Calculus 2

Chapter 3

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Contents

Techniques of Integration

3.1 Integration by Parts

Definition 1:

Many students want to know whether there is a product rule for integration. There isn't, but there is a technique based on the product rule for differentiation that allows us to exchange one integral for another. We call this technique **integration by parts.**

The Integration-by-Parts Formula

If, $h(x) = f(x)g(x)$, then by using the product rule, we obtain $h'(x) = f'(x)g(x) + g'(x)f(x)$. Although at first it may seem counterproductive, let's now integrate both sides of this equation:

$$\int h'(x)\,dx = \int \left(g(x)f'(x) + f(x)g'(x)\right)\,dx.$$

This gives us

$$h(x) = f(x)g(x) = \int g(x)f'(x)\,dx + \int f(x)g'(x)\,dx.$$

Now we solve for $\int f(x)g'(x)\,dx$:

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx.$$

By making the substitutions $u = f(x)$ and $v = g(x)$, which in turn make $du = f'(x)\,dx$ and $dv = g'(x)\,dx$, we have the more compact form

$$\int u\,dv = uv - \int v\,du.$$

Theorem 1: Integration by Parts

Let $u = f(x)$ and $v = g(x)$ be functions with continuous derivatives. Then, the integration-by-parts formula for the integral involving these two functions is:

$$\int u\,dv = uv - \int v\,du.$$

Example 1: Using Integration by Parts

Use integration by parts with $u = x$ and $dv = \sin x\,dx$ to evaluate

$$\int x \sin x\,dx.$$

Solution: So to use the formula:

$$\int \quad u\ dv = uv - \int v\ du.$$

We need:

$$\begin{aligned} u &= x \quad du = dx \\ dv &= \sin x dx \quad v = -\cos x. \end{aligned}$$

Thus:

$$\begin{aligned} \int x \sin x\ dx &= -x\cos x - \int -\cos x\ dx \\ &= -x\cos x + \sin x + C. \end{aligned}$$

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The natural question to ask at this point is: How do we know how to choose u and dv? Sometimes it is a matter of trial and error; however, the acronym **LIATE** can often help to take some of the guesswork out of our choices. This acronym stands for

- **L**ogarithmic Functions
- **I**nverse Trigonometric Functions
- **A**lgebraic Functions
- **T**rigonometric Functions
- **E**ponential Functions

This mnemonic serves as an aid in determining an appropriate choice for u.