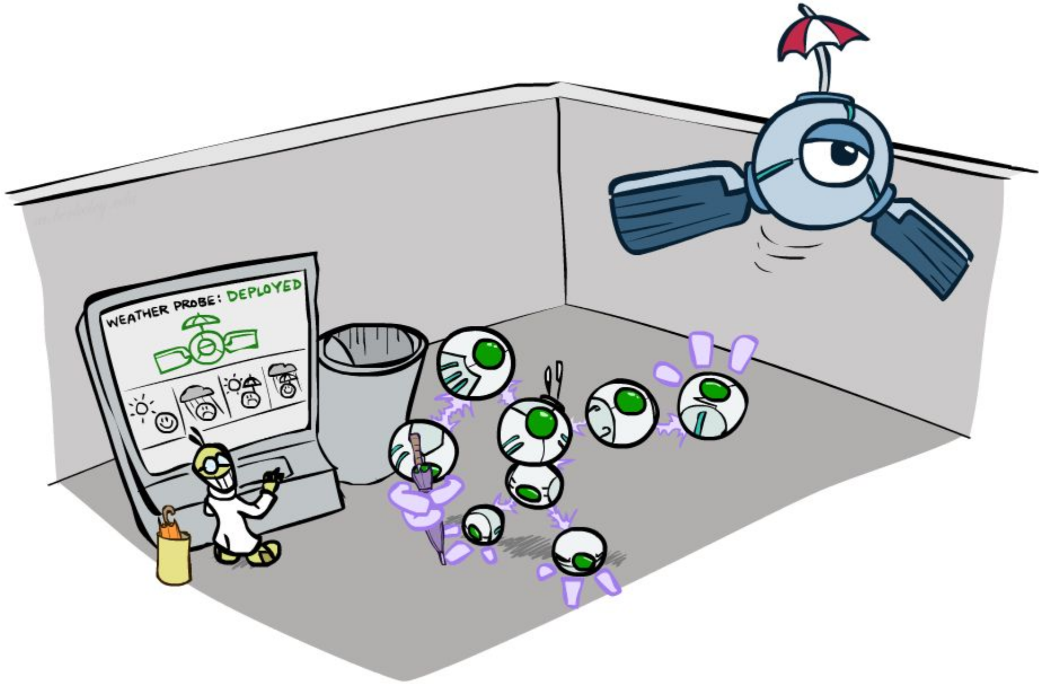
CS 188: Artificial Intelligence

Decision Networks and VPI



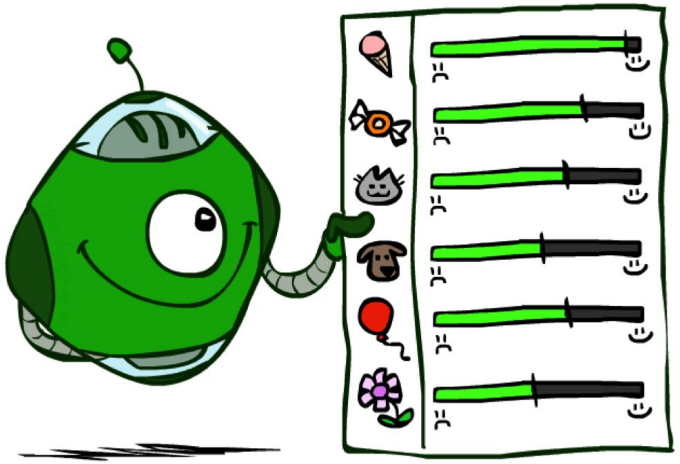
Instructor: Saagar Sanghavi—University of California, Berkeley

[Slides Credit: Dan Klein, Pieter Abbeel, Anca Dragan, and many others.]

Recap: Utilities and Rationality

- Utilities and Rationality
- Rational Preferences

Orderability: $(A > B) \vee (B > A) \vee (A \sim B)$
Transitivity: $(A > B) \wedge (B > C) \Rightarrow (A > C)$
Continuity: $(A > B > C) \Rightarrow \exists p\ [p, A;\ 1\text{-}p, C] \sim B$
Substitutability: $(A \sim B) \Rightarrow [p, A;\ 1\text{-}p, C] \sim [p, B;\ 1\text{-}p, C]$
Monotonicity: $(A > B) \Rightarrow$
$(p \geq q) \Leftrightarrow [p, A;\ 1\text{-}p, B] \geq [q, A;\ 1\text{-}q, B]$



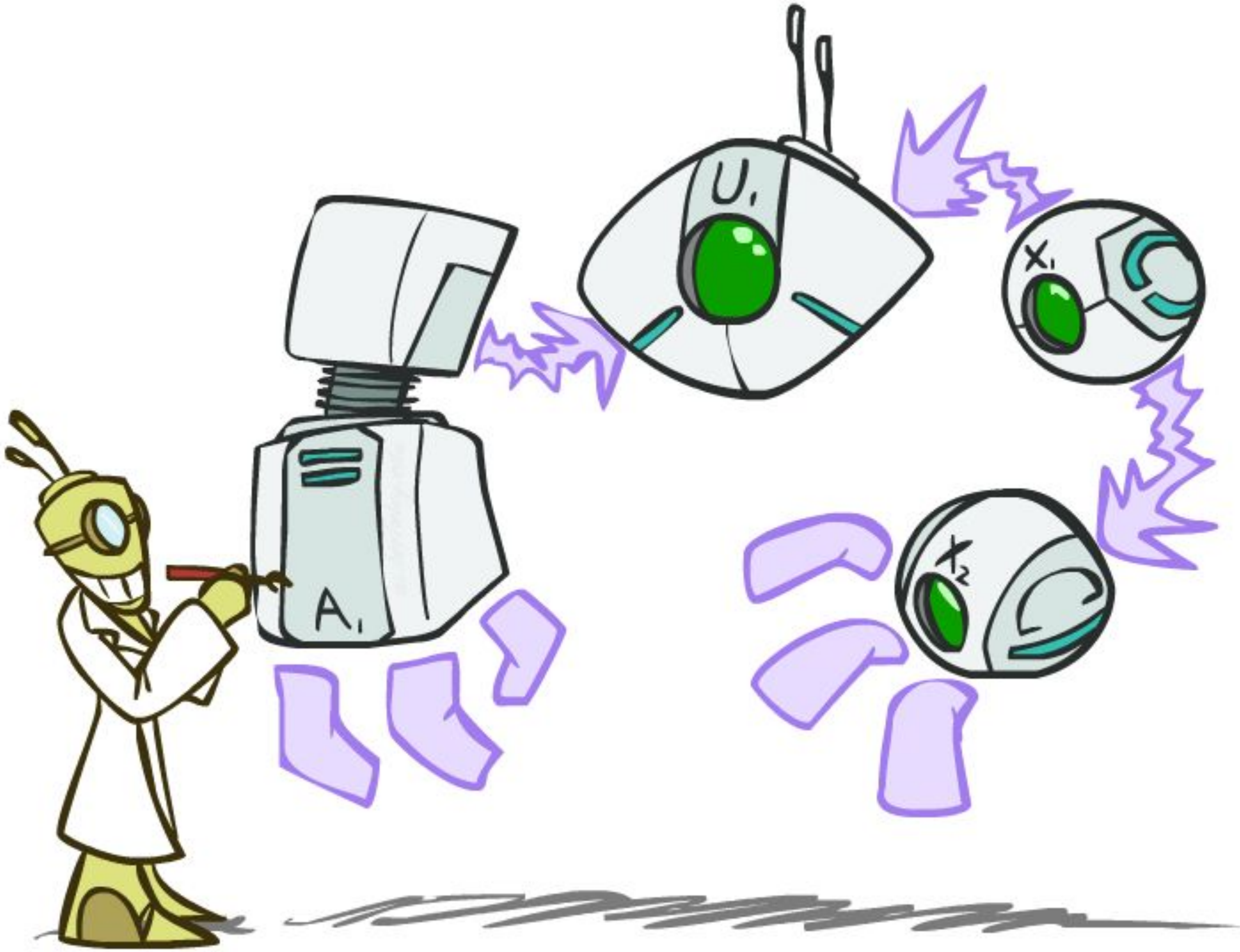
MEU Principle:

Given Rational Preferences, Exists U(X) s.t.

