Chapter 5: Quiz 4

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1.) Use midpoints with the given value of n to approximate the integral

$$\int_0^4 (x-1)^2 \, dx, \quad n = 4.$$

Compute Δx:

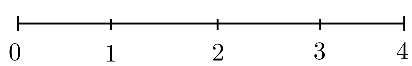
If:

$$\Delta x = \frac{b-a}{n}.$$

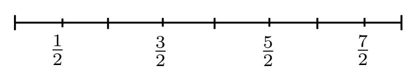
Then:

$$\begin{aligned}\Delta x &= \frac{4-0}{4} \\ &= 1.\end{aligned}$$

Now that we have Δx, we can construct a numberline with right endpoints:



From here if we divide Δx by 2 and add this number to each point we can construct the number line for our midpoints:



Now by the Riemann sum, which states:

$$M_4 = \sum_{i=1}^{n} \Delta x f(x_i)\ .$$

We have:

$$\begin{aligned} & 1\left(f\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right)\right) \\ &= \left(\left(\frac{1}{2}-1\right)^2 + \left(\frac{3}{2}-1\right)^2 + \left(\frac{5}{2}-1\right)^2 + \left(\frac{7}{2}-1\right)^2\right) \\ &= \left(\frac{1}{4} + \frac{1}{4} + \frac{9}{4} + \frac{25}{4}\right) \\ &= \frac{36}{4} \\ &\boxed{= 9}. \end{aligned}$$

2.) Given that

$$\int_0^\pi \sin^2 x \ dx, = \frac{\pi}{2}.$$

Find:

$$\int_0^\pi (x + \sin^2 x) \ dx.$$

Using the property of integrals, which states:

$$\int_a^b \left[f(x) + g(x)\right] \ dx = \int_a^b f(x) \ dx + \int_a^b g(x) \ dx.$$

We can write the equation as:

$$\begin{aligned} \int_0^\pi x \ dx + \int_0^\pi (\sin^2 x) \ dx \\ = \int_0^\pi x \ dx + \frac{\pi}{2}. \end{aligned}$$

And if we use the fundemental theorem of calculus to evaluate $\int_0^\pi x \ dx$, we get:

$$\begin{aligned} \frac{1}{2}x^2\Big]_0^\pi \\ = \left(\frac{1}{2}(\pi)^2\right) - \left(\frac{1}{2}(0)^2\right) \\ = \frac{\pi^2}{2}. \end{aligned}$$

Therefore:

$$\begin{aligned} \int_0^\pi x \ dx + \int_0^\pi (\sin^2 x) \ dx \\ \boxed{\frac{\pi^2}{2} + \frac{\pi}{2}}. \end{aligned}$$

3.) Given

$$\int_1^4 f(t)\ dt = -5 \ and \ \int_1^2 2f(t)\ dt = -1.$$

Use the properties of integrals to compute

$$\int_2^4 f(t)\ dt.$$

To start, we can utilize the property:

$$\int_a^b cf(x)\ dx = c \cdot \int_a^b f(x)\ dx.$$

To see that

$$\begin{aligned}\int_1^2 2f(t)\ dt &= -1 \\ &= 2(-1) \\ &= -2.\end{aligned}$$

Using the property, which states:

$$\int_a^c f(x)\ dx = \int_a^b f(x)\ dx + \int_b^c f(x)\ dx.$$

We can deduce that $a = 1$ $b = 2$ and $c = 4$, from this logic we can see that our first integral is $\int_a^c$, and our second integral is $\int_a^b$, and we are asked to find $\int_b^c$

Therefore:

$$-5 = -2 + \int_b^c f(t)\ dt.$$

And if we let $\int_b^c f(t)\ dt = x$, and solve for x:

$$\begin{aligned}-5 &= -2 + x \\ \boxed{x = -3}&.\end{aligned}$$

4.) Use part one of the fundemental theorem of calculus to find the derivative of:

$$h(x) = \int_t^3 \frac{1}{1+x^2}\, dx.$$

To start, we must use the property:

$$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx.$$

To flip the limits of integration such that the upper limit is a funtion of x, so:

$$-\int_3^t \frac{1}{1+x^2}\, dx.$$

From here we can use part one of the fundemental theorem of calculus to find the derivative, so:

$$\begin{aligned} h'(x) &= \frac{d}{dt} - \int_3^t \frac{1}{1+x^2}\, dx \\ &\boxed{= -\frac{1}{1+t^2}}. \end{aligned}$$

5.) Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral

$$\int_{-1}^{1} (1 - x^2)\ dx.$$

If first we find the indefinite integral:

$$\begin{aligned} &\int (1 - x^2)\ dx \\ &= 1x - \frac{1}{3}x^3. \end{aligned}$$

We can then use the fundemental theorem of calculus to evalute the integral

So:

$$\begin{aligned} & 1x - \frac{1}{3}x^3 \Big]_{-1}^{1} \\ &= \left(1(1) - \frac{1}{3}\left(1\right)^3\right) - \left(1(-1) - \frac{1}{3}\left(-1\right)^3\right) \\ &= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) \\ &= \frac{2}{3} - \left(-\frac{2}{3}\right) \\ &\boxed{= \frac{4}{3}}. \end{aligned}$$