

Math 170: Quiz 3

Nathan Warner

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1. Using the graph of the function f , find the following. Circle your final answers.

1.a: Local Min: $f(2) = 3$

1.b: Local Max $f(4) = 5$

1.c: Absolute Min: None

1.d: Absolute Max: $f(0) = 5$ and $f(4) = 5$

3. The height in feet of a given body about the surface of the earth at time t seconds is given.

$$y = 1000 + 160t - 16t^2, \quad 3 \leq t \leq 6.$$

Find:

- a.) The maximum and minimum height of the body
- b.) The maximum and minimum velocity v of the body, and
- c.) The maximum and minimum speed s of the body

During the given time interval.

Closed Interval: $[3, 6]$

3.a:

y' :

$$y' = 160t - 32t.$$

Set $y' = 0$

$$160 - 32t = 0$$

$$-32t = -160$$

$$t = \frac{-160}{-32}$$

$$t = 5.$$

Since $5 \in [3, 6]$, 5 is a critical value

y' DNE

There are no values of t that make this function undefined

Plug critical values into y :

$$y(5) = 1000 + 160(5) - 16(5)^2$$

$$\boxed{= 1400}.$$

Find $y(a)$ and $y(b)$:

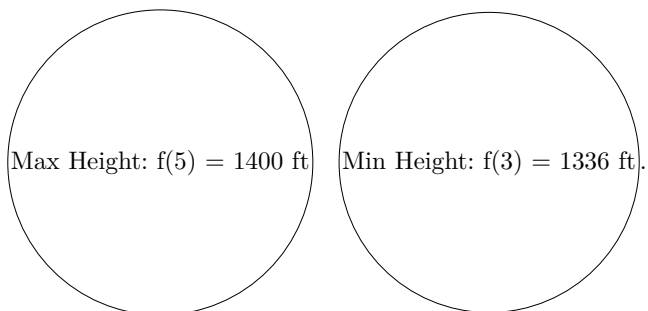
$$y(3) = 1000 + 160(3) - 16(3)^2$$

$$\boxed{= 1336}.$$

$$y(6) = 1000 + 160(6) - 16(6)^2$$

$$\boxed{= 1384}.$$

Therefore:



3.b:

Find $v(t)$:

$$v(t) = 160 - 32t.$$

$v'(t)$:

$$v'(t) = -32.$$

Set $v'(t) = 0$:

$$-32 = 0.$$

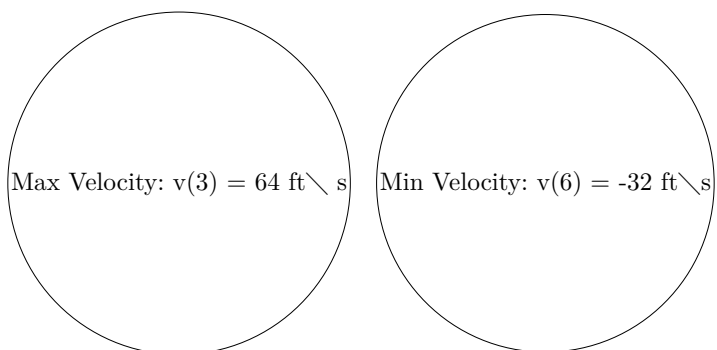
Since $-32 \notin [3, 6]$, -32 is not a critical value

Find $v(a)$ and $v(b)$:

$$\begin{aligned} v(3) &= 160 - 32(3) \\ &= 64 \text{ ft/s} \end{aligned}$$

$$\begin{aligned} v(6) &= 160 - 32(6) \\ &= -32 \text{ ft/s} \end{aligned}$$

Therefore:



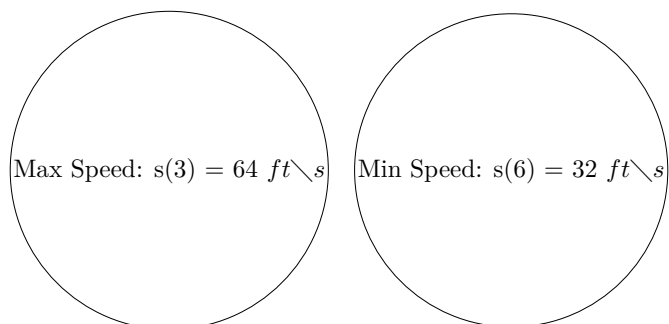
3.c:

Since $s(t) = |v(t)|$, we can take the absolute value of our values from 3.b and retrieve that maximum and minimum speeds:

$$\begin{aligned} s(3) &= |160 - 32(3)| \\ &= 64 \text{ ft/s} \end{aligned}$$

$$\begin{aligned} s(6) &= |160 - 32(6)| \\ &= 32 \text{ ft/s} \end{aligned}$$

Therefore:



4. Find the limit using l'Hospital's Rule, if the rule is appropriate.

4.a:

$$\lim_{x \rightarrow 1} \frac{3x^2 + 2x - 5}{x^4 + 3x^2 - 4}.$$

$$\lim_{x \rightarrow 1} 3x^2 + 2x - 5 = 0 \quad \text{and} \quad \lim_{x \rightarrow 1} x^4 + 3x^2 - 4 = 0.$$

Since we have an indeterminate form of the type $\frac{0}{0}$, we can use L'Hospital's Rule to evaluate the limit.

So:

$$\begin{aligned} L'H &= \lim_{x \rightarrow 1} \frac{6x + 2}{4x^3 + 6x} \\ &= \frac{8}{10} \end{aligned}$$

$$\left(\frac{4}{5} \right)$$

4.b:

$$\lim_{x \rightarrow \infty} \frac{e^{2+\ln x}}{3x + 4}.$$

$$\lim_{x \rightarrow \infty} e^{2+\ln x} = \infty \quad \text{and} \quad \lim_{x \rightarrow \infty} 3x + 4 = \infty.$$

Since we have an indeterminate form of the type $\frac{\infty}{\infty}$, We must use L'Hospital's Rule.

$$\begin{aligned} L'H &= \lim_{x \rightarrow \infty} \frac{e^{2+\ln x} \cdot \frac{1}{x}}{3} \\ L'H &= \lim_{x \rightarrow \infty} \frac{\frac{e^{2+\ln x}}{x}}{3} \\ L'H &= \lim_{x \rightarrow \infty} \frac{e^{2+\ln x}}{3x} \end{aligned}$$

At it's current state, we are stuck in a loop of applying L'Hospital's Rule to no avail, therefore we must try and simplify the equation

$$\begin{aligned} L'H &= \lim_{x \rightarrow \infty} \frac{e^{2+\ln x}}{3x} \\ L'H &= \lim_{x \rightarrow \infty} \frac{e^2 \cdot e^{\ln x}}{3x} \\ L'H &= \lim_{x \rightarrow \infty} \frac{e^2 \cdot x}{3x} \\ L'H &= \lim_{x \rightarrow \infty} \frac{e^2}{3} \end{aligned}$$

$$= \frac{e^2}{3}$$

4.c:

$$\lim_{x \rightarrow \pi} \frac{\sin^2 x}{(x - \pi)^2}.$$

$$\lim_{x \rightarrow \pi} \sin^2 x = 0 \text{ and } \lim_{x \rightarrow \pi} (x - \pi)^2 = 0.$$

Since we have an indeterminate form of the type $\frac{0}{0}$, we can use L'Hospital's Rule

$$\begin{aligned} L'H &= \lim_{x \rightarrow \pi} \frac{2 \sin x \cos x}{2(x - \pi)(1)} \\ L'H &= \lim_{x \rightarrow \pi} \frac{2 \sin x \cos x}{2x - 2\pi} \end{aligned}$$

$$\lim_{x \rightarrow \pi} 2 \sin(x) \cos(x) = 0 \text{ and } \lim_{x \rightarrow \pi} 2x - 2\pi = 0.$$

We still have the indeterminate form of the type $\frac{0}{0}$, so once again, we must use L'Hospital's Rule

$$\begin{aligned} L'H &= \lim_{x \rightarrow \pi} \frac{2[(\sin x)(-\sin x) + (\cos x)(\cos x)]}{2} \\ L'H &= \lim_{x \rightarrow \pi} \frac{2[-\sin^2 x + \cos^2 x]}{2} \\ L'H &= \lim_{x \rightarrow \pi} \frac{-2 \sin^2 x + 2 \cos^2 x}{2}. \end{aligned}$$

From here if we evaluate:

$$\lim_{x \rightarrow \pi} -2 \sin^2 x + 2 \cos^2 x.$$

We get:

$$\begin{aligned} &-2 \sin^2 \pi + 2 \cos^2 \pi \\ &= -2(0)^2 + 2(-1)^2 \\ &= 2. \end{aligned}$$

Therefore we have

$$\frac{2}{2}$$

$$\textcircled{1}$$