

Calculus 2
Problem Set 1.

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1.a Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(6)$

$$g(0)$$

$$\begin{aligned} g(0) &= \int_0^0 f(t) \, dt \\ &= 0 \quad (\text{By integral with the same bounds equals zero}). \end{aligned}$$

$$g(1)$$

$$\begin{aligned} g(1) &= \int_0^1 f(t) \, dt \\ &= lw \\ &= 1 \cdot 2 \\ &= 2. \end{aligned}$$

$$g(2)$$

$$\begin{aligned} g(2) &= \int_0^2 f(t) \, dt \\ &= \left(lw \right) + \left(\frac{1}{2}bh \right) \\ &= \left(2 \cdot 2 \right) + \left(\frac{1}{2}(1)(2) \right) \\ &= 4 + 1 \\ &= 5. \end{aligned}$$

$$g(3)$$

$$\begin{aligned} g(3) &= \int_0^3 f(t) \, dt \\ &= g(2) + \frac{1}{2}bh \\ &= 5 + \frac{1}{2}(1)(4) \\ &= 5 + 2 \\ &= 7. \end{aligned}$$

$$g(6)$$

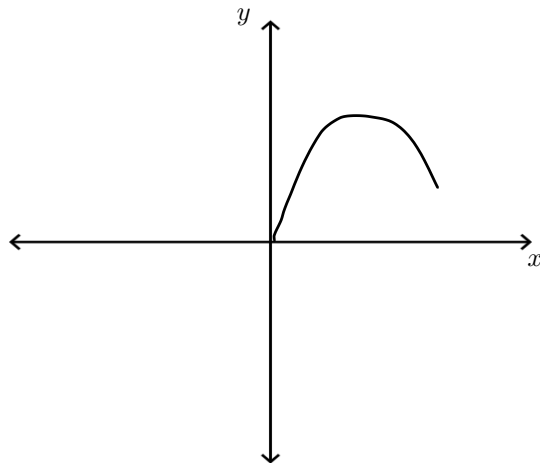
$$\begin{aligned} g(6) &= \int_0^6 f(t) \, dt \\ &= g(3) - \left(\int_3^6 f(t) \, dt \right) \\ &= 7 - \left(\int_3^5 f(t) \, dt + \int_5^6 f(t) \, dt \right) \\ &= 7 - \left(\frac{1}{2}(2)(2) + 1 \cdot 2 \right) \\ &= 7 - (2 + 2) \\ &= 7 - 4 \\ &= 3. \end{aligned}$$

1.b g has a maximum value at $g(3)$

1.c g is increasing on the interval $(0, 3)$

1.d Rough sketch of g :

Figure:



2. Use part 1 of the Fundamental Theorem of Calculus to find the derivative of the functions

Remark. Part 1: $\int_a^x f(t) dt = f(x)$, $a \leq x \leq b$

2.a

$$\begin{aligned} \text{If : } g(x) &= \int_3^x \sqrt{9-t^2} dt \\ \text{Then : } g'(x) &= \frac{d}{dx} \int_3^x \sqrt{9-t^2} dt \\ &= \sqrt{9-x^2}. \end{aligned}$$

2.b

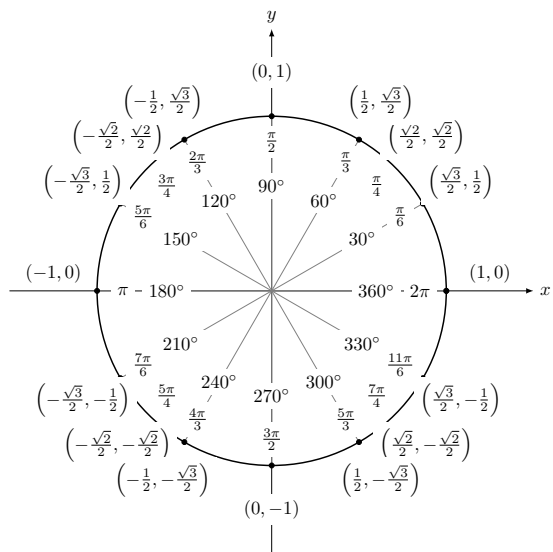
$$\begin{aligned} \text{If : } y &= \int_0^{\ln x} e^t dt \\ \text{Then : } y' &= \frac{d}{dx} \int_0^{\ln x} e^t dt \\ &= e^{\ln x} \cdot \frac{d}{dx} \ln x \\ &= e^{\ln x} \cdot \frac{1}{x} \\ &= \frac{e^{\ln x}}{x}. \end{aligned}$$

