

# Conclusion

December 7, 2022

# Today

- Course Wrap-up and focus on evaluating research
- Brief discussion of logistic regression
  - Not expected to know logit, but should know basic interpretation (and how it differs from OLS) if you see it
- Evaluating research
  - 8 questions to ask to evaluate hypothesis testing
- Discussion of final exam

# Regression with non-interval DVs

- OLS Regression is generally advisable only with an interval DV
  - Although, the linear probability model (LPM) can be used
- Can use regression with non-interval DV, but it's not linear
- Dichotomous DV: Logit or Probit
  - Ex: civil war vs. no civil war
  - Ex: win before Supreme Court vs. loss before Court
- Nominal DV: Multinomial Logit or Multinomial Probit
  - Ex: violence, non-violent direct action, conventional politics, nothing
  - Ex: liberal decision, conservative decision, neither, holdover to next term
- Ordinal DV: Ordered Logit or Ordered Probit
  - Ex: no conflict, minor internal armed conflict, civil war, large-scale civil war
  - Ex: liberal decision, mixed decision, conservative decision

# Logistic Regression

- Basic logic of OLS regression: examine how changes in  $X$  affect the (expected) mean value of  $Y$
- Basic logic of logit: examine how changes in  $X$  affect probability of observing a “1” on a dichotomous variable
  - Ex—impact of an individual’s income on probability he/she will vote
  - Ex—impact of a country’s GDP (per capita) on the probability it will experience a civil war
- Key difference between OLS regression and logistic regression:
  - OLS generates predicted values of the DV
    - Predicted vote share for President Biden in a state based on the number of college graduates
  - Logistic regression generates predicted probabilities:
    - Probability an individual will vote (or not) based on her/his income, etc.

# Maximum Likelihood Estimation

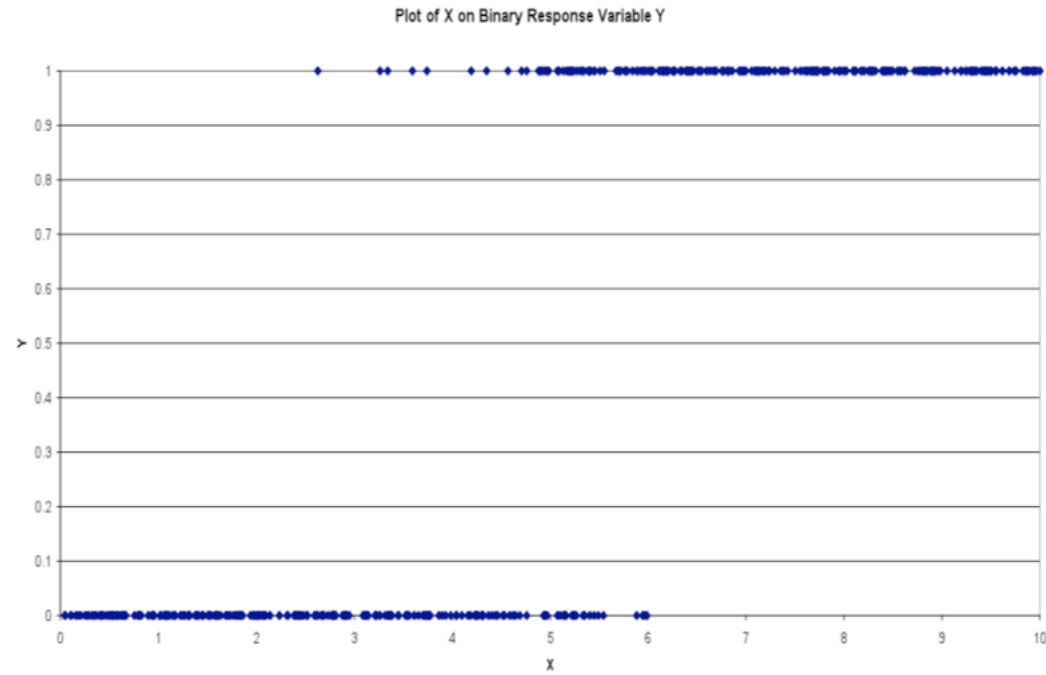
## Intuition

- Steps & Intuition
  - Start by thinking about the process that generated the data—that is, how was our dependent variable generated?
  - Then, we specify the probability distribution that describes the dependent variable
  - Our regression estimates are the values that maximize the probability (i.e., likelihood) that we observe the dependent variable,  $Y$ , that we actually observed
- Full details in GVPT 729a