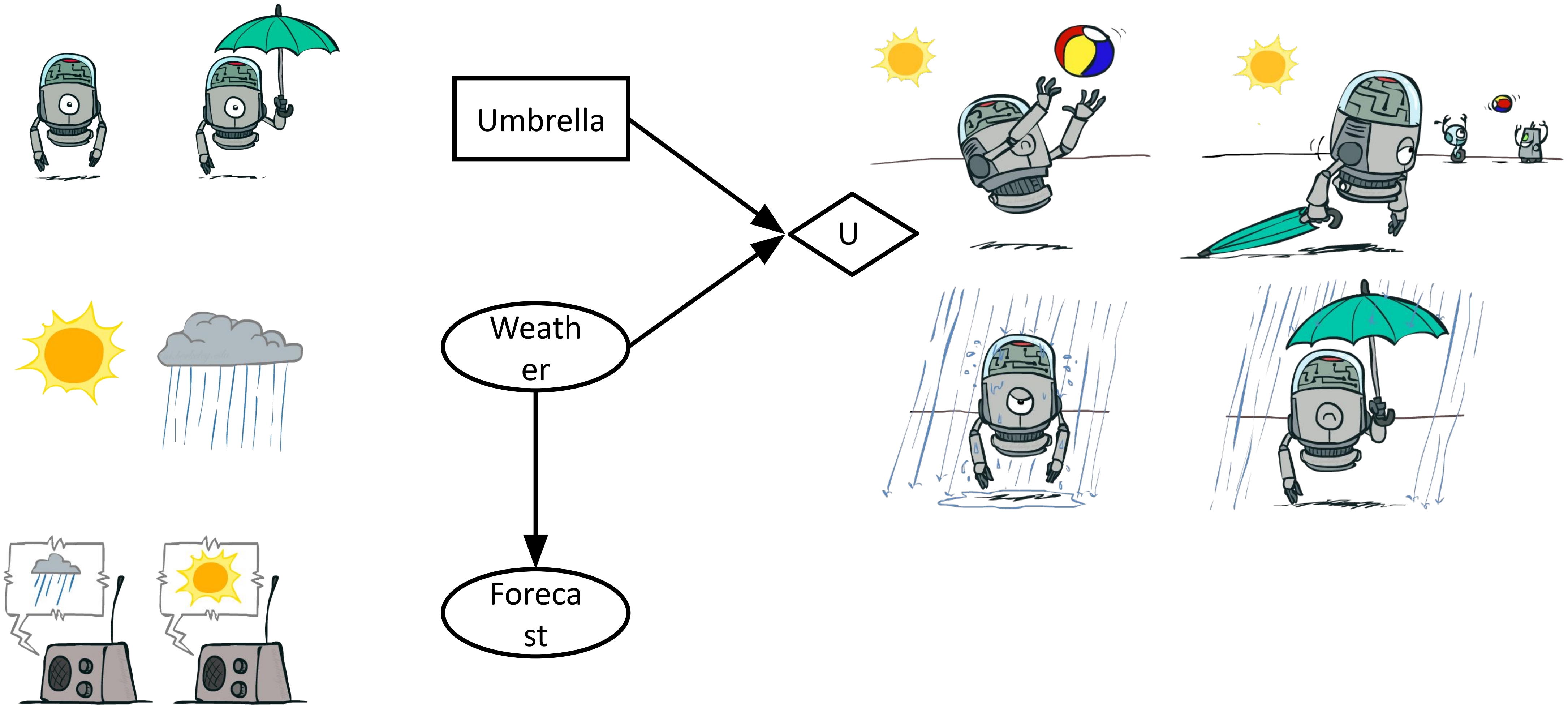
$U(A) \geq U(B) \ \Leftrightarrow\ A \geq B$

$$U([p_1,S_1;\ \dots\ ;p_n,S_n]) = p_1U(S_1) + \dots + p_nU(S_n)$$

Decision Networks



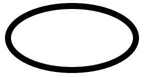
Decision Networks



Decision Networks

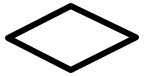
- **MEU: choose the action which maximizes the expected utility given the evidence**

  - Can directly operationalize this with decision networks
    - Bayes nets, with new node types for utilities and actions
    - Lets us calculate the expected utility for each action
  - New node types:

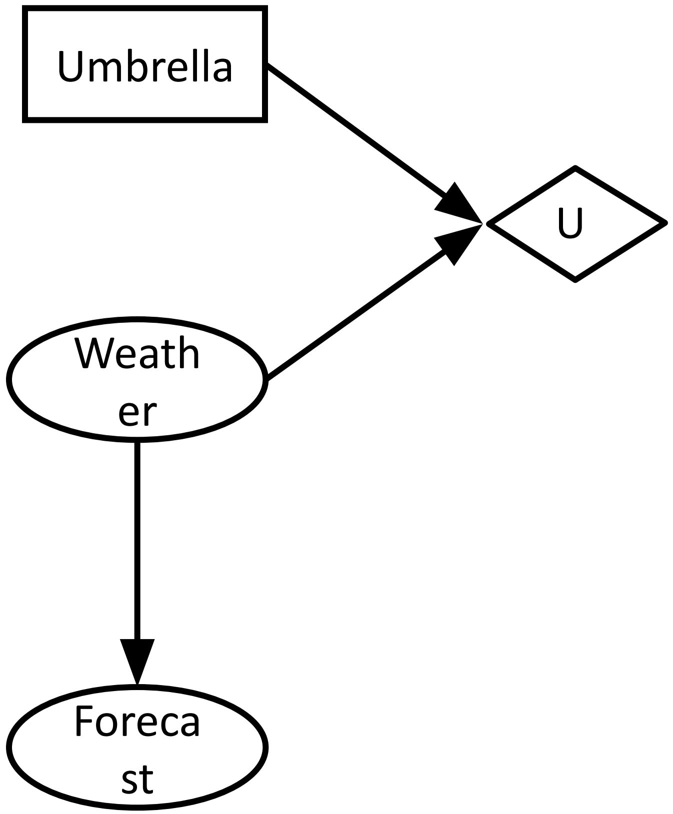


    - Chance nodes (just like Bayes Nets)

    - Actions (rectangles, cannot have parents, act as observed evidence)

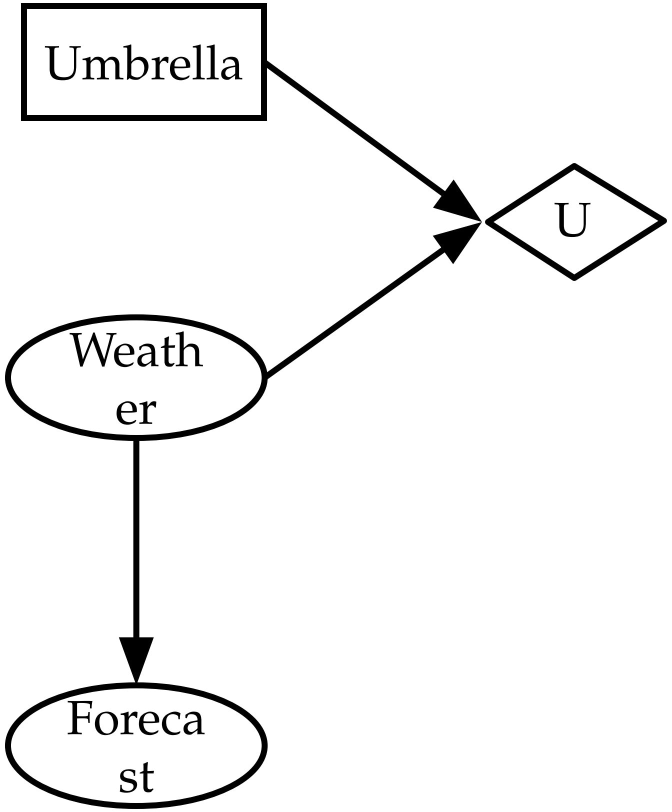


    - Utility node (diamond, depends on action and chance nodes)



