Hypothesis Testing

October 26, 2022

Today

- Hypothesis testing in two different contexts:
  - Difference of means tests
  - Difference of proportions tests
- Will talk about chi-square next week

Difference of Means

```
. tab married, sum(tvhours)
```

Is R married?	Summary of Hours per day watching TV Mean	Std. Dev.	Freq.
No	3.2383268	2.5831093	1028
Yes	2.611691	1.8652487	958
Total	2.9360524	2.2864047	1986

Inferences about Population mean

- Interested in population mean (μ), but we do not know μ, so we make inferences based on the sample mean ($\bar{x}$)
- Spread of sampling distribution (of $\bar{x}$) affected by standard deviation of population (σ)
- But, we don't know σ, so we use the standard deviation of the sample (s)
- Standard Error
  - When the standard deviation of a statistic is estimated from the data, called standard error
  - $SE_{\bar{x}} = \frac{s}{\sqrt{n}}$

T-statistic

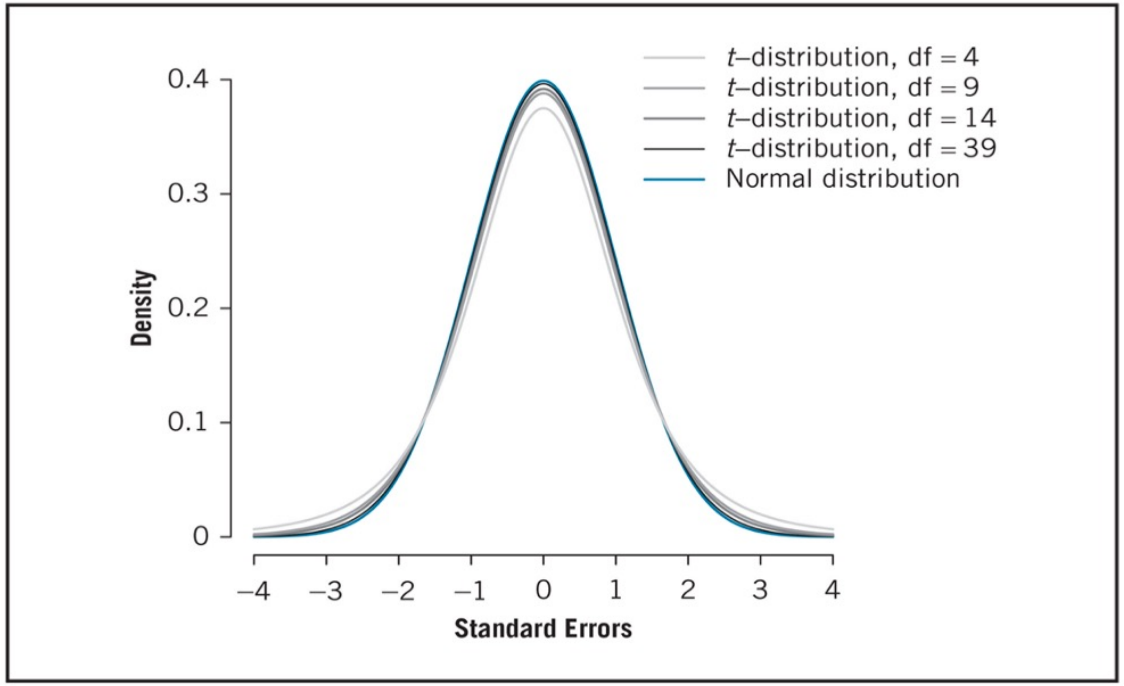
- If we know σ, and have a reasonable sample size, we can assume that sampling distribution is normal
- If we do not know σ, (and thus use our sample estimate s), we use the t-distribution
  - Varies based on degrees of freedom (n-1)
  - As *N* gets large (near 1000), very similar to a normal distribution
- The one-sample t statistic:
  - $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$
    - has the t-distribution with n-1 degrees of freedom

