Pendulum Periods

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Section 253D, 6:00 PM Tuesday

Contents

1 Theory

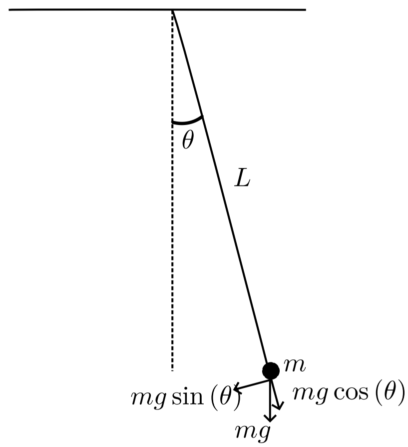


Figure 1: *Shows a simple pendulum*

In Figure 1, we observe the fundamental behavior of a pendulum. By securing one end of a string to a mass, m, and the other end to a ceiling point, it becomes clear through physical intuition that the pendulum will exhibit a consistent back-and-forth motion. Indeed, it does. When displaced at an angle, θ, from the vertical (represented by the dashed line in Figure 1), the mass follows an arc. The arc length, denoted as x, is defined by

$$x = L\theta$$

where x is the arc length, θ is the angle from the vertical in radians, and L is the string's length. Conveniently positioning our axes, we let the y-axis align with the string. This orientation simplifies breaking down the gravitational force into x and y-components, as depicted in Figure 1. The x-component acts leftward, serving as a "restoring force" that drives the system back to its initial position. This force is quantified as

$$F = -mg\sin(\theta)$$

The negative sign in Equation 2 indicates the nature of the restoring force. Given the dependency of Equations 1 and 2 on θ, we solve for θ in Equation 1:

$$\theta = \frac{x}{L}$$

Substituting this back into Equation 2 yields:

$$F = -mg\sin\left(\frac{x}{L}\right)$$

Here, we apply the "small angle approximation," valid for angles up to about 15°, where $\sin(\theta) \approx \theta$. This approximation simplifies our understanding of pendular motion and is demonstrated at the experiment's onset. Using this approximation, the force equation becomes:

$$F = -\frac{mg}{L}x$$

Remarkably, this mirrors Hooke's Law for harmonic motion, where $F = -kx$, and defining k as $\frac{mg}{L}$ confirms the analogy. To determine the pendulum's period, or the time for a complete cycle, we utilize the general formula for simple harmonic motion:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

With $k = \frac{mg}{L}$ from Equation 3, the formula simplifies, canceling out the mass, leaving the period dependent solely on L:

$$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{mg/L}} \\ &= 2\pi\sqrt{\frac{L}{g}} \end{aligned}$$

This reveals that neither mass nor displacement θ affects the period. Only L influences T, specifically through its square root, indicating a linear relationship between T^2 and L:

$$T^2 = \frac{4\pi^2}{g}L$$

1.1 Experimental Procedure

1. The pendulum is currently at a length of about 1 meter long with a 100 gram mass attached to it. The LoggerPro setup should already have a program set up to measure the period as this pendulum swings. Run a test trial by displacing the pendulum and letting it swing. When you press collect, it should start displaying the period during each of the swings.

2. Once you collect five data points on the graph, take the average of the five periods and record that average into the proper column in the table provided.

3. Repeat step 2 for 150 gram, 200 gram, and 250 gram masses.

4. Once complete with the masses, remove any excess mass from the bob, and start measuring the angle by which you displace the pendulum for each trial. You should have a measurement of 5°, 10°, 15°, and 20°. Again, record the average period for each of these displacements into the proper column in the table provided.

5. Adjust the length of the pendulum by wrapping it around a pole. Measure from the bottom of the pole to the top of the mass: this will be your new length L. Find the average over 5 periods and record the average for this new length.

6. Repeat Step 5 so that you get the average period of five different lengths. Record all of your data in the tables below.