3. Use part 2 of the Fundamental Theorem of Calculus to evaluate the integrals.

3.a $\int_1^4 \frac{2-x^{\frac{1}{2}}}{x^2} dx$

$$\begin{aligned}
 & \int_1^4 \frac{2-x^{\frac{1}{2}}}{x^2} dx \\
 &= \int_1^4 \frac{2}{x^2} - \frac{x^{\frac{1}{2}}}{x^2} dx \\
 &= \int_1^4 2x^{-2} - x^{-\frac{3}{2}} dx \\
 &= \left. -2x^{-1} + 2x^{-\frac{1}{2}} \right]_1^4 \\
 &= \left(-2(4)^{-1} + 2(4)^{-\frac{1}{2}} \right) - \left(-2(1)^{-1} + 2(1)^{-\frac{1}{2}} \right) \\
 &= \left(-\frac{2}{4} + \frac{2}{\sqrt{4}} \right) - \left(-2 + 2 \right) \\
 &= -\frac{1}{2} + 1 \\
 &= \frac{1}{2}.
 \end{aligned}$$

3.b



$$\begin{aligned}
 & \int_0^\pi (\sin x - 3\sqrt{x}) dx \\
 &= \int_0^\pi (\sin x - 3x^{\frac{1}{2}}) dx \\
 &= \left. -\cos x - 2x^{\frac{3}{2}} \right]_0^\pi \\
 &= \left(-\cos \pi - 2(\pi)^{\frac{3}{2}} \right) - \left(-\cos(0) - 2(0)^{\frac{3}{2}} \right) \\
 &= \left(-(-1) - 2\pi^{\frac{3}{2}} \right) - (-1) \\
 &= 1 - 2\pi^{\frac{3}{2}} + 1 \\
 &= 2 - 2\pi^{\frac{3}{2}}.
 \end{aligned}$$

4. Evaluate the following integrals

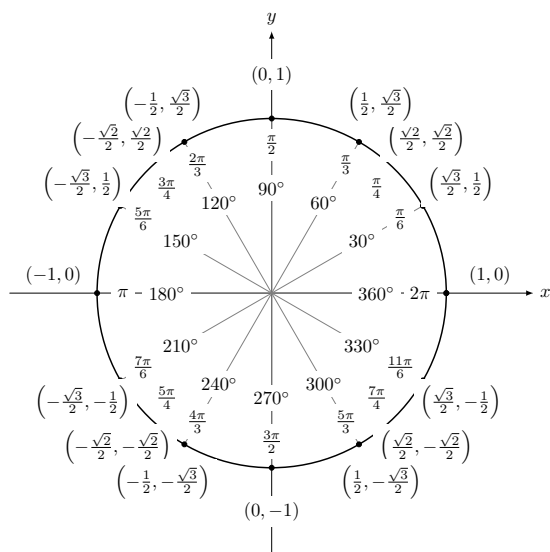
4.a

$$\begin{aligned}
 & \int x^{\frac{1}{2}}(x^2 + 5x + 2) \, dx \\
 &= \int x^{\frac{1}{2}+2} + 5x^{1+\frac{1}{2}} + 2x^{\frac{1}{2}} \, dx \\
 &= \int x^{\frac{5}{2}} + 5x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \, dx \\
 &= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + C.
 \end{aligned}$$

