Discrete Structures

Functions

Nathan Warner

Computer Science
Northern Illinois University
August 22, 2023
United States

Contents

Functions

Preface

Much of the information covered in this chapter has been purposely omitted, as most of this chapter is trivial for people with a background in algebra.

1 Vocabulary

- A function $f:\ A \to B$ is **One-to-One (injective)** is injective if every element of A has a unique image in B
- A function $f:\ A \to B$ is **Onto (surjective)** is surjective if every element of B is the image of at least one element of A.
- A function $f:\ A \to B$ is **Bijective** if it is both injective and surjective.
- The **Inverse** of a function reverses the direction of the original function. A function $f:\ A \to B$ has an inverse $f^{-1}:\ B \to A$ iff
  - f is bijective (both injective and surjective).
  - $\forall\ a \in A,\ b \in B,\ f(a) = b \iff f^{-1}(b) = a$

Note:-

$\mathcal{D}$ and $\mathcal{R}$ flip for the inverse function

2 Notation

- **Domain of a function**: Denoted $\mathcal{D}$ or $\mathcal{D}(f)$
- **Range of a function**: Denoted $\mathcal{R}$ or $\mathcal{R}(f)$ Consider we have some function with $\mathcal{D}(f) = \mathbb{R}$ and $\mathcal{R}(f) = (2, \infty)$, then we can say

$$\begin{aligned} f:\ \mathbb{R} \to (2, \infty):\ x \mapsto f(x) \\ Or:\ f(x) \in (2, \infty),\ \forall\ x \in \mathbb{R}. \end{aligned}$$

- **Functional Notation (Set-Builder)**

$$f:\ A \to B : x \mapsto f(x)$$

.

Where $A \to B$ is used to indicate the domain and codomain of the function, and $x \mapsto f(x)$ is used to indicate how individual elements are mapped under the function.

$$Ex:\ f:\ \mathbb{R} \to \mathbb{R}:\ x \mapsto x^2 - 6.$$

- **Exclude elements in functional notation**

$$f:\ \mathbb{R} \setminus \{2\} \mapsto \mathbb{R}:\ x \mapsto \frac{x+3}{x-2}.$$

- **Injective (one-to-one)**:

$$\forall\ x_1,\ x_2 \in A, (f(x_1) = f(x_2 \implies x_1 = x_2)).$$

- **Subjective**

$$f:\ X \to Y\ onto \iff \forall y \in Y,\ \exists\ x \in X \mid f(x) = y.$$