Decision Networks

- Action selection
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action



Maximum Expected Utility

Umbrella = leave

$$\mathrm{EU}(\text{leave}) = \sum_w P(w)U(\text{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

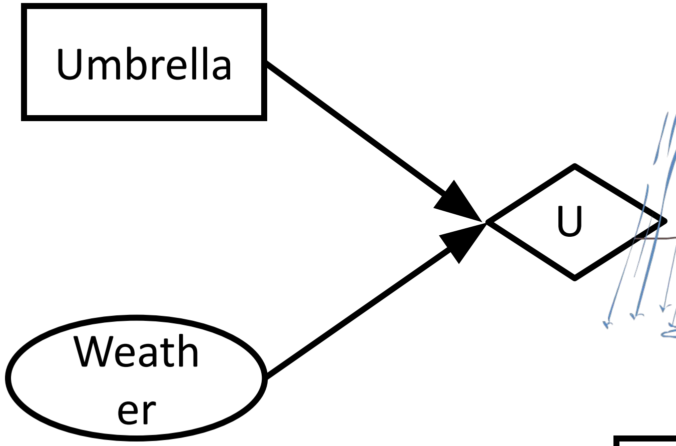
Umbrella = take

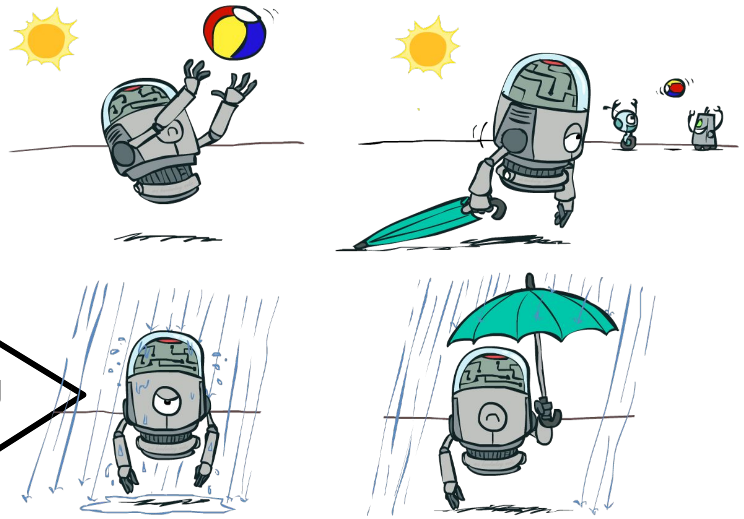
$$\mathrm{EU}(\text{take}) = \sum_w P(w)U(\text{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\mathrm{MEU}(\varnothing) = \max_a \mathrm{EU}(a) = 70$$

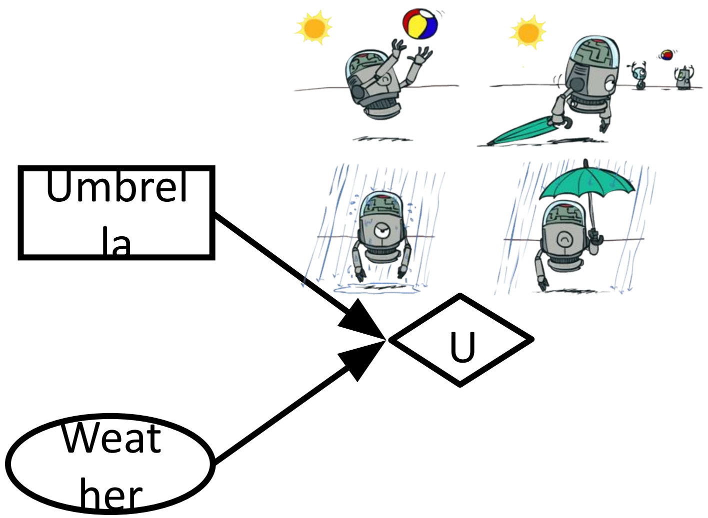


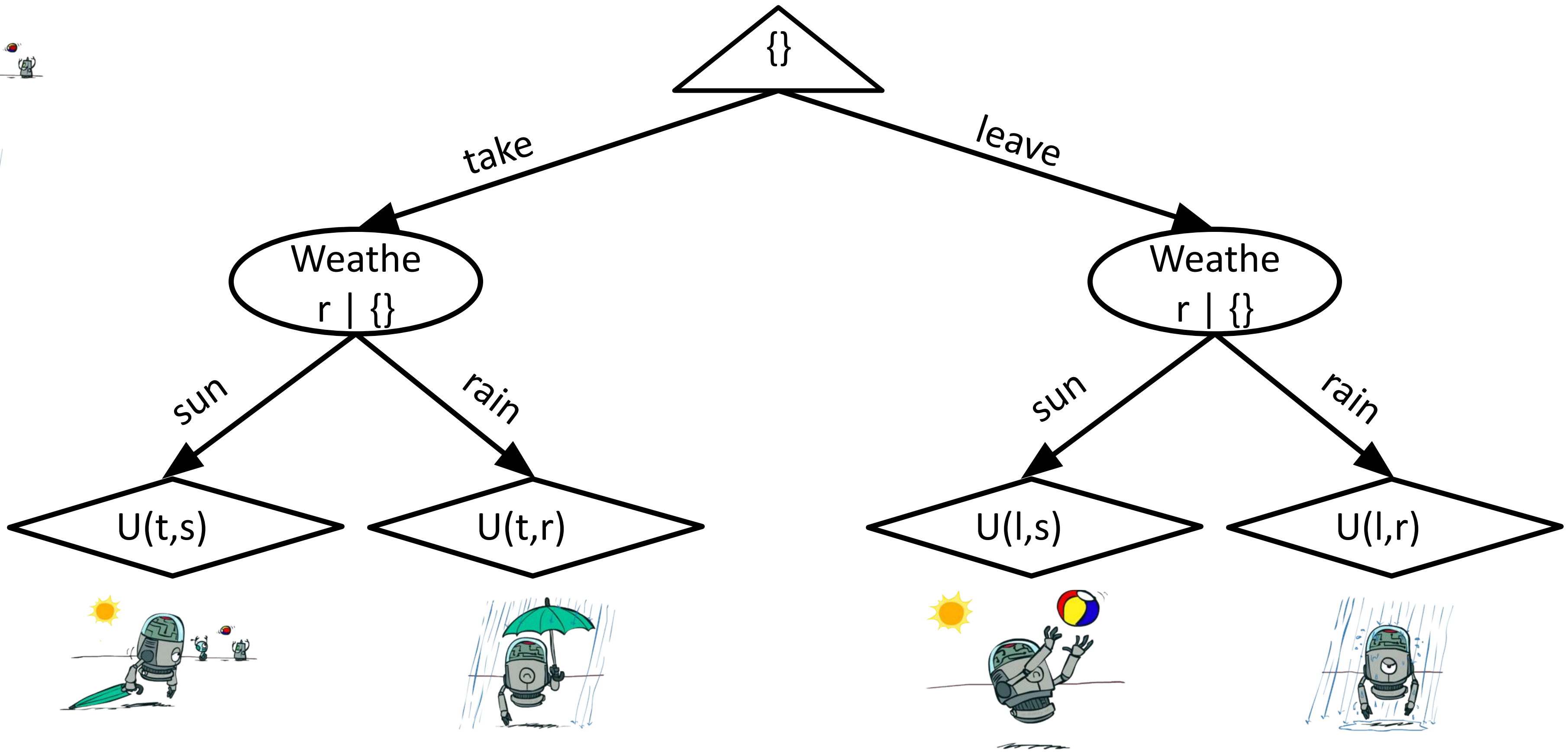


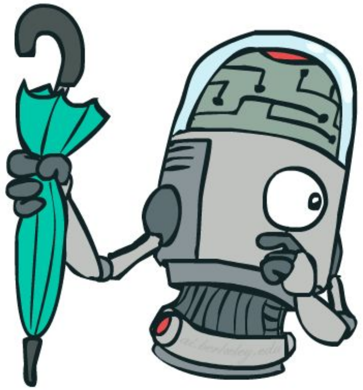
W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decisions as Outcome Trees