4.b

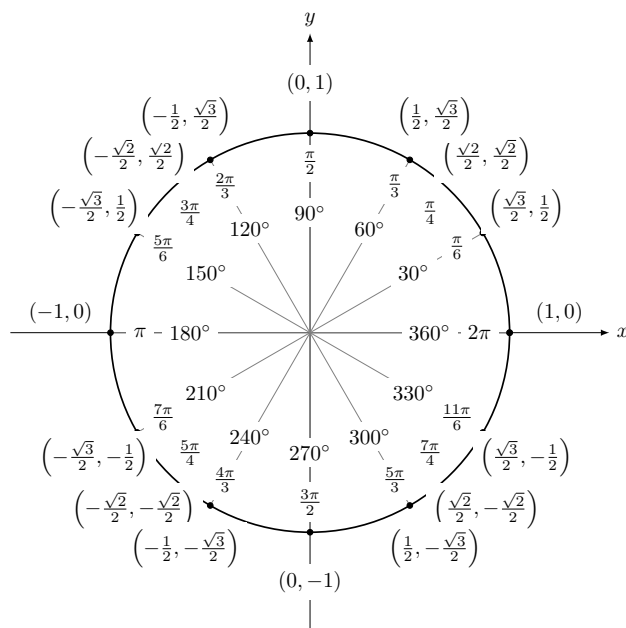
$$\begin{aligned}
 & \int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^3} \right) dx \\
 &= \int_1^2 x^{-2} - x^{-3} \, dx \\
 &= \left. -x^{-1} + \frac{1}{2}x^{-2} \right]_1^2 \\
 &= \left(-(2)^{-1} + \frac{1}{2}(2)^{-2} \right) - \left(-(1)^{-1} + \frac{1}{2}(1)^{-2} \right) \\
 &= \left(-\frac{1}{2} + \frac{1}{8} \right) - \left(-1 + \frac{1}{2} \right) \\
 &= -\frac{3}{8} - \left(-\frac{1}{2} \right) \\
 &= -\frac{3}{8} + \frac{1}{2} \\
 &= \frac{1}{8}.
 \end{aligned}$$

4.c



$$\begin{aligned}
 & \int_0^\pi (2 \sin x - 3 \sec^2 x) \, dx \\
 &= \left. -2 \cos x - 3 \tan x \right]_0^\pi \\
 &= \left(-2 \cos \pi - 3 \tan \pi \right) - \left(-2 \cos 0 - 3 \tan 0 \right) \\
 &= \left(-2(-1) - 3(0) \right) - \left(-2(1) - 3(0) \right) \\
 &= 2 - (-2) \\
 &= 4.
 \end{aligned}$$

4.d



$$\begin{aligned}
 & \int_0^{\pi/4} \sec x \tan x \, dx \\
 &= \sec x \Big|_0^{\pi/4} \\
 &= \sec \frac{\pi}{4} - \sec 0 \\
 &= \frac{1}{\frac{\sqrt{2}}{2}} - 1 \\
 &= \frac{2\sqrt{2}}{2} - 1 \\
 &= \sqrt{2} - 1.
 \end{aligned}$$

5. Find displacement and distance traveled of $t^2 - 3t - 18$, $0 \leq t \leq 6$

Remark. Displacement: $\int_a^b v(t) \, dt$ and Distance Traveled: $\int_a^b |v(t)| \, dt$

Displacement:

$$\begin{aligned}
 & \int_0^6 t^2 - 3t - 18 \, dt \\
 &= \left[\frac{1}{3}t^3 - \frac{3}{2}t^2 - 18t \right]_0^6 \\
 &= \left(\frac{1}{3}(6)^3 - \frac{3}{2}(6)^2 - 18(6) \right) - \left(\frac{1}{3}(0)^3 - \frac{3}{2}(0)^2 - 18(0) \right) \\
 &= 72 - 54 - 108 \\
 &= -90.
 \end{aligned}$$

Distance Traveled:

$$\int_0^6 |t^2 - 3t - 18| \, dt.$$

Find where the function turns negative

$$\begin{aligned}
 t^2 - 3t - 18 &= 0 \\
 (t+3)(t-6) & \\
 t &= -3, 6.
 \end{aligned}$$

Rewrite Piecewise

$$v(t) = \begin{cases} t^2 - 3t - 18 & \text{if } t > 6 \\ -(t^2 - 3t - 18) & \text{if } t \leq 6 \end{cases} \quad (1)$$

Thus:

$$\begin{aligned} & \int_0^6 -(t^2 - 3t - 18) \, dt \\ &= \int_0^6 -t^2 + 3t + 18 \, dt \\ &= -\frac{1}{3}t^3 + \frac{3}{2}t^2 + 18t \Big|_0^6 \\ &= \left(-\frac{1}{3}(6)^3 + \frac{3}{2}(6)^2 + 18(6) \right) - \left(-\frac{1}{3}(0)^3 + \frac{3}{2}(0)^2 + 18(0) \right) \xrightarrow{0} \\ &= 90. \end{aligned}$$