Nate Warner MATH 336 Spring 2026

Problem set 3 - Due: Monday, Febuary 2

1. Consider the differential equation

$$y\frac{dy}{dx} = 4x.$$

   (a) Verify that $4x^2 - y^2 = C$ gives a one-parameter family of implicit solutions.

   (b) State the existence and uniqueness theorem for the IVP

$$\begin{cases} \frac{dy}{dx} = f(x,y), \\ y(x_0) = y_0 \end{cases}.$$

   (c) If $x_0 \neq 0$, does this theorem guarantee the existence of a solution to the IVP

$$\begin{cases} y\frac{dy}{dx} = 4x, \\ y(x_0) = 0 \end{cases}.$$

   (d) Give two distinct solutions to

$$\begin{cases} y\frac{dy}{dx} = 4x, \\ y(0) = 0 \end{cases}.$$

   Hint: consider $y = kx$ for some constant k).

a.) We differentiate the proposed solution implicitly,

$$\frac{d}{dx}\left(4x^2 - y^2\right) = \frac{d}{dx}C$$
$$\implies 8x - 2y\frac{dy}{dx} = 0$$
$$\implies y\frac{dy}{dx} = \frac{-8x}{-2} = 4x$$
$$.$$

Thus, verified.

2. Solve the following differential equation

$$\frac{dy}{dx} = \frac{(x-1)y^5}{x^2(2y^3 - y)}.$$

3. Solve the following differential equation

$$\frac{dy}{dx} = 2xy^2 + 3x^2y^2, \quad y(1) = -1.$$

4. Solve the following differential equation

$$\frac{dy}{dx} = 1 + x + y + xy.$$

5. Solve the following differential equation

$$\frac{dy}{dx} = 6e^{2x-y}, \quad y(0) = 0.$$