- Almost exactly like expectimax

Maximum Expected Utility Given Evidence

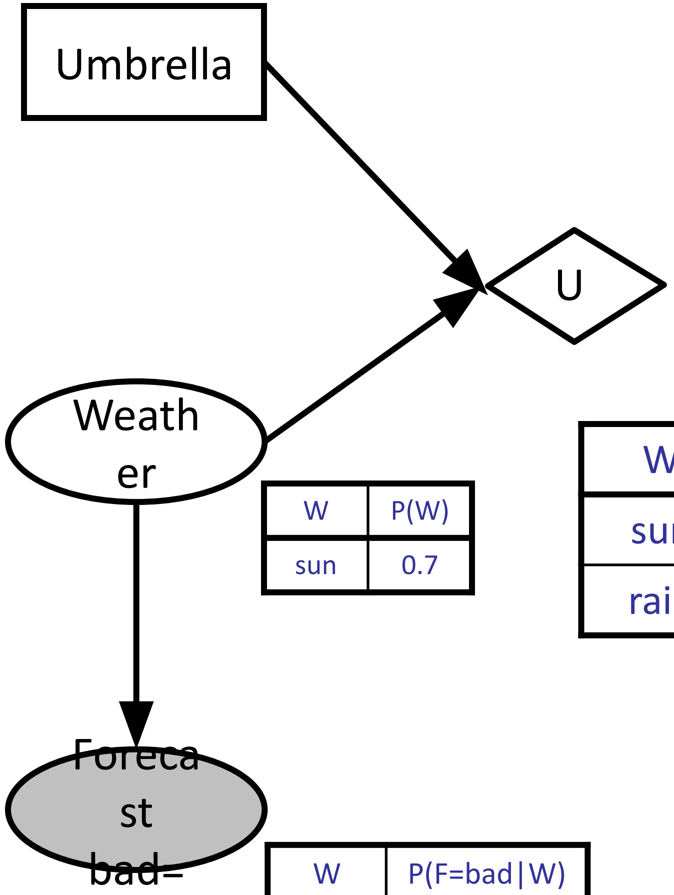
Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_{w} P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$P(W) \qquad\qquad P(F|W)$$

$$P(W|F) = \frac{P(W,F)}{\sum_w P(w,F)}$$

$$= \frac{P(F|W)P(W)}{\sum_w P(F|w)P(w)}$$

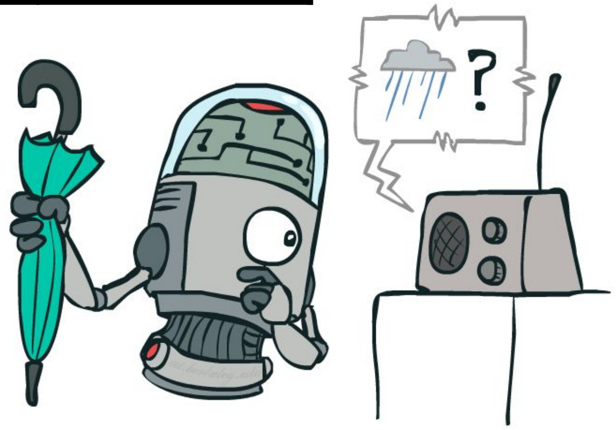


A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

W	P(W)
sun	0.7

W	P(W|F=bad)
sun	0.34
rain	0.66

W	P(F=bad|W)
sun	0.17
rain	0.77



Maximum Expected Utility Given Evidence

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

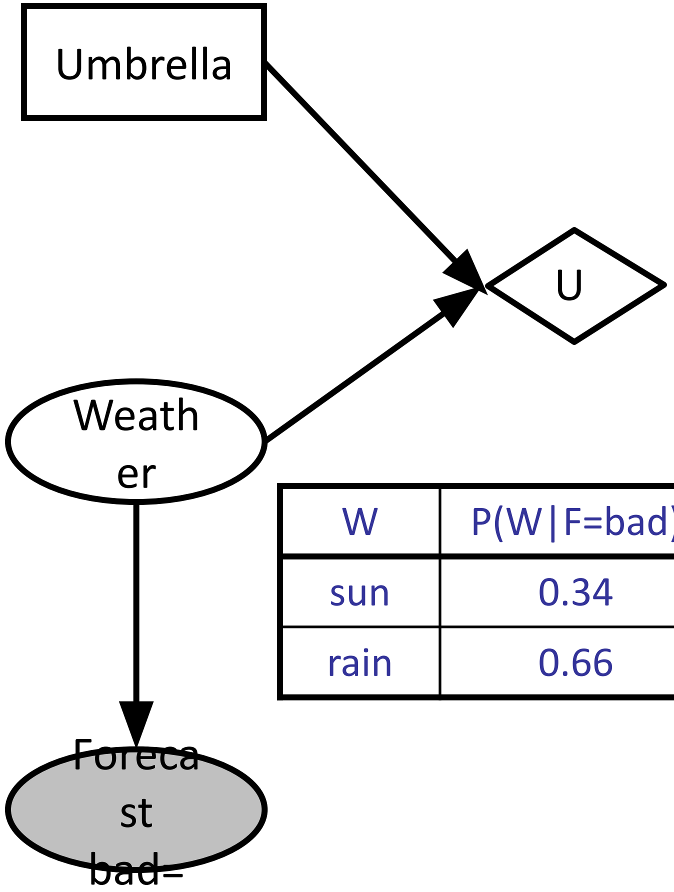
Umbrella = take

$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

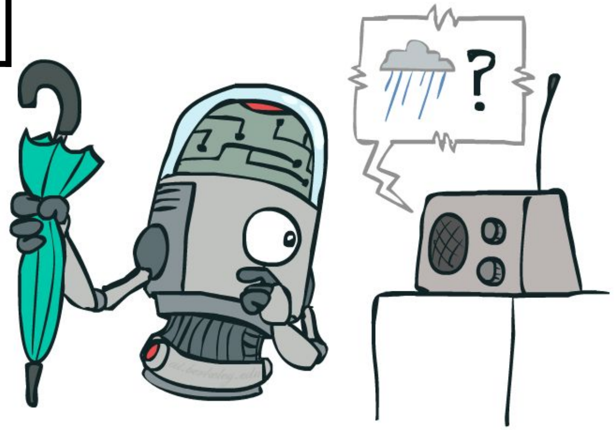
Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

