Homework/Worksheet 2 - Due: Wednesday, January 31

1. Convert the rectangular equation $y^2 = 4x$ to polar form and sketch its graph.

Remark. Given a point P with cartesian coordinates (x, y), and polar coordinates (r, θ) the following conversion formulas are true.

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ x^2 + y^2 &= r^2 \\ \tan\theta &= \frac{y}{x}. \end{aligned}$$

With the formulas mentioned above, we can convert $y^2 = 4x$ to polar form.

$$\begin{aligned} y^2 &= 4x \\ y &= 4 \cdot \frac{x}{y} \\ r\sin\theta &= 4\cot\theta \\ r &= 4\cot\theta\csc\theta. \end{aligned}$$

To graph this equation, we first make a table of points

θ	r
0	undefined
$\frac{\pi}{2}$	0
π	undefined
$\frac{3\pi}{2}$	0
2π	undefined

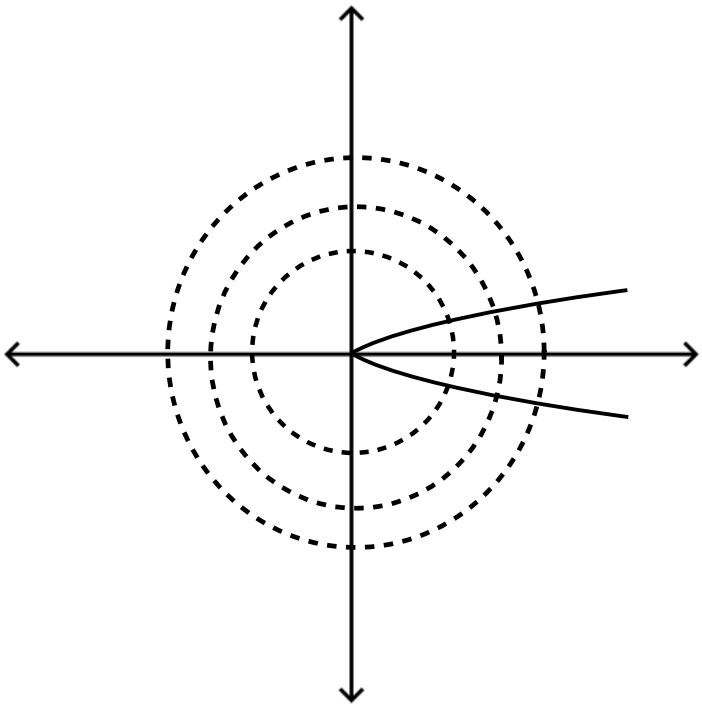
Additionally, we know $\csc\theta$ has period 2π, and $\cot\theta$ has period π. Thus, $4\cot\theta\csc\theta$ will have period π. Since both functions have vertical asymptotes at $x = k\pi, k \in \mathbb{R}$, we know the graph of $4\cot\theta\csc\theta$ will also have these asymptotes. Moreover, we can find the zeros by setting the equation equal to zero and solving for θ

$$\begin{aