Nate Warner MATH 336 Spring 2026

**Problem set 3 - Due: Monday, Febuary 2**

1. Consider the differential equation

$$y\frac{dy}{dx} = 4x.$$

(a) Verify that $4x^2 - y^2 = C$ gives a one-parameter family of implicit solutions.

(b) State the existence and uniqueness theorem for the IVP

$$\begin{cases} \frac{dy}{dx} = f(x,y), \\ y(x_0) = y_0 \end{cases}.$$

(c) If $x_0 \neq 0$, does this theorem guarantee the existence of a solution to the IVP

$$\begin{cases} y\frac{dy}{dx} = 4x, \\ y(x_0) = 0 \end{cases}.$$

(d) Give two distinct solutions to

$$\begin{cases} y\frac{dy}{dx} = 4x, \\ y(0) = 0 \end{cases}.$$

**Hint:** consider $y = kx$ for some constant $k$).

a.) We differentiate the proposed solution implicitly,

$$\begin{aligned} \frac{d}{dx}\left(4x^2 - y^2\right) &= \frac{d}{dx}C \\ \implies 8x - 2y\frac{dy}{dx} &= 0 \\ \implies y\frac{dy}{dx} &= \frac{-8x}{-2} = 4x. \end{aligned}$$

Thus, verified.

b.) If $f(x,y)$ is continuous around the initial point $(x_0, y_0)$, then a solution exists on an open interval containing $x_0$. If $\frac{\delta f}{\delta y}$ is continuous around $(x_0, y_0)$, then the solution is unique on that interval.

c.) No, since $f(x,y) = \frac{4x}{y}$ is not continuous at the initial point $(x_0, 0)$ for any $x_0$, the theorem does not guarantee a solution on an open interval containing $x_0$.

d.) If we consider $y = kx$, for some constant $k$, then

$$\frac{dy}{dx} = k.$$

Using the differential equation $y\frac{dy}{dx} = 4x$, we see that

$$(kx)(k) = 4x \implies k^2x = 4x \implies k^2 = 4 \implies k = \pm 2.$$

Thus, two distinct solutions are

$$y(x) = 2x, \quad y(x) = -2x,$$

which both satisfy the initial condition $y(0) = 0$.

2. Solve the following differential equation

$$\frac{dy}{dx} = \frac{(x-1)y^5}{x^2(2y^3 - y)}.$$

We have

$$\frac{dy}{dx} = \frac{x-1}{x^2}\left(\frac{y^5}{2y^3 - y}\right).$$

So,

$$\begin{aligned}
\frac{2y^3 - y}{y^5}\,dy &= \frac{x-1}{x^2}\,dx \\
\implies \int \frac{2y^3 - y}{y^5}\,dy &= \int \frac{x-1}{x^2}\,dx \\
\implies \int (2y^{-2} - y^{-4})\,dy &= \int (x^{-1} - x^{-2})\,dx \\
\implies -\frac{2}{y} + \frac{1}{3y^3} &= \ln|x| + \frac{1}{x} + C.
\end{aligned}$$

3. Solve the following differential equation

$$\frac{dy}{dx} = 2xy^2 + 3x^2y^2, \quad y(1) = -1.$$

We have

$$\frac{dy}{dx} = 2xy^2 + 3x^2y^2 = y^2(2x + 3x^2).$$

So,

$$\begin{aligned} \frac{1}{y^2}\,dy &= 2x + 3x^2\,dx \\ \implies \int \frac{1}{y^2}\,dy &= \int (2x + 3x^2)\,dx \\ \implies -\frac{1}{y} &= x^2 + x^3 + C. \end{aligned}$$

Using the initial condition $y(1) = -1$,

$$-\frac{1}{-1} = 1^2 + 1^3 + C \implies 1 = 1 + 1 + C \implies C = -1.$$

Thus,

$$-\frac{1}{y} = x^2 + x^3 - 1.$$

4. Solve the following differential equation

$$\frac{dy}{dx} = 1 + x + y + xy.$$

We have

$$\frac{dy}{dx} = 1 + x + y + xy = 1 + x + y(1 + x) = (1 + x)(1 + y).$$

So,

$$\begin{aligned}
\frac{1}{1+y}\ dy = 1 + x\ dx &\implies \int \frac{1}{1+y}\ dy = \int 1 + x\ dx \\
&\implies \ln|1+y| = x + \frac{1}{2}x^2 + C_0 \\
&\implies |1+y| = e^{x+\frac{1}{2}x^2+C_0} = C_1 e^{x+\frac{1}{2}x^2} \\
&\implies 1 + y = Ce^{x+\frac{1}{2}x^2} \\
&\implies y = Ce^{x+\frac{1}{2}x^2} - 1.
\end{aligned}$$

5. Solve the following differential equation

$$\frac{dy}{dx} = 6e^{2x-y}, \quad y(0) = 0.$$

We have

$$\begin{aligned} \frac{dy}{dx} &= 6e^{2x-y} = \frac{6e^{2x}}{e^y} \\ \implies e^y \, dy &= 6e^{2x} \, dx \\ \implies \int e^y \, dy &= \int 6e^{2x} \, dx \\ \implies e^y &= 3e^{2x} + C. \end{aligned}$$

With $y(0) = 0$,

$$e^0 = 3e^0 + C \implies 1 = 3 + C \implies C = -2.$$

Thus,

$$e^y = 3e^{2x} - 2.$$