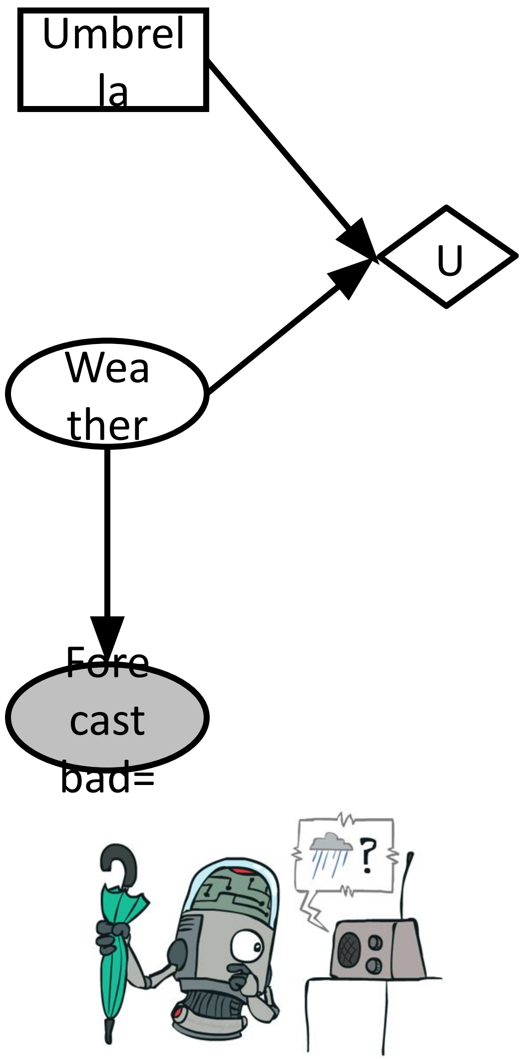


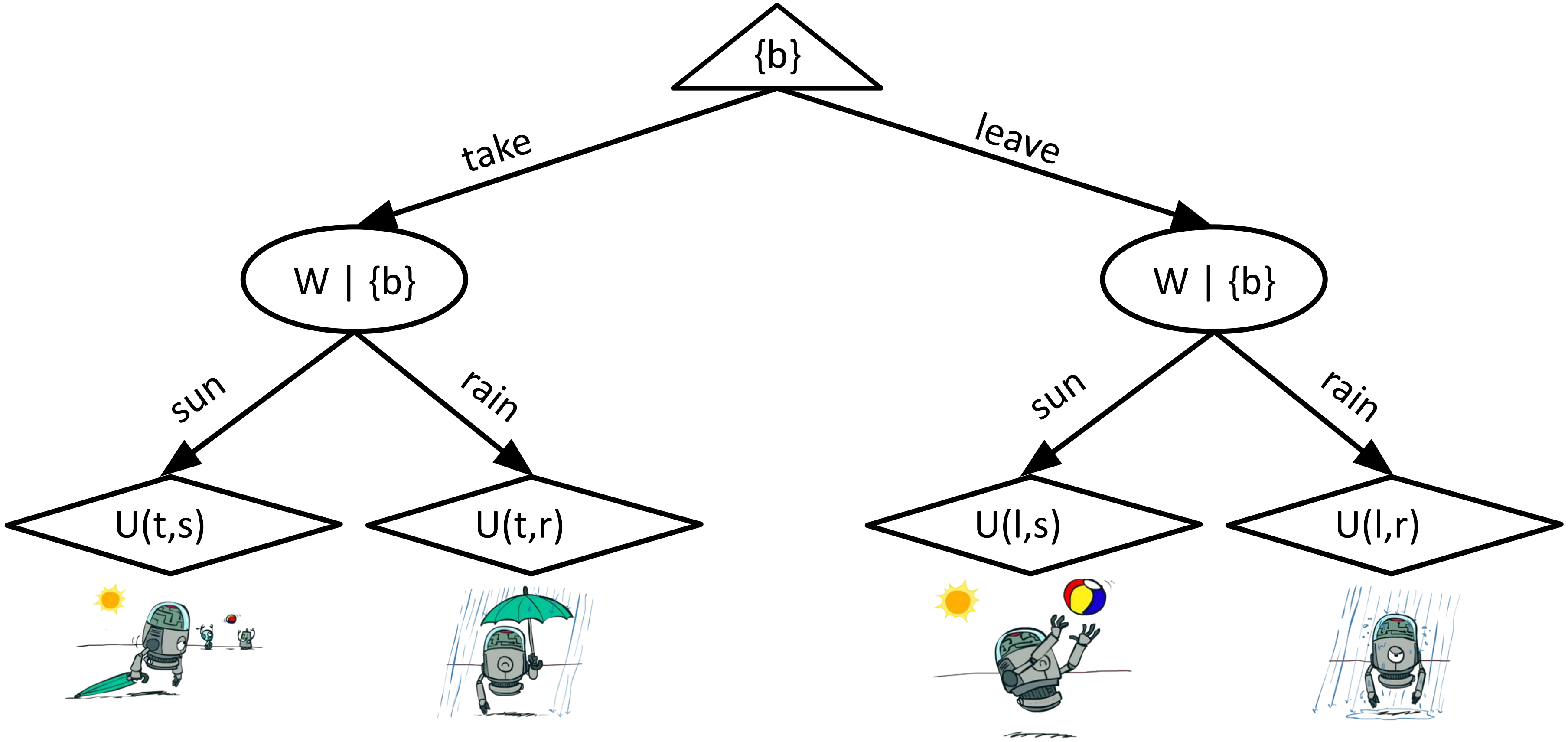
A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

W	P(W|F=bad)
sun	0.34
rain	0.66

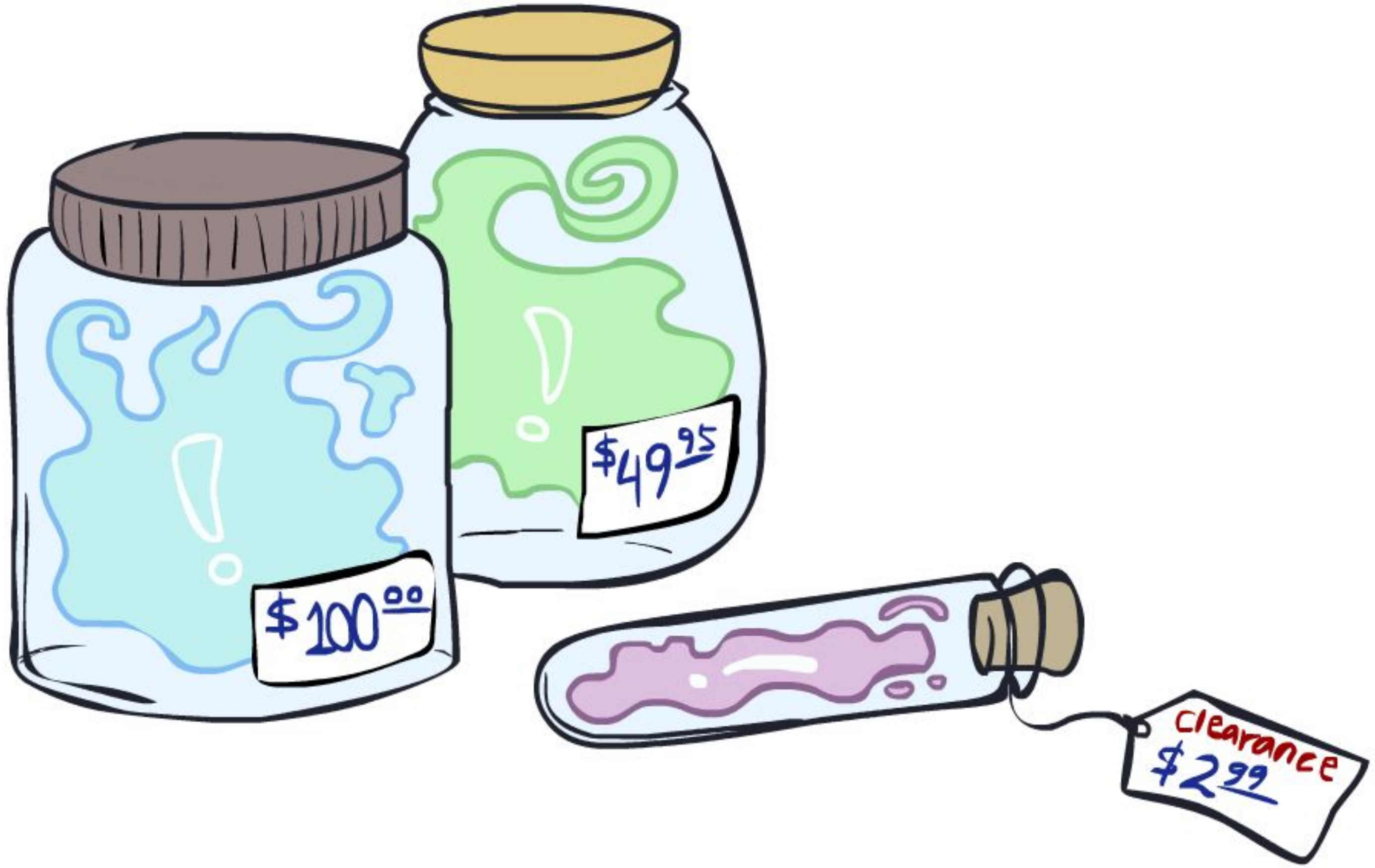


Decisions as Outcome Trees





Value of Information



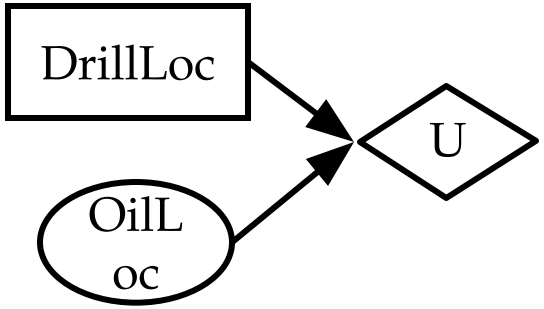
Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network

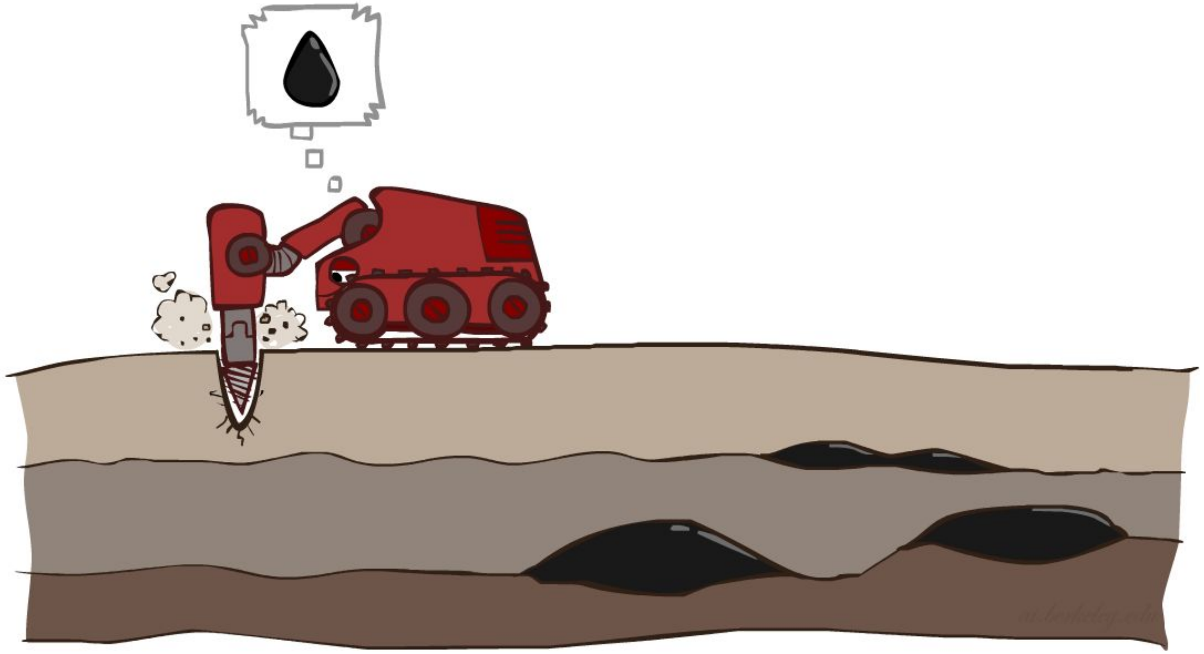
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2

- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b," prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2

O	P
a	1/2
b	1/2



D	O	U
a	a	k
a	b	0
b	a	0
b	b	k



Value of Perfect Information

MEU with no evidence

$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$

MEU if forecast is bad

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

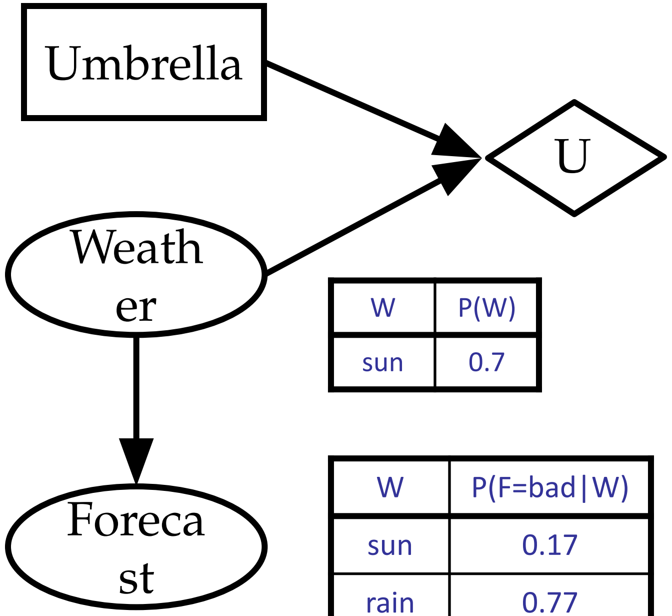
MEU if forecast is good

$$\mathrm{MEU}(F = \mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 89.4$$

Forecast distribution

F	P(F)
bad	0.35
good	0.65

VPI = 0.35 (53) + 0.65 (89.4) – 70
VPI = 6.66

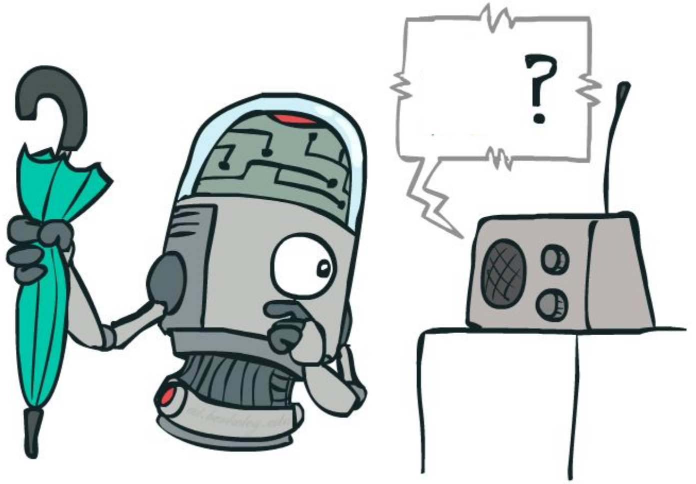


W	P(W)
sun	0.7

W	P(F=bad|W)
sun	0.17
rain	0.77

A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

$$\mathrm{VPI}(E'|e) = \left(\sum_{e'} P(e'|e)\mathrm{MEU}(e, e')\right) - \mathrm{MEU}(e)$$



Value of Information

- Assume we have evidence E=e. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

- We see new evidence E′ = e′. Value if we act then:

$$\text{MEU}(e,e') = \max_a \sum_s P(s|e,e')\, U(s,a)$$

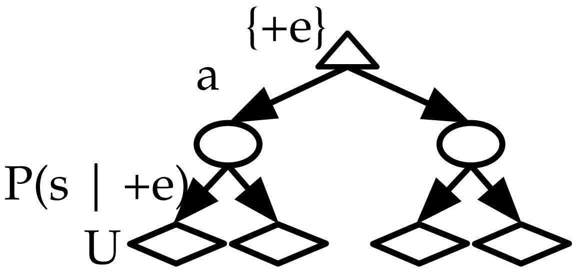
- BUT E′ is a random variable whose value is unknown, so we don't know what e′ will be.