T-Distribution

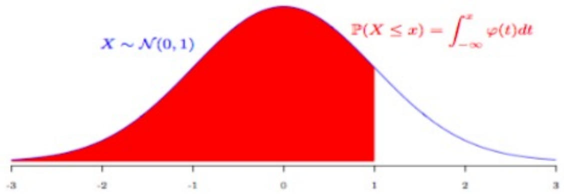
- The T-distribution is symmetric around 0
- The T-distribution has thicker tails and is more spread out than the normal distribution
- The shape depends on the "degrees of freedom" (*df*).
    - $df = n-1$

Student's t-distribution

Figure 6-11 *t*-Distributions Compared to a Standard Normal Distribution



Z-table

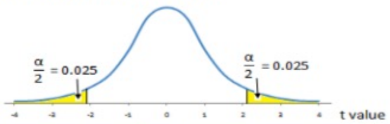


	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

T-table

Student's t Distribution Table

For example, the t value for 18 degrees of freedom is 2.101 for 95% confidence interval (**2-Tail** $\alpha = 0.05$).



	90%	95%	97.5%	99%	99.5%	99.95%	1-Tail Confidence Level
	80%	90%	95%	98%	99%	99.9%	2-Tail Confidence Level
	0.100	0.050	0.025	0.010	0.005	0.0005	1-Tail Alpha
df	0.20	0.10	0.05	0.02	0.01	0.001	2-Tail Alpha
1	3.0777	6.3138	12.7062	31.8205	63.6567	636.6192	
2	1.8856	2.9200	4.3027	6.9646	9.9248	31.5991	
3	1.6377	2.3534	3.1824	4.5407	5.8409	12.9240	
4	1.5332	2.1318	2.7764	3.7469	4.6041	8.6103	
5	1.4759	2.0150	2.5706	3.3649	4.0321	6.8688	
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.9588	
7	1.4149	1.8946	2.3646	2.9980	3.4995	5.4079	
8	1.3968	1.8595	2.3060	2.8965	3.3554	5.0413	
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.7809	
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.5869	
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.4370	
12	1.3562	1.7823	2.1788	2.6810	3.0545	4.3178	
13	1.3502	1.7709	2.1604	2.6503	3.0123	4.2208	
14	1.3450	1.7613	2.1448	2.6245	2.9768	4.1405	
15	1.3406	1.7531	2.1314	2.6025	2.9467	4.0728	
16	1.3368	1.7459	2.1199	2.5835	2.9208	4.0150	
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.9651	
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.9216	
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.8834	
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.8495	
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.8193	
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.7921	
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.7676	
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.7454	
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.7251	
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.7066	
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.6896	
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.6739	
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.6594	
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.6460	

Confidence Intervals

- We can use the t-statistic to generate a confidence interval for an unknown population mean (μ) based on $\bar{x}$
  - Margin of error when we use s instead of σ:
    - m=(t)($\frac{s}{\sqrt{n}}$)
- So, a level-C confidence interval for μ is:
  - $\bar{x}$ +/- (t)($\frac{s}{\sqrt{n}}$)

One-sample t test

- We can use a t-test to tell whether $\bar{x}$ is significantly different from some population mean (μ)
- Step 1: determine what μ would be if H_0 were true: μ_0
- Step 2: determine t-statistic
  - $t = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$
- Then, we can develop a p-value based on a random variable, T, having the $t(n\text{-}1)$ distribution:
  - If H_a is that $\mu>\mu_0$: $p(T\geq t)$
  - If H_a is that $\mu<\mu_0$: $p(T\leq t)$
  - If H_a is that $\mu\neq\mu_0$: $2p(T\geq|t|)$

Example

- H_0: μ=4
- H_a: μ>4
- n=9
- $\bar{x}$ =6
- s=3
- What's the p-value?

Comparing Two Means

- We are often not interested in whether the population mean is different from some specified value, but rather whether two groups have significantly different means
  - Ex—average income among men and women
- We can treat these groups as two different populations
- Then, we want to determine whether μ_1 is significantly different from μ_2.
- If we take a sample from each population, then have $\bar{x}_1$ and $\bar{x}_2$, which are estimates of μ_1 and μ_2.
- To determine likelihood that μ_1 is different from μ_2, need standard error of the difference
  - Variance of the difference ($\bar{x}_1$- $\bar{x}_2$) is the sum of their variances:
    - $\frac{\sigma^2{}_1}{n_1} + \frac{\sigma^2{}_2}{n_2}$

Z-statistic for difference of means

- If we know σ, we can compare the difference in the sample to the difference in the population by:
  - Subtracting the difference in the sample from the difference in the population and then dividing that by the square root of difference in variance
    - $\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma^2{}_1}{n_1} + \frac{\sigma^2{}_2}{n_2}}}$
  - That will give you a z-score, can use the normal distribution to determine p-value
- But, we usually don’t know population means (and the spread of their distributions)

T-statistic for difference of means

- If we don't know the population-mean, we need to use the *t*-statistic
- For the t-statistic, we substitute s_1 (the standard deviation of $\bar{x}_1$) for σ_1 and s_2 (the standard deviation of $\bar{x}_2$) for σ_2
- Need to determine degrees of freedom (*k*), but remember *n* for each group can be different
- Two choices:
  - Use value of *k* that is calculated from the data (if using statistical software)
  - Use *k* equal to the smaller of (n_1-1) and (n_2-1)

Two-sample t-test

- Suppose an SRS of size n_1 is drawn from a population with unknown mean μ_1 and an independent SRS of size n_2 is drawn from another population with unknown mean μ_2
- Null hypothesis is μ_0=0, so determine the p-value that we would get difference between $\bar{x}_1$ and $\bar{x}_2$ if the null were true
  - $t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{{s^2}_1}{n_1} + \frac{{s^2}_2}{n_2}}}$
- Can also determine a confidence interval by taking the difference between $\bar{x}_1$ and $\bar{x}_2$ and multiplying t times the square root of the sum of the variances
  - $(\bar{x}_1 - \bar{x}_2)$ +/- $(t)(\sqrt{\frac{{s^2}_1}{n_1} + \frac{{s^2}_2}{n_2}})$

Two-sample t-test

```
. ttest tvhours, by(married)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
      No |    1028    3.238327     .080565    2.583109    3.080236    3.396418
     Yes |     958    2.611691    .0602634    1.865249    2.493427    2.729955
---------+--------------------------------------------------------------------
combined |    1986    2.936052    .0513054    2.286405    2.835434    3.036671
---------+--------------------------------------------------------------------
    diff |            .6266358    .1017324                .4271222    .8261494
------------------------------------------------------------------------------
    diff = mean(No) - mean(Yes)                                   t =   6.1596
Ho: diff = 0                                     degrees of freedom =     1984

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

Difference of Proportions

cappun2	R resides in South Nonsouth	South	Total
0	587 33.87	343 31.70	930 33.04
1	1,146 66.13	739 68.30	1,885 66.96
Total	1,733 100.00	1,082 100.00	2,815 100.00

Inference about Population Proportion

- We are interested in the population proportion (p), but we do not observe p, so we make inferences about p based on sample proportion ($\hat{p}$)

- Standard deviation of $\hat{p}$ is based on p
  - $\sigma_{\hat{p}} = \sqrt{\frac{(p)(1-p)}{n}}$

- If we do not know p, can estimate the standard error of p using the sample proportion ($\hat{p}$)
  - $SE_{\hat{p}} = \sqrt{\frac{(\hat{p})(1-\hat{p})}{n}}$

- If the sample size is large enough, then the sampling distribution is approximately normal

- So, the margin of error for confidence-level C is:
  - m=(z)($SE_{\hat{p}}$)

Significance test for a single proportion

- We can use the normal distribution (and a z-score) to test whether $\hat{p}$ is significantly different from some population proportion p
- Step 1: Determine what p would be if null hypothesis were true (p_0)
- Step 2: Determine z-score
  - $z = \frac{\hat{p} - p_0}{\sqrt{\frac{(p_0)(1-p_0)}{n}}}$
- Then, can develop a p-value based on a standard Normal random variable Z:
  - If H_a is that $p > p_0$: $p(Z \geq z)$
  - If H_a is that $p < p_0$: $p(Z \leq z)$
  - If H_a is that $p \neq p_0$: $2p(Z \geq |z|)$

Example

- H_0: p=0.5
- H_a: p>0.5
- N=25
- $\hat{p}$ =0.6
- What's the p-value?

Comparing Two Proportions

- We are often interested in whether two groups have different proportions
  - Ex—proportion of men and women intending to vote for specific candidate
- Again, we can treat these groups as two different populations
- Then, we are interested in whether p_1 is significantly different from p_2.
- If we take a sample from each population, then we have $\hat{p}_1$ and $\hat{p}_2$, which are estimates of p_1 and p_2.
- To determine likelihood p_1 is different from p_2, we need the standard error of the difference
  - Variance of the difference ($\hat{p}_1$- $\hat{p}_2$) is the sum of their variances
    - $\sigma_D = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

Confidence Interval for a difference in proportions

- Confidence interval for difference in proportions is an interval estimate for how large the difference is between proportions
- Estimate of difference in population proportions is:
  - $D = \hat{p}_1 - \hat{p}_2$
- Standard error of D is:
  - $SE_D = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
- Margin of error for confidence level-C is
  - $m = (z)(SE_D)$
- So, an approximate level-C confidence interval for $p_1 - p_2$ is:
  - D +/- m

Significance Test for Difference in Proportions

- If H_a is that there is some difference in the proportion of successes between the two populations, then
- H_0: $p_1=p_2$
- If $p_1=p_2$, then we can estimate it using $\hat{p}$, which is the pooled estimate such that
  - $\hat{p} = \frac{x_1+x_2}{n_1+n_2}$
- The standard error for D, assuming that the null hypothesis is true, is:
  - $SE_{D_p} = \sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}$

Significance Test for Difference in Proportions

- Using $\hat{p}$ and the standard error, can then compute z statistic:
  - $z = \frac{\hat{p}_1 - \hat{p}_2}{SE_{D_p}}$
- Can use z-statistic to find p-value
- Using a standard Normal random variable Z, the p-value for H_0 is:
  - If H_a is that $p_1 > p_2$: $p(Z \geq z)$
  - If H_a is that $p_1 < p_2$: $p(Z \leq z)$
  - If H_a is that $p_1 \neq p_2$: $2p(Z \geq |z|)$

Two-sample Difference of Proportions test

```
. prtest cappun2, by(south)

Two-sample test of proportions                  Nonsouth: Number of obs =      1733
                                                   South: Number of obs =      1082

    Variable |       Mean   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+------------------------------------------------------------------
    Nonsouth |    .661281   .0113688                      .6389986    .6835634
       South |   .6829945   .0141458                      .6552691    .7107198
-------------+------------------------------------------------------------------
        diff |  -.0217134   .0181481                     -.0572831    .0138562
             |  under Ho:   .0182241    -1.19   0.233
------------------------------------------------------------------------------
        diff = prop(Nonsouth) - prop(South)                       z =  -1.1915
    Ho: diff = 0

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(Z < z) = 0.1167         Pr(|Z| < |z|) = 0.2335          Pr(Z > z) = 0.8833
```

Next steps

- Next week, will discuss chi-square
- Will also focus on an application article that uses these approaches

- Midterm exam #2—week of November 7th
  - Everything since first midterm—e.g., probability theory, sampling distributions, difference of means & proportions, statistical significance & hypothesis testing, etc.