2 Data

The data collected during the lab is split among three tables

Mass (g)	Avg Period (s)
100	1.882
150	1.879
200	1.879
250	1.879

Table 1: Average Periods for Different Masses

Amplitude (degrees)	Avg Period (s)
5	1.125
10	1.879
15	1.885
20	1.889

Table 2: Average Periods for Different Amplitudes

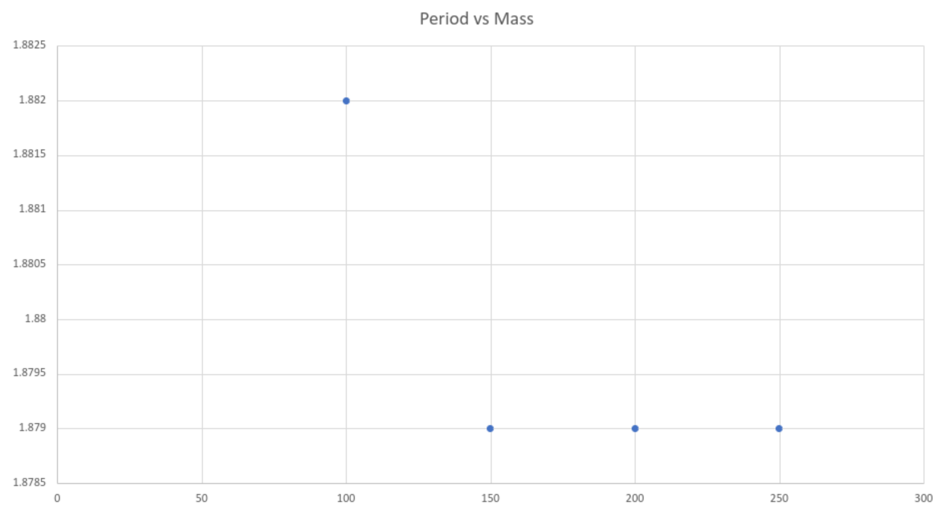
Length (m)	Avg Period (s)	Period Squared (sec^2)
0.81	1.879	3.53
0.73	1.8	3.24
0.62	1.669	2.786
0.5	1.516	2.298
0.44	1.453	2.11

Table 3: Average Periods and Period Squared for Different Lengths

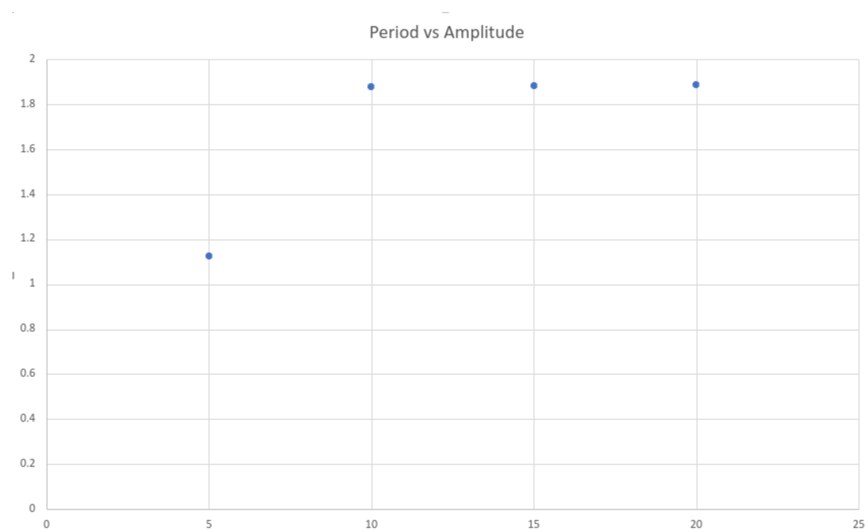
3 Results

We start the data analysis by contructing a scatter plot for the three tables listed above.

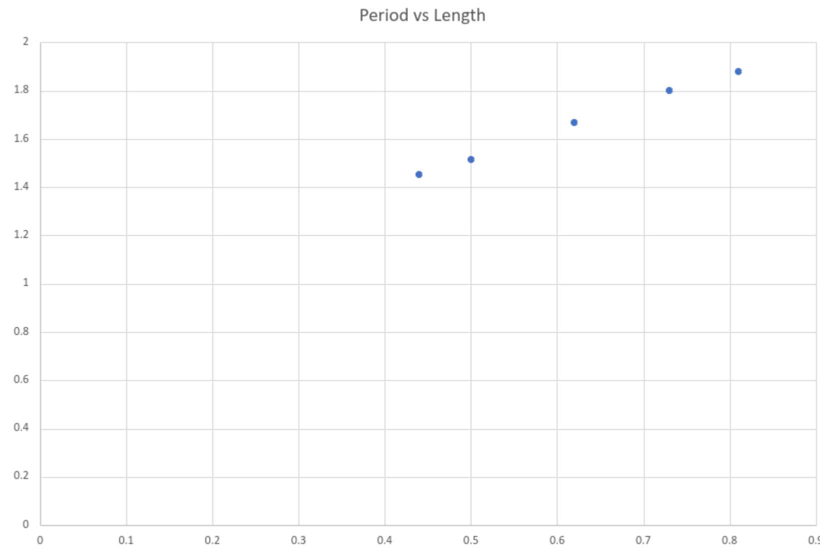
3.1 Scatter plot: Mass vs Average Period



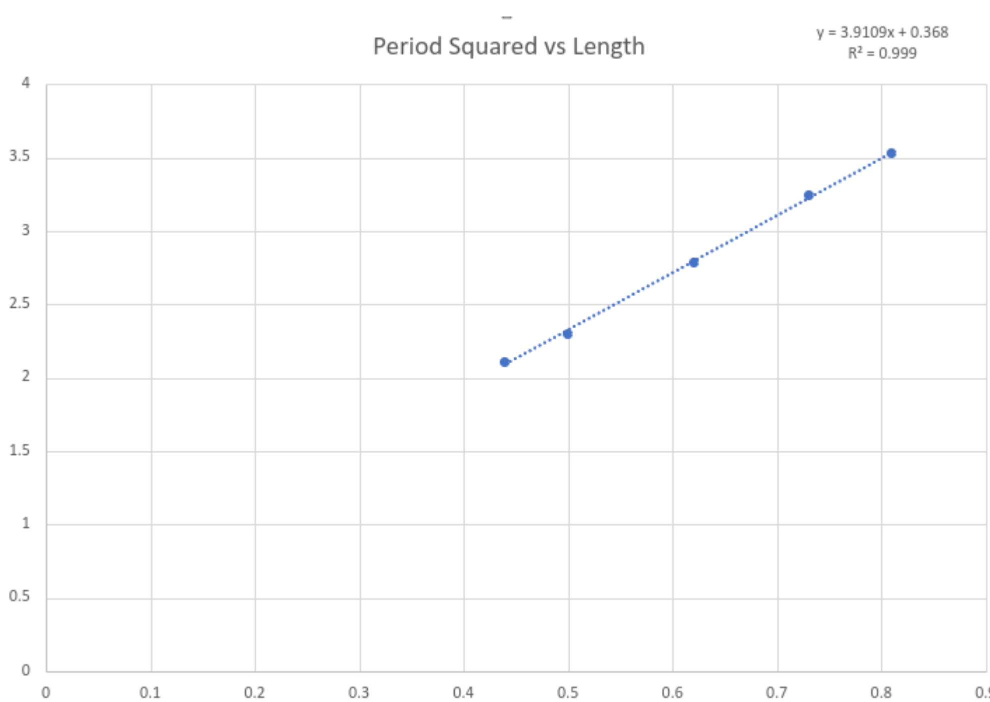
3.2 Scatter plot: Period vs Amplitude



3.3 Scatter plot: Period vs Length



3.4 Scatter plot: Period^2 vs Length



Equation of fitted line:

$$y = 3.9109x + 0.368, \quad \text{with } r^2 = 0.999.$$

3.5 Analysis of the period^2 vs length plot

Using equation six, and the slope found from the fitted line equation, we find gravity (g) to be

$$\begin{aligned} m = \frac{4\pi^2}{g} &\implies g = \frac{4\pi^2}{m} \\ &= \frac{4\pi^2}{3.9109} \\ &= 10.0945. \end{aligned}$$

Thus, in this lab we have calculated the quantity of gravity to be $10.0945\frac{m}{s^2}$. Using the accepted value $9.8\frac{m}{s^2}$, we compute the percent error with

$$\begin{aligned} \%\text{Err} &= \frac{|A-B|}{A} \cdot 100\% \\ &= \frac{|9.8 - 10.0945|}{9.8} \cdot 100\% \\ &= 3.005\%. \end{aligned}$$

4 Discussion

The experiments conducted on pendulum periods provided insights into the fundamental properties of harmonic motion. Notably, our results confirm that the mass of the pendulum does not significantly affect the period of the pendulum, consistent with the theoretical predictions for simple harmonic oscillators. As shown in the data and reinforced by the scatter plots, the period values remained relatively constant despite varying masses (1.882 s for 100 g and 1.879 s for 250 g). This observation supports the hypothesis that, for small angles, the period of a simple pendulum is independent of its mass, primarily influenced by the length of the string and the acceleration due to gravity.

Displacement from the vertical (angle of amplitude) also showed minimal influence on the period until reaching larger angles, which is expected under the small angle approximation. The theory suggests that $\sin(\theta) \approx \theta$ should hold true for angles up to about 15°, which our data corroborated, as periods did not vary drastically up to 20°. Regarding the value of gravity derived from the slope of the Period Squared versus Length plot ($10.0945\,\mathrm{m/s^2}$), it demonstrated a minor percent error (3.005%) relative to the accepted value of $9.8\,\mathrm{m/s^2}$. This discrepancy likely arises from errors in the experimental setup, such as the sensitivity of the photogate sensor or minor alignment issues in the pendulum's release mechanism, which could affect the initial angle measurement and the resultant period calculation.

5 Conclusion

In this laboratory experiment, we investigated the periodic behavior of a simple pendulum and explored how various factors such as mass, amplitude of swing, and length of the pendulum influence its period. Our findings show the classical theory of simple harmonic motion, demonstrating that the pendulum's period is independent of its mass and mostly not affected by the amplitude of swing, provided the swings are within small angles.