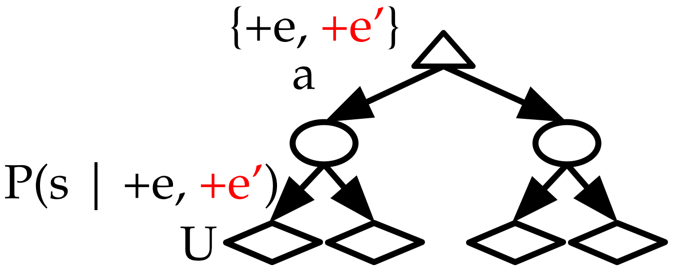
- Expected value if E′ is revealed and then we act:

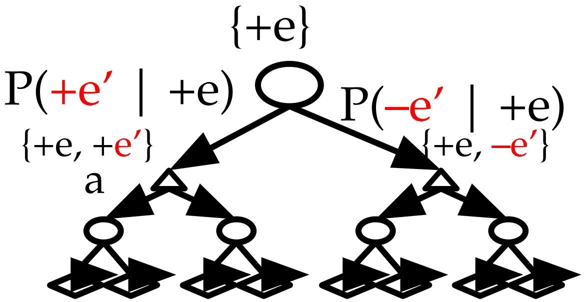
$$\text{MEU}(e,E') = \sum_{e'} P(e'|e)\text{MEU}(e,e')$$

- Value of information: how much MEU goes up by revealing E′ first then acting, as opposed to acting now:

$$\text{VPI}(E'|e) = \text{MEU}(e,E') - \text{MEU}(e)$$







Value of Information

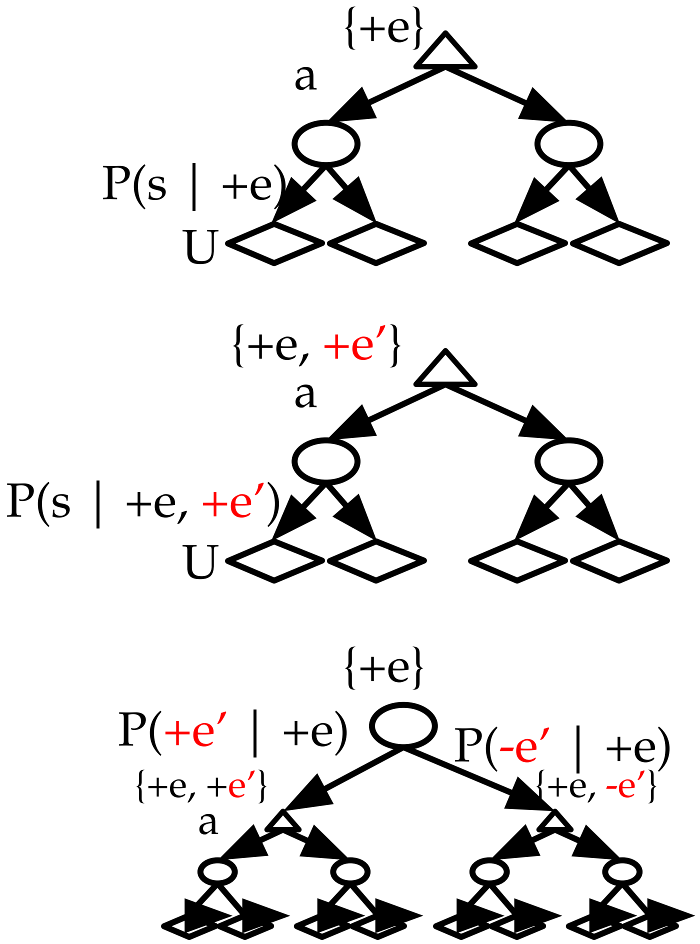
$$\text{MEU}(e, E') = \sum_{e'} P(e'|e)\text{MEU}(e, e')$$

$$= \sum_{e'} P(e'|e) \max_a \sum_s P(s|e, e')U(s, a)$$

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s, a)$$

$$= \max_a \sum_{e'} \sum_s P(s, e'|e)U(s, a)$$

$$= \max_a \sum_{e'} P(e|e') \sum_s P(s|e, e')U(s, a)$$



VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$

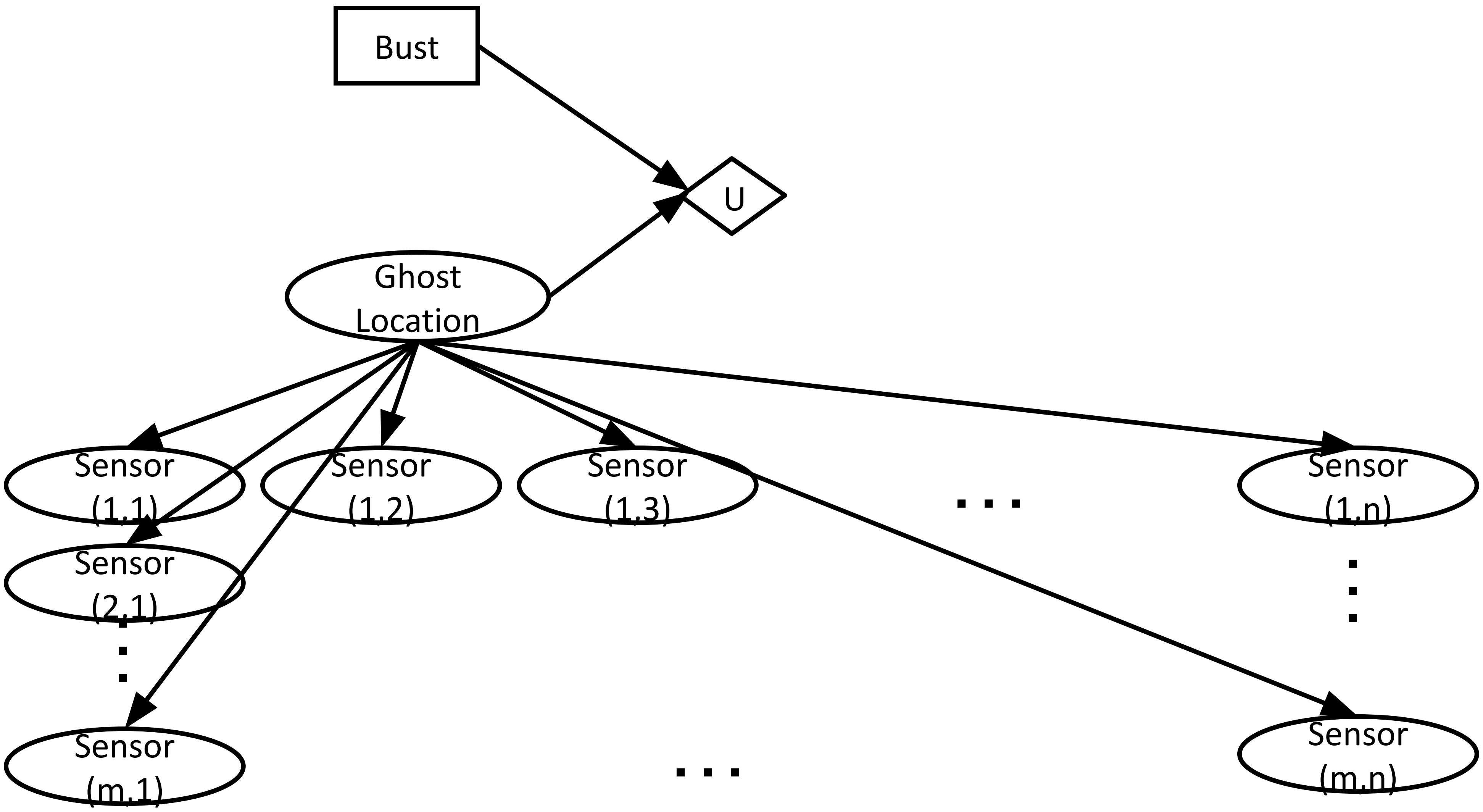
- Nonadditive
  (think of observing E_j twice)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$

