Problem 1. Use the Comparison Test or Limit Comparison Test to determine if the series converges.

$$\sum_{n=1}^{\infty} \frac{n^4+6}{n^5+4}.$$

Let $b_n = \frac{1}{n}$, which we know diverges. Since $\frac{n^4+6}{n^5+4} < \frac{1}{n}$. The simple comparison test will prove fruitless. Thus, we use the limit comparison test

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \frac{\frac{n^4+6}{n^5+4}}{\frac{1}{n}} \\
&= \lim_{n\to\infty} \frac{n(n^4+6)}{n^5+4} \\
&= \lim_{n\to\infty} \frac{n^5+6n}{n^5+4} \\
&= 1.
\end{aligned}$$

Since $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = L \neq 0,$ or ∞. Whatever b_n does, a_n will follow. Thus, $\sum\limits_{n=1}^{\infty} \frac{n^4+6}{n^5+4}$ Diverges

Problem 2. Use the Comparison Test or Limit Comparison Test to determine if the series converges.

$$\sum_{n=1}^{\infty} \frac{5^n-5}{5^n}.$$

Let $b_n = 1^n$, which we know diverges. Since $\frac{5^n-5}{5^n} < 1^n$. The simple comparison test will not be of use. So we try the limit comparison test.

$$\begin{aligned}
&\lim_{n\to\infty} \frac{a_n}{b_n} \\
&= \lim_{n\to\infty} \frac{\frac{5^n-5}{5^n}}{1^n} \\
&= \lim_{n\to\infty} \frac{5^n-5}{5^n} \\
&= \lim_{n\to\infty} \frac{\frac{5^n}{5^n}-\frac{5}{5^n}}{\frac{5^n}{5^n}} \\
&= 1.
\end{aligned}$$

Since $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = L \neq 0,$ or ∞. Whatever b_n does, a_n will follow. Thus, $\sum\limits_{n=1}^{\infty} \frac{5^n-5}{5^n}$ Diverges

Problem 3. Use the Comparison Test or Limit Comparison Test to determine if the series converges.

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5+5}}\ .$$

Let $b_n = \frac{1}{n^{\frac{3}{2}}}$ which, by the $p-series$, converges. Since $\frac{n}{\sqrt{n^5+5}} < \frac{1}{n^{\frac{3}{2}}}$. We can indeed use the simple comparison test, which tells us that $\frac{n}{\sqrt{n^5+5}}$ will also converge.

Problem 4. We want to use the Basic Comparison Test (sometimes called the Direct Comparison Test or just the Comparison Test) to determine if the series converges or diverges

$$\sum_{k=1}^{\infty} \frac{k^2}{k^3+11}\ .$$

Let $b_n = \frac{1}{k}$, which we know diverges. Because $\frac{k^2}{k^3+11} < \frac{1}{k}$, the simple comparison test is inconclusive (question didn't ask but limit comparison would show that they both diverge)

Problem 5. Use the Limit Comparison Test to determine if the series converges.

$$\sum_{k=1}^{\infty} \frac{k+12}{k(k-1)(k+2)}\ .$$

Let $b_n = \frac{1}{k^2}$ which, by the p-series, we know will converge. By the limit comparison test

$$\begin{aligned}
&\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{k+12}{k^3-k^2-2k}}{\frac{1}{k^2}} \\
&= \lim_{n\to\infty} \frac{k^2(k+12)}{k^3-k^2-2k} \\
&= \lim_{n\to\infty} \frac{k^3+12k^2}{k^3-k^2-2k} \\
&= 1.
\end{aligned}$$

Since $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = L \neq 0,$ or ∞. Whatever b_n does, a_n will follow. Thus, $\sum\limits_{n=1}^{\infty} \frac{k+12}{k(k-1)(k+2)}$ converges

Problem 6. Use the Limit Comparison Test to determine if the series converges.

$$\sum_{k=1}^{\infty} \frac{k+19}{k(k-3)}\ .$$

Let $b_n = \frac{1}{k}$, which we know diverges. By the limit comparison test

$$\begin{aligned}\lim_{n\to\infty} \frac{a_n}{b_n} &= \frac{\frac{k+19}{k^2-3k}}{\frac{1}{k}} \\ &= \lim_{n\to\infty} \frac{k(k+19)}{k^2-3k} \\ &= \lim_{n\to\infty} \frac{k^2+19k}{k^2-3k} \\ &= 1.\end{aligned}$$

Since $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = L \neq 0,$ or ∞. Whatever b_n does, a_n will follow. Thus, $\sum\limits_{n=1}^{\infty} \frac{k+19}{k(k-3)}$ diverges