Discrete Structures
Functions

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Contents

Functions

Preface

Much of the information covered in this chapter has been purposely omitted, as most of this chapter is trivial for people with a background in algebra.

1 Vocabulary

- A function $f:\ A \to B$ is **One-to-One (injective)** is injective if every element of A has a unique image in B
- A function $f:\ A \to B$ is **Onto (surjective)** is surjective if every element of B is the image of at least one element of A.
- A function $f:\ A \to B$ is **Bijective** if it is both injective and surjective.
- The **Inverse** of a function reverses the direction of the original function. A function $f:\ A \to B$ has an inverse $f^{-1}:\ B \to A$ iff
  - f is bijective (both injective and surjective).
  - $\forall\ a \in A,\ b \in B,\ f(a) = b \iff f^{-1}(b) = a$

Note:-

$\mathcal{D}$ and $\mathcal{R}$ flip for the inverse function

2 Notation

- **Domain of a function**: Denoted $\mathcal{D}$ or $\mathcal{D}(f)$
- **Range of a function**: Denoted $\mathcal{R}$ or $\mathcal{R}(f)$ Consider we have some function with $\mathcal{D}(f) = \mathbb{R}$ and $\mathcal{R}(f) = (2, \infty)$, then we can say

$$\begin{aligned} f:\ \mathbb{R} \to (2, \infty):\ x \mapsto f(x) \\ Or:\ f(x) \in (2, \infty),\ \forall\ x \in \mathbb{R} \\ Or:\ \forall\ x \in \mathbb{R},\ f(x) \in (2, \infty). \end{aligned}$$

- **Functional Notation (Set-Builder)**

$$f:\ A \to B : x \mapsto f(x)$$

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Where $A \to B$ is used to indicate the domain and codomain of the function, and $x \mapsto f(x)$ is used to indicate how individual elements are mapped under the function.

$$Ex:\ f:\ \mathbb{R} \to \mathbb{R}:\ x \mapsto x^2 - 6.$$

- **Exclude elements in functional notation**

$$f:\ \mathbb{R} \setminus \{2\} \to \mathbb{R}:\ x \mapsto \frac{x+3}{x-2}.$$

- **Injective (one-to-one)**:

$$\forall\ x_1,\ x_2 \in A, (f(x_1) = f(x_2) \implies x_1 = x_2).$$

- **Subjective**

$$f:\ X \to Y\ onto \iff \forall y \in Y,\ \exists\ x \in X \mid f(x) = y.$$