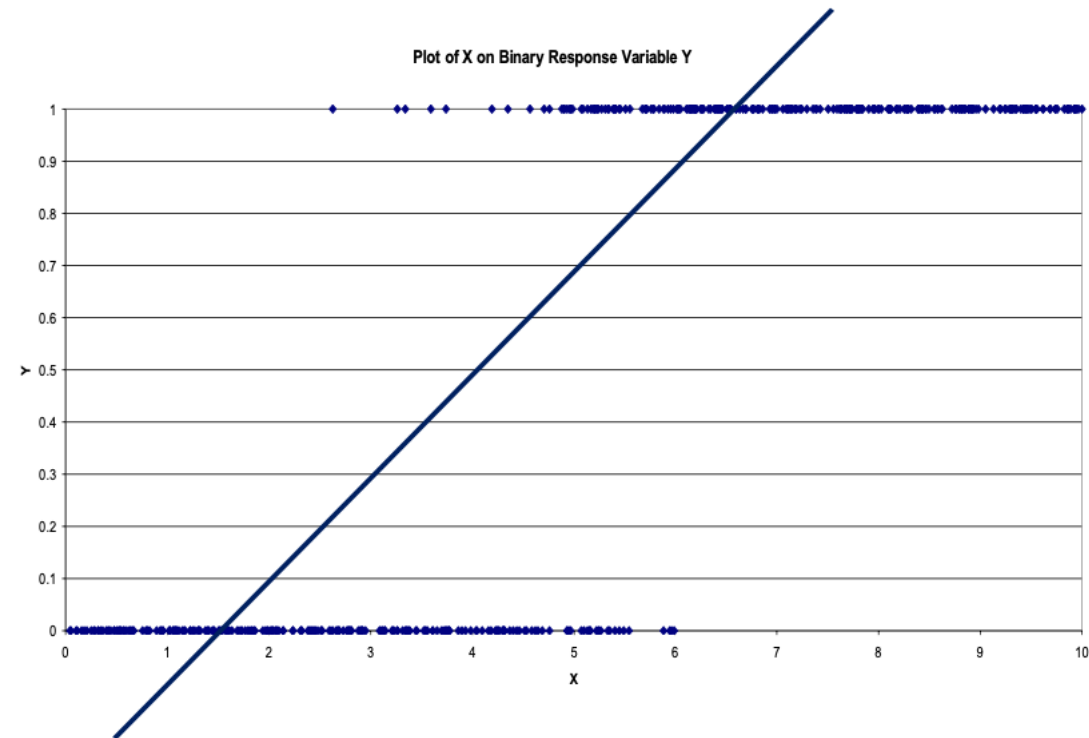
# Empirical Set-up

- For convenience, code the dependent variable to “1” (success) and “0” (failure)
  - This is generally arbitrary & we set the outcome of greatest interest to “1”
- Example—individual turnout (i.e., did a survey respondent vote?)
  - 1=reported voting
  - 0=reported not voting
- What might the data look like?