- Order-independent

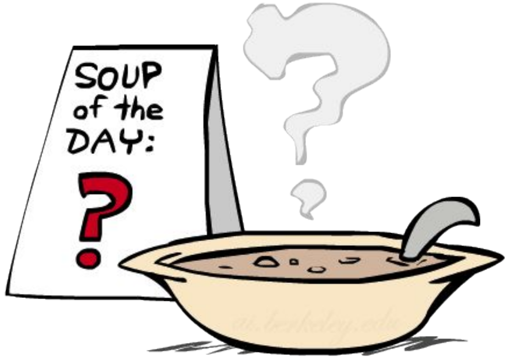
$$\text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j)$$
$$= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k)$$

Ghostbusters Decision Network

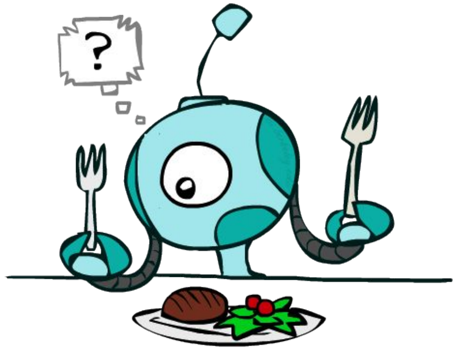


Quick VPI Questions

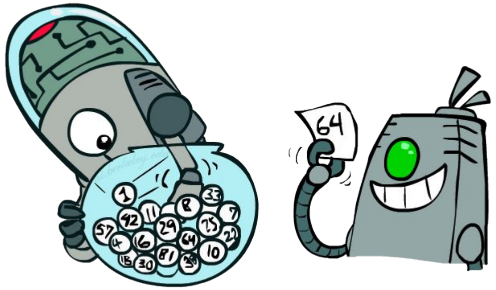
- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?



- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?



- You're playing the lottery. The prize will be \$0 or \$100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?

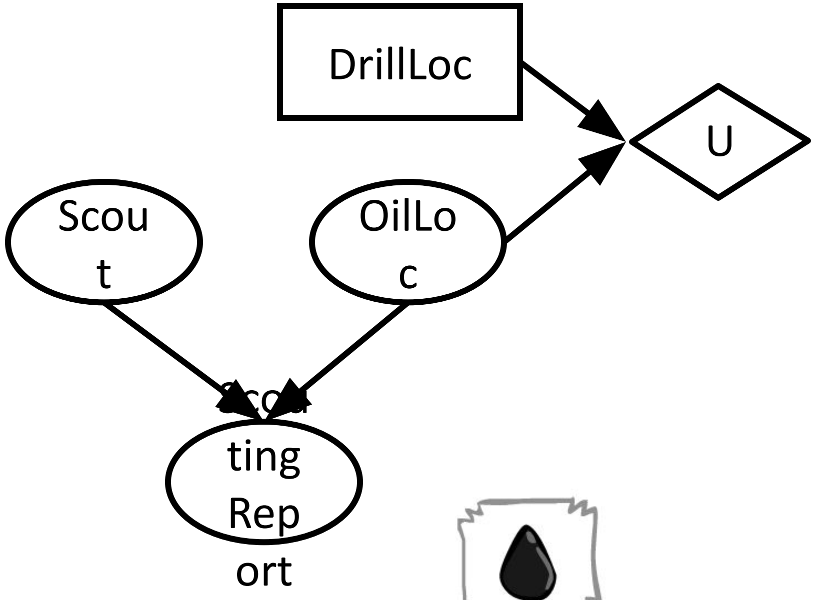


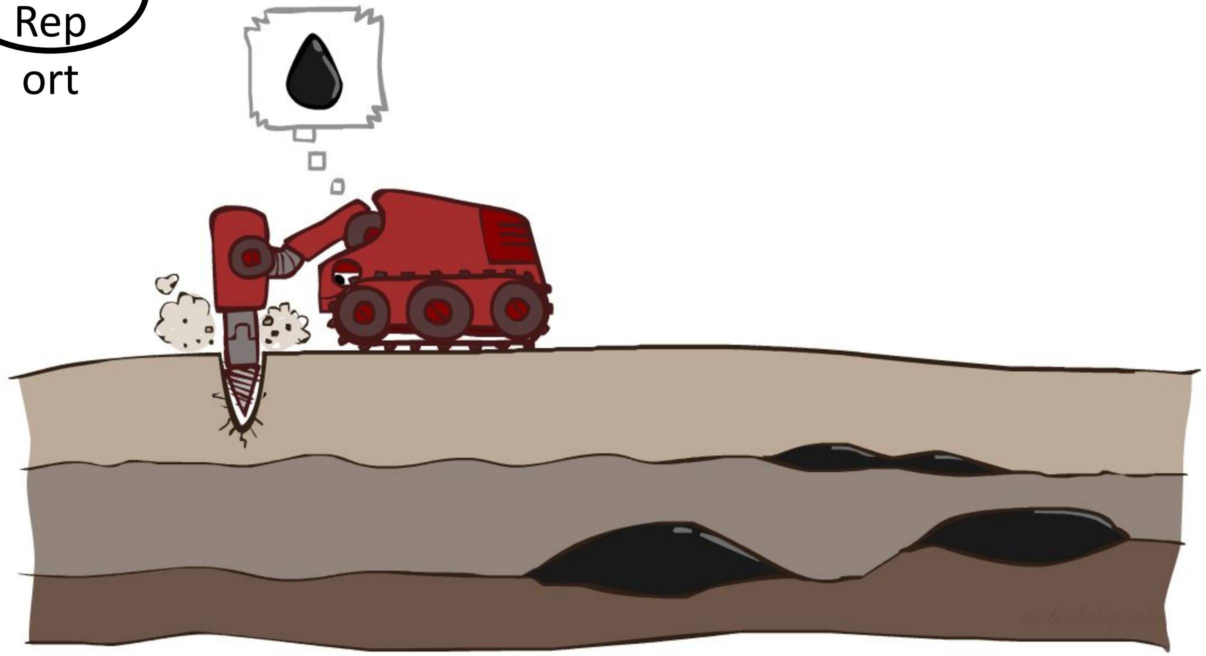
Value of Imperfect Information?

- No such thing
- Information corresponds to the observation of a node in the decision network
- If data is "noisy" that just means we don't observe the original variable, but another variable which is a noisy version of the original one

VPI Question

- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Scout) ?
- VPI(Scout | ScoutingReport) ?





- Generally:

  If $\quad \text{Parents}(U) \perp\!\!\!\perp Z \mid \text{CurrentEvidence}$
  Then $\quad \text{VPI}(Z \mid \text{CurrentEvidence}) = 0$