Discrete Structures
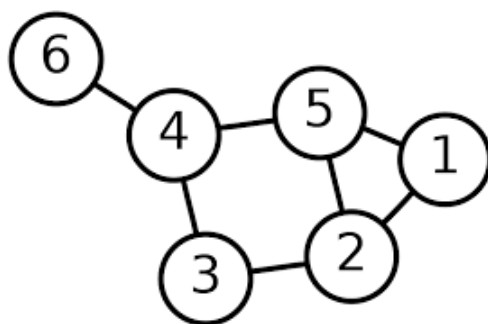
Notes

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Contents

1   Set Theory

1.1   Definition of a set

> **Definition:** A **set** is a collection of elements
> We denote sets with the following syntax:
>
> $$A = \{1, 2, 3, 4\}.$$
>
> Where in this case A is the identifier and it's elements are delimited by commas and encapsulated among braces.
> **Note:** The identifier for sets are commonly represented with capital letters
>
> We can also indicate *infinitely many* elements in a set by use of the **ellipsis**, which would look like:
>
> $$A = \{1, 2, 3, ...\}$$
> $$Generally: \ A = \{A_1, A_2, A_3, ..., A_n\}.$$

More Notation: We can indicate that an object is an **element** of a set with the following syntax:

$$A = \{1, 2, 3, 4\}$$
$$3 \in A.$$

1.2   Number Sets

The set of **Natural Numbers** (whole numbers) is denoted by $\mathbb{N}$:

$$\mathbb{N} : 1, 2, 3, ...$$

The set of **Integers** is denoted by $\mathbb{Z}$:

$$\mathbb{Z} : -5, -4, -3-, 2, -1, 0, 1, 2, 3, 4, 5, ...$$

So you can see the set of all integers is similar to that of the natural numbers, however this set includes *negative numbers*

The set of **Rational numbers**, (ratio of two integers), is denoted by $\mathbb{Q}$:

$$\mathbb{Q} : \frac{1}{6}, \frac{1}{4}, \frac{1}{2}, ...$$

The set of **Irrational numbers**, is denoted by $\bar{\mathbb{Q}}$:

$$\bar{\mathbb{Q}} : \pi, e, \sqrt{2}, \ etc.$$

> **Note:-**
>
> for a number to be considered irrational, they cannot be exactly represented as fractions of integers and have non-repeating, non-terminating decimal representations. Thus, the following condition must hold:
>
> $$x \text{ is irrational} \iff \frac{a}{b}, \quad \text{where } a \wedge b \notin \mathbb{Z} \text{ and } \gcd(a, b) = 1..$$

The set of all **Real numbers** is denoted by $\mathbb{R}$:

$$\mathbb{R}: \text{ Both rational and irrational numbers.}$$

The set of all **imaginary numbers** is denoted by $\mathbb{I}$:

$$\mathbb{I}: \ i^2 = -1, \ i = \sqrt{-1}$$
$$Ex: \ \sqrt{-9} = \sqrt{9} \cdot \sqrt{-1} = 3i.$$

The set of **Complex numbers**, which describes numbers that are comprised of two components, one real and one imaginary, and is denoted by:

$$\mathbb{C}: 2 + 3i.$$

In summary:

- $\mathbb{N}$: Denotes the set of all **Natural Numbers**

- $\mathbb{Z}$: Denotes the set of all **Integers**

- $\mathbb{Q}$: Denotes the set of all **Rational Numbers**

- $\bar{\mathbb{Q}}$: Denotes the set of all **Irrational Numbers**

- $\mathbb{I}$: Denotes the set of all **Imaginary Numbers**

- $\mathbb{C}$: Denotes the set of all **Complex Numbers**

Figure:
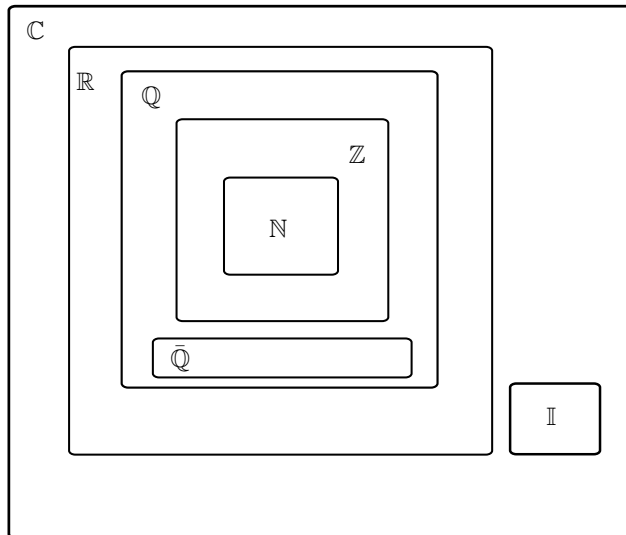
1.3 Set Equality

> **Definition:** An **axiom** is a rule or statement that is generally accepted to be true without proof.
> An **Axiom of Extension** is a set determined by what its elements are - not the order in which they might
> be listed or the fact that some elements might be listed more than once.

Consider the sets:

$$A = \{1, 3, 5, 1, 5, 5, 3\}$$
$$B = \{1, 3, 5\}.$$

Because of the **Axiom of Extension**, which states that a set is not determined by the order or possible repetitions, we can conclude that $A = B$.

Furthermore, we can conclude that we only have 3 elements amongst set A, although it may seem like we have 7.

1.4 Set-Builder Notation

Set-Builder is a convention we can use when dealing with sets to imply the elements of a set without listing all of its values.

Suppose we have:

$$x = -5, 4, 3, -10, -5, 2, 0.$$

Then:

$$\{x | x < 0\} \text{ Reads: "The set of all x's such that (pipe) x is less than zero"}$$
$$= \{-10, -5\}.$$

So naturally you can infer that this set would be all x's from are defined pool of x values that are negative.

Additionally, we can utilize *Number Sets*:

$$\{x \in \mathbb{R} | -2 < x < 5\}.$$

1.5 Types of Sets

- **Universal Set**: Denoted $\mathbb{U}$, represents the collection of all possible elements or objects that are under consideration for a particular context or problem.

- **Empty Set (Null set)**: Denoted $\emptyset$ (phi), represents a set that contains no elements

- **Singleton Set**: Represents a set that only has one element

- **Finite Set**: Represents a set that has a countable number of elements

- **Infinite Set**: Represents a set that has an infinite amount of elements

- **Subset**: A set in which all elements are part of a larger set

> **Definition: Cardinal Number of a Set:** is the number of elements in a set, denoted $n(A)$. Where, in this case, A represents the name of the set.

Consider the set:

$$A = \{1, 2, 3\}$$
$$Then: \ n(A) = 3.$$

Where $n(A) = 3$ represents the cardinal number of the set

> **Definition: Equivalent Set:** Represents sets that have the same *Cardinal Number*. To show that two sets are equivalent, we can use the notation:
>
> $$A \sim B.$$
>
> Which shows that the cardinality of A equals the cardinality of B

Consider the sets:

$$A = \{1, 4, 5\}$$
$$B = \{6, 8, 10\}.$$

Then we can say:

$$A \sim B.$$