Comprehensive Compendium:
Calculus II

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Contents

1 Calc II

1.1 Chapter 1 Key Equations

- **Mean Value Theorem For Integrals**: If $f(x)$ is continuous over an interval [a,b], then there is at least one point $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a} \int f(x)\ dx.$$

- **Integrals resulting in inverse trig functions**

1.

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{|a|} + C.$$

2.

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C.$$

3.

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{|a|} \sec^{-1} \frac{|x|}{a} + C.$$

1.2 Chapter 2 Key Terms / Ideas

- **Finding limits of integration for region between two functions**: Usually, we want our limits of integration to be the points where the functions intersect
- A **"complex region"** between curves usually refers to an area that is not easily described by a single, continuous function over the interval of interest.
- **compound regions** are regions bounded by the graphs of functions that cross one another
- **Cross-section:** The intersection of a plane and a solid object.
- a **cylinder** is a three-dimensional shape that has two parallel, congruent bases connected by a curved surface. The bases are usually circles, but they can be other shapes as well
- The line segment connecting the centers of the two bases is called the **"axis" of the cylinder.**
- **Slicing method:** A method of calculating the volume of a solid that involves cutting the solid into pieces, estimating the volume of each piece, then adding these estimates to arrive at an estimate of the total volume; as the number of slices goes to infinity, this estimate becomes an integral that gives the exact value of the volume.
    1. Examine the solid and determine the shape of a cross-section of the solid. It is often helpful to draw a picture if one is not provided.
    2. Determine a formula for the area of the cross-section.
    3. Integrate the area formula over the appropriate interval to get the volume.
- **Solid of revolution:** A solid generated by revolving a region in a plane around a line in that plane.
- **Disk method:** A special case of the slicing method used with solids of revolution when the slices are disks.
- A **Washer (Annuli)** is a disk with holes in the center.
- **Washer method:** A special case of the slicing method used with solids of revolution when the slices are washers.
- **Method of cylindrical shells:** A method of calculating the volume of a solid of revolution by dividing the solid into nested cylindrical shells; this method is different from the methods of disks or washers in that we integrate with respect to the opposite variable.
- **Arc length:** The arc length of a curve can be thought of as the distance a person would travel along the path of the curve.
- **Surface area:** The surface area of a solid is the total area of the outer layer of the object; for objects such as cubes or bricks, the surface area of the object is the sum of the areas of all of its faces.

1.3 Chapter 2 Key Equations

- **Area between two curves, integrating on the x-axis**

$$A = \int_a^b [f(x) - g(x)]\, dx \tag{1}$$

Where $f(x) \geqslant g(x)$

$$A = \int_a^b \ [g(x) - f(x)]\ dx.$$

for $g(x) \geqslant f(x)$

- **Area between two curves, integrating on the y-axis**

$$A = \int_c^d [u(y) - v(y)]\, dy \tag{2}$$

- **Areas of compound regions**

$$\int_a^b \ |f(x) - g(x)|\ dx.$$

- **Area of complex regions**

$$\int_a^b \ f(x)\ dx + \int_b^c \ g(x)\ dx.$$

- **Slicing Method**

$$V(s) = \sum_{i=1}^{n} \ A(x_i^*)\ \Delta x = \int_a^b \ A(x)\ dx.$$

- **Disk Method along the x-axis**

$$V = \int_a^b \pi[f(x)]^2\, dx \tag{3}$$

- **Disk Method along the y-axis**

$$V = \int_c^d \pi[g(y)]^2\, dy \tag{4}$$

- **Washer Method along the x-axis**

$$V = \int_a^b \pi[(f(x))^2 - (g(x))^2]\, dx \tag{5}$$

- **Washer Method along the y-axis**

$$V = \int_c^d \pi[(u(y))^2 - (v(y))^2]\, dy \tag{6}$$

- **Radius if revolved around other line (Washer Method)**

$$\begin{aligned} If:&\ \ x = -k \\ Then:&\ \ r = Function + k. \end{aligned}$$

$$\begin{aligned} If:&\ \ x = k \\ Then:&\ \ r = k - Function. \end{aligned}$$

- **Method of Cylindrical Shells (x-axis)**

$$V = \int_a^b 2\pi x f(x)\, dx \tag{7}$$

- **Method of Cylindrical Shells (y-axis)**

$$V = \int_c^d 2\pi y g(y)\, dy \tag{8}$$

- **Region revolved around other line (method of cylindrical shells):**

$$If:\ x = -k$$
$$Then:\ V = \int_a^b\ 2\pi(x+k)(f(x))\ dx.$$

$$If:\ x = k$$
$$Then:\ V = \int_a^b\ 2\pi(k-x)(f(x))\ dx.$$

- **A Region of Revolution Bounded by the Graphs of Two Functions (method cylindrical shells)**

$$V = \int_a^b\ 2\pi x\left[f(x) - g(x)\right]\ dx.$$

- **Arc Length of a Function of x**

$$\text{Arc Length} = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx \tag{9}$$

- **Arc Length of a Function of y**

$$\text{Arc Length} = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy \tag{10}$$

- **Surface Area of a Function of x**

$$\text{Surface Area} = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx \tag{11}$$

- **Natural logarithm function**

$$\ln x = \int_1^x \frac{1}{t}\, dt\ Z \tag{12}$$

- **Exponential function**

$$y = e^x, \quad \ln y = \ln(e^x) = x\ Z \tag{13}$$