cdot \left(\frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial w_0} \right) = w_0 - \alpha \cdot \left(\frac{\partial L}{\partial y} \cdot g(z)(1 - g(z)) \cdot (x_0) \right)$$

Note that we are using minus sign here because the algorithm is running with gradient descent.

This completes Problem 6.3.

Problem 7.2. Notice we use uppercase O's to distinguish letters, but they should be lowercase as in the problem description.

$$
\begin{aligned}
\frac{\partial y}{\partial w_5} &= \frac{\partial}{\partial w_5}(w_5 \cdot O_d) \\
&= O_d \\
\frac{\partial y}{\partial w_4} &= \frac{\partial y}{\partial O_d}\frac{\partial O_d}{z_d}\frac{\partial z_d}{w_4} \\
&= w_5 \cdot \gamma \cdot O_c \\
&= 3 \cdot 0.1 \cdot O_c \\
&= 0.3O_c \\
\frac{\partial y}{\partial w_3} &= \frac{\partial y}{\partial O_d}\frac{\partial O_d}{\partial z_d}\frac{\partial z_d}{\partial O_c}\frac{\partial O_c}{\partial z_c}\frac{\partial z_c}{\partial w_3} \\
&= 0.3 \cdot w_4 \cdot 1 \cdot O_b \\
&= 0.3 \cdot (-5) \cdot O_b \\
&= -1.5O_b \\
\frac{\partial y}{\partial w_2} &= \frac{\partial y}{\partial O_d}\frac{\partial O_d}{\partial z_d}\frac{\partial z_d}{\partial O_c}\frac{\partial O_c}{\partial z_c}\frac{\partial z_c}{\partial O_b}\frac{\partial O_b}{\partial z_b}\frac{\partial z_b}{\partial w_2} \\
&= -1.5 \cdot w_3 \cdot (1) \cdot O_a \\
&= -1.5 \cdot (1) \cdot (1) \cdot O_a \\
&= -1.5O_a \\
\frac{\partial y}{\partial w_1} &= \frac{\partial y}{\partial O_d}\frac{\partial O_d}{\partial z_d}\frac{\partial z_d}{\partial O_c}\frac{\partial O_c}{\partial z_c}\frac{\partial z_c}{\partial O_b}\frac{\partial O_b}{\partial z_b}\frac{\partial z_b}{\partial O_a}\frac{\partial O_a}{\partial z_a}\frac{\partial z_a}{\partial w_1} \\
&= -1.5 \cdot w_2 \cdot 1 \cdot x \\
&= -1.5(2)x \\
&= -3x
\end{aligned}
$$

As computed above, we've derived the partial derivatives required. Technically we could plug in the values of O_a, O_b, O_c, O_d, x inside the equation for the exact values, but we were asked by the question to express answers in terms of x, O_a, O_b, O_c, O_d.

This completes Problem 7.2.