# Binary Response



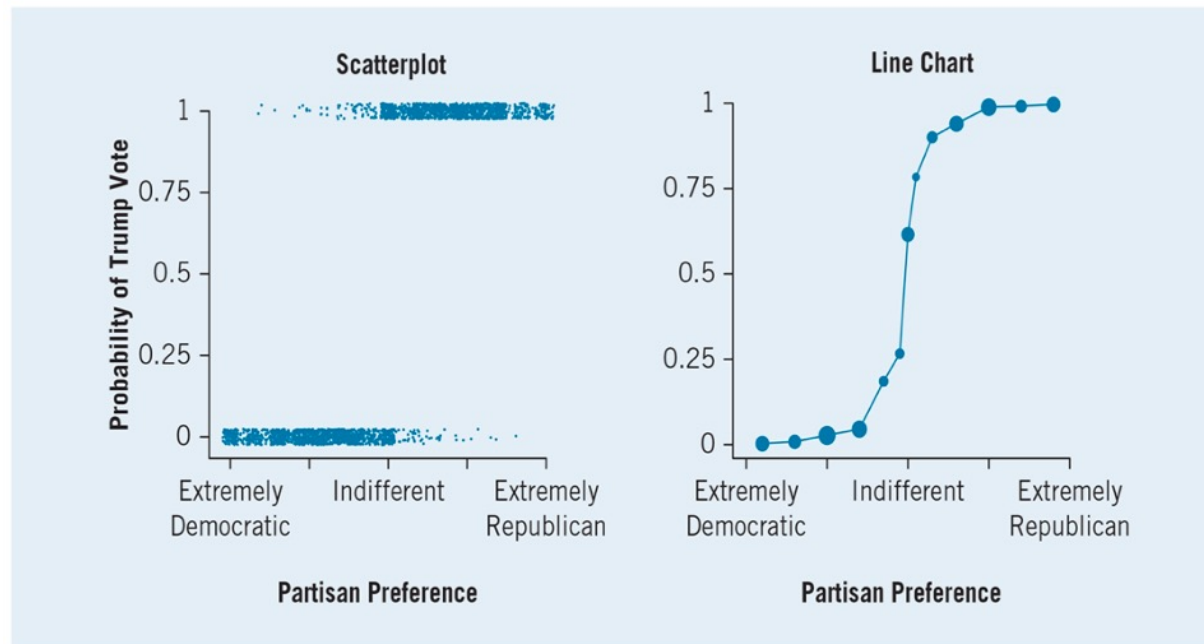
# Binary Response





# Binary Response

Figure 9-1 Observed Probabilities of Trump Vote by Partisan Preference



*Note:* The size of the line chart points is proportional to the number of observations in each group.

# Challenges with the Basic Linear Probability Model

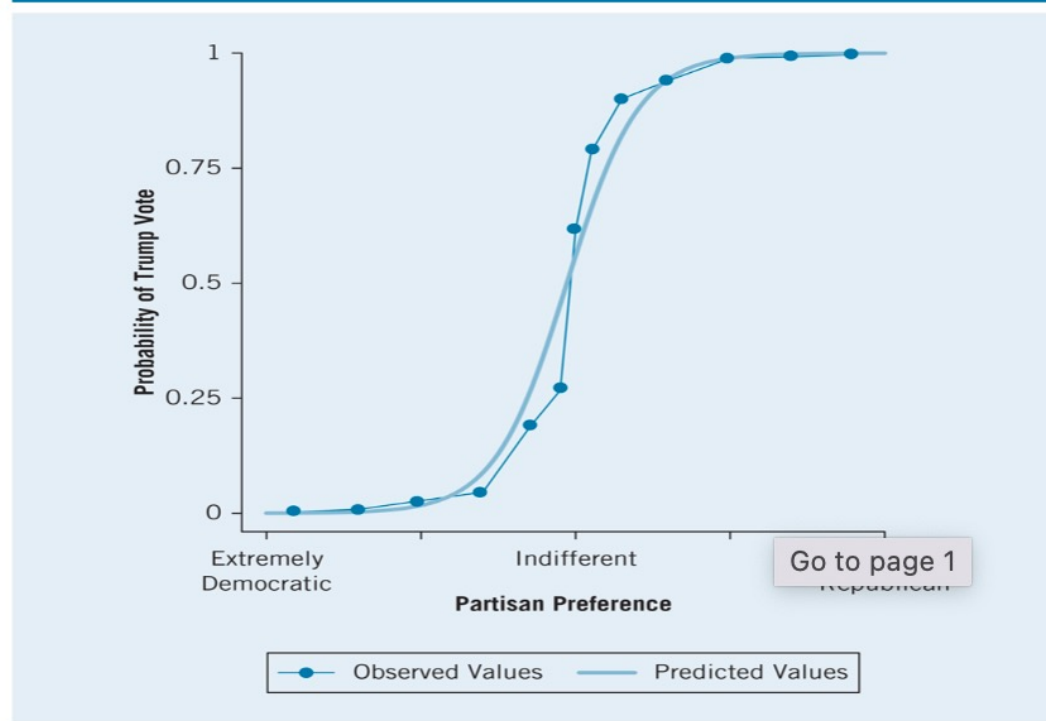
- OLS may produce nonsensical predictions—recall that OLS assumes continuous interval-level data
  - Predictions that exceed the bounds of 0 and 1
- Disturbances are not normally distributed—they follow the Bernoulli distribution
  - More problematic with smaller samples—i.e., hypothesis testing & inference
- $Y$  is distributed with a mean of  $P_i$  and variances of  $P_i(1 - P_i)$  and  $P_i$  is a function of the  $X$ 's—thus we have heteroscedasticity with variance related to the  $X$ 's
  - Although, we can adjust the standard errors accordingly, or use WLS
- Linearity assumption of OLS is suspect
  - A potentially serious problem—i.e. functional form issues
- Goodness of fit measures are even less helpful

# Logistic Regression

- Linear regression assumes that a one-unit change in  $X$  has the same effect on  $Y$  across all values of  $X$ 
  - But, this is not generally true for probabilities
  - Ex—a country going from \$1,000 to \$2,000 GDPpc reduces its risk of civil war more than a country going from \$20,000 to \$21,000 GDPpc
  - Logic—the probability of civil war at \$20,000 (per capita) is already low, hard to decrease much more

# Binary Response

**Figure 9-2 Predicted Probabilities of Trump Vote by Partisan Preference**



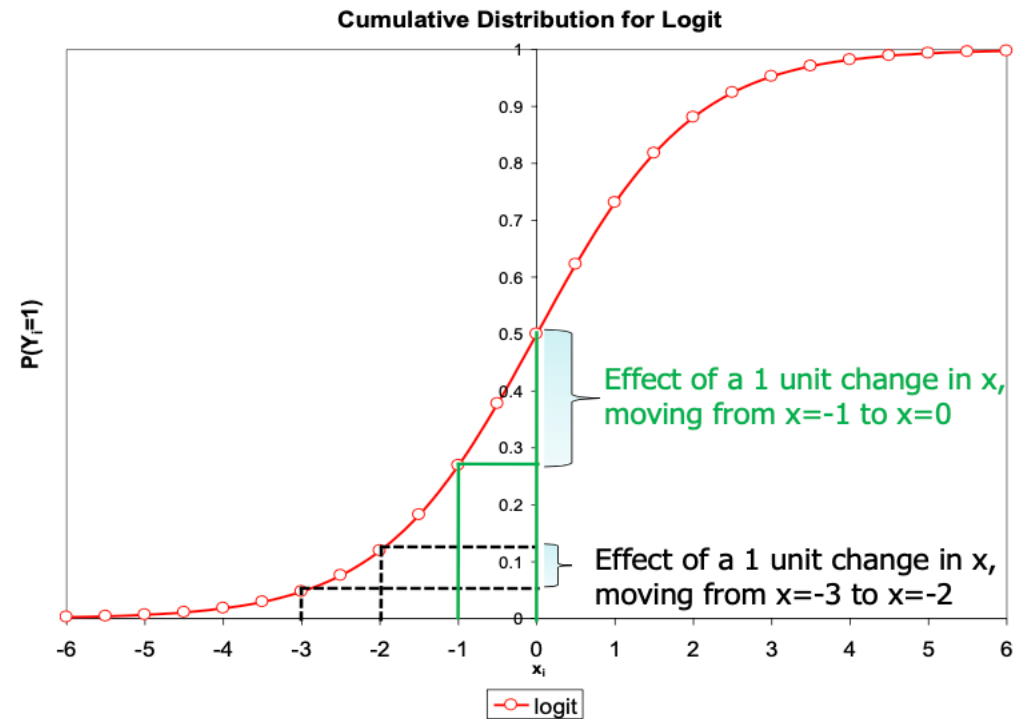
# An Important Point

- The model is inherently interactive in all of the variables
  - This is critical—the effect of a one-unit change in  $X$  is **NOT CONSTANT**—it depends on where we are on the curve
  - Compare this to OLS—the effect of  $\beta_k$  is constant across the entire range of the predictor
- The output tells us (assuming no interaction terms)
  - The sign of the effect of  $X_k$  on the probability of success
  - Overall statistical significance of the effect
- But, we do NOT directly interpret the coefficients as we did with OLS
  - We need to transform them in order to determine substantive significance

# Intuition Behind These Nonlinear Effects

- Note in the following CDF for logit that:
  - A one-unit shift in  $X$  when  $P_i$  is close to 0 results in a small change
  - The effect gets larger as we move closer to  $P_i = 0.5$
  - The effect then tapers as we get closer to  $P_i = 1$
- Thus, the initial probability of success plays an important role in the determination of the effect of changes in the independent variables
- Back to turnout example:
  - For those almost sure not to vote, mobilization will do little
  - For those close to 50-50, mobilization will have a large effect
  - For those almost sure to vote, mobilization will do little

# Cumulative Distribution Function for Logit





# Probability and Odds

- Logistic regression uses odds, which are a function of probabilities
  - Odds=probability of something happening/probability of it not happening
    - Probability of someone voting: .8
    - Probability of someone not voting: .2
    - Odds: 4 (i.e., 4 to 1 odds that the person will vote)
- Logistic regression makes predictions about the odds of observing the DV at different values of the IV
- Interested in the odds ratio—that is, the odds of observing the DV at one value of the IV compared to the next value of the IV
  - Ex. odds of a country with a GDPpc of \$5,000 having a civil war compared to a country with a GDPpc of \$4,000



# Odds Ratios

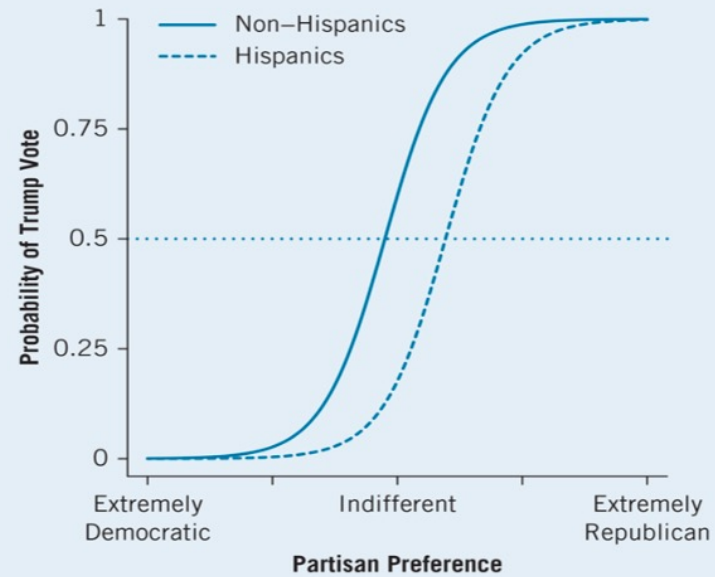
- The difference between OLS regression and logistic regression:
  - OLS regression—changes in the value of the IV have a consistent effect on the values of the DV
  - Logistic regression—changes in the value of the IV have a consistent effect on the odds ratio of the DV
- The logistic regression equation is:
  - $\text{Log odds}(Y) = \beta_0 + \beta_1(X_1)$
- We can use the logistic regression equation to make predictions about the probability of observing the DV based on specific values of the IV
  - Find the odds (by taking the exponent of the log-odds) to determine the probability
    - $e^{\log Odds} = \text{Odds}$
    - $\text{Probability} = \frac{\text{Odds}}{1 + \text{Odds}}$

# Logistic Regression with Multiple I.V.s

- Just like OLS, Logistic regression is easily extended to use multiple IVs
  - $\text{Log odds}(Y) = \beta_0 + \beta_1(X_1) + \beta_2(X_2) + \beta_3(X_3) + \dots + \beta_k(X_k)$
- In multiple logistic regression, the predicted log odds are a function of the values of all of the IVs
- The coefficient on each IV tells us the impact of a change in that variable on the log odds of the DV, controlling for the other variables
- Like OLS, separate IVs are assumed to be additive (even though the functional form is inherently non-linear)
- If effect is interactive, need to include a multiplicative term

# Binary Response

**Figure 9-4 Predicted Probabilities at Representative Values of an Independent Variable**



Source: 2016 American National Election Study

# Inference and Logistic Regression

- Statistical inference in logit is very similar to OLS
  - Null hypothesis:  $\beta_1 = 0$
  - Each coefficient has a standard error
  - Can use standard error to find the p-value
- In logit, we can use the normal distribution, so not affected by degrees of freedom
  - So, we find z-statistics (and not t-statistics)
- Can also use the standard error to estimate a confidence interval for  $\beta_1$

# Logistic Regression Output from R

```
Call:
glm(formula = own_dog ~ prochoice_scale + dem_age6 + obama_therm,
    family = "binomial", data = nes)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-1.5525	-1.0352	-0.8837	1.2476	1.7858

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	0.730192	0.181920	4.014	5.97e-05 ***
prochoice_scale	-0.005875	0.003192	-1.841	0.06568 .
dem_age630-39	0.125267	0.154726	0.810	0.41816
dem_age640-49	0.224402	0.160981	1.394	0.16333
dem_age650-59	0.035993	0.156161	0.230	0.81771
dem_age660-69	-0.202141	0.176782	-1.143	0.25285
dem_age670-older	-0.613725	0.212812	-2.884	0.00393 **
obama_therm	-0.011200	0.001719	-6.516	7.21e-11 ***

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

# Evaluating Research

- We have focused a lot on hypothesis testing
- Much of political science designed to test hypotheses
- Let's discuss several big-picture questions to ask about hypothesis testing when evaluating research
- First steps:
  - Is the theory logically consistent?
  - Do hypotheses flow logically from the theory?
- Several other questions to ask...

# Hypothesis Testing

(1) Do the variables properly measure the concepts?

- Remember—a hypothesis is prediction about the relationship between concepts
- Hypothesis testing will evaluate the relationship between measures
  - If measures and concepts are not highly correlated, results of analysis don't tell you much
- Want to ask—is a measure reliable? Is it valid?
- Also important to think about level of measurement—is it appropriate?

# Hypothesis Testing

(2) Is the sample unbiased?

- Hypothesis predicts relationship between variables in population
- Generally will only observe a sample, not the population
- Are some members of the population systematically more likely to end up in sample?
- Related question—what can you generalize to?
  - Ex—relationship between democracy and war?
    - Should it apply during 19<sup>th</sup> century?
    - In the future?



# Hypothesis Testing

(3) Does study control for other factors that could affect relationship between IV and DV?

- Remember, test of hypothesis will examine correlation between IV and DV
- In non-experimental design consider what determines the distribution of values of the IV
- If factors that affect distribution of the IV also affect distribution of the DV, can lead to omitted variable bias
  - Ex—Ice cream sales and drownings

# Hypothesis Testing

(4) Are the statistical techniques appropriate?

- Remember—appropriate techniques a function of measurement of variables
  - Ex—OLS is only appropriate if DV is interval (IV can be interval or dichotomous)
  - Ex—Difference of proportions only appropriate if DV is dichotomous, IV is dichotomous
- Also want to consider distribution of variable
  - Ex—mean is less useful if variable is skewed
  - Regression focused on mean—if mean is not useful, regression may not be appropriate

# Hypothesis Testing

(5) Are the assumptions underlying the statistical models met?

- All statistical tests have assumptions, important to consider whether they are violated and the potential consequences
- Ex—linear regression assumes a linear relationship between the IV and DV
  - If not, linear regression is not appropriate
- Ex—regression assumes errors are uncorrelated
  - May not be true, if not, need to do something about it

# Hypothesis Testing

(6) Is the interpretation of the analysis correct?

- Are there basic mistakes?
  - This can happen surprisingly frequently
- Interpretation can be complicated
  - Ex-Interaction terms in non-linear models (such as logit)

# Hypothesis Testing

(7) How big (and substantively meaningful) is the effect?

- Many studies focus excessively on statistical significance
- Important—mostly interested in substantive effect
  - Ex—getting a Ph.D. increases average income by \$1000 ( $p=0.001$ )
  - Ex—for every additional year of schooling, average incoming increases by \$50000 ( $p=0.001$ )
- Some factors that are statistically significant have little substantive effect/importance

# Hypothesis Testing

(8) Would the results hold up to replication?

- Remember, 95% significance level means 5% chance we will reject the null hypothesis when it is actually correct
  - True even if sample is representative of population, concepts perfectly measured, etc.
- Important to replicate studies—chances of committing Type I error twice are dramatically lower

# Final Exam

- Final exam posted online by Wednesday December 14
- Due by Monday, December 19<sup>th</sup>, 12:00 noon
- Same format as first two exams
- Not cumulative, focused only on material since 2<sup>nd</sup> midterm
- Questions?