PSET 2 - Due: Wednesday, June 26

Consider an experiment with the sample space $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and the events $A = \{0, 1, 2, 3\}$, $B = \{2, 3, 4, 5, 6\}$, $C = \{7, 8\}$, and $D = \{1, 3, 7\}$. Find each of the following events.

(a) A^C

(b) B^C

(c) $A \cup B$

(d) $(A \cup B)^C$

(e) $A^C \cap B^C$

(f) $B \cap C$

(g) $C \cap D^C$

(h) $A \cup B \cup C$

(i) $A \cap B \cap D$

(j) Compare parts (d) and (e). What do you notice?

(a) $\{4, 5, 6, 7, 8, 9\}$

(b) $\{0, 1, 7, 8, 9\}$

(c) $\{0, 1, 2, 3, 4, 5, 6\}$

(d) $\{7, 8, 9\}$

(e) $\{7, 8, 9\}$

(f) $\varnothing$

(g) $\{8\}$

(h) $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

(i) $\{3\}$

(j) They are the same (De Morgan's law)

A mutual fund company offers its customers a variety of funds. Among customers who own shares in just one fund, the percentages of customers in the different funds are given below.

- Money-market 20%
- High-risk stock 18%
- Short bond 15%
- Moderate-risk stock 25%
- Intermediate bond 10%
- Balanced ??%
- Long bond 5%

Suppose that a customer who owns shares in just one fund is selected at random. Find each of the following probabilities.

(a) The individual owns shares in the balanced fund.

(b) The individual owns shares in a bond fund.

(c) The individual does not own shares in a stock fund.

a.) To find the missing probability (Balanced fund), we use the following axiom.

$$P(\mathcal{S}) = \sum P(E) = 1.$$

If we denote the proportion of customers who own shares in the balanced fund λ, we can use the above axiom to solve for it

$$\begin{aligned}
1 &= 0.2 + 0.18 + 0.15 + 0.25 + 0.1 + \lambda + 0.05 \\
&= 1 - .2 - .18 - .15 - .25 - .1 - .05 \\
\lambda &= 0.07 \\
\implies P(\lambda) &= P(\text{balanced}) = 7\%.
\end{aligned}$$

Thus, the probability that a customer selected randomly is in the balanced fund is 7%

b.) The probability the individual owns shahres in a bond fund is the summation of the probabilities of the three bond funds. Thus,

$$\begin{aligned}
&P(\text{Long bond or Short bond or Intermediate bond}) \\
&= P(\text{Long bond}) + P(\text{Short bond}) + P(\text{Intermediate bond}) \\
&= 0.05 + 0.1 + 0.15 \\
&= 0.3 = 30\%.
\end{aligned}$$

c.) The probability that the individual does not own share in a stock fund is the complement of the probability that the individual does own shares in a stock fund. That is

$$
\begin{aligned}
&P((\text{Stock fund})^C) \\
&= 1 - P(\text{High-risk stock or Moderate-risk stock}) \\
&= 1 - (P(\text{High-risk stock} + P(\text{Moderate-risk stock}) \\
&= 1 - (0.18 + 0.25) \\
&= 0.57 = 57\%.
\end{aligned}
$$

The three most popular options on a certain type of new car at a dealership are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). Suppose that $P(A) = 0.40$, $P(B) = 0.55$, $P(C) = 0.70$, $P(A \cup B) = 0.63$, and $P(B \cap C) = 0.45$. Suppose that a customer at the dealership is selected at random. Find the probability of each of the following events.

(a) The customer wants a built-in GPS and a sunroof.

(b) The customer wants a sunroof or an automatic transmission.

(c) The customer does not want a sunroof.

(d) Consider the event "the customer wants neither a built-in GPS nor a sunroof".

  (i) Write the event in symbols (i.e. using A, B, C, $\cup$, $\cap$, etc.).

  (ii) Find the probability of the event.

(e) Consider the event "the customer does not want a sunroof but does want an automatic transmission".

  (i) Write the event in symbols (i.e. using A, B, C, $\cup$, $\cap$, etc.).

  (ii) Find the probability of the event.

Note: These events are independent, we handle $P(E_1 \cup E_2 \cup ... \cup E_n)$ accordingly

Remark. Given two independent events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

a.)

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B) \\ &= 0.4(0.55) = 0.22 = 22\%. \end{aligned}$$

b.)

$$\begin{aligned} P(B \cup C) &= P(B) + P(C) - P(B \cap C) \\ &= 0.55 + 0.7 - 0.45 \\ &= 0.80 = 80\%. \end{aligned}$$

c.)

$$\begin{aligned} P(B') &= 1 - P(B) \\ &= 1 - 0.55 \\ &= 0.45 = 45\%. \end{aligned}$$

d.)

$$\begin{aligned} P(A' \cap B') &= (1 - P(A))(1 - P(B)) \\ &= (1 - 0.4)(1 - 0.55) \\ &= 0.27. \end{aligned}$$

e.)

$$\begin{aligned} P(B' \cap C) &= (1 - P(B))(P(C)) \\ &= (1 - 0.55)(0.7) \\ &= 0.315 = 31.5\%. \end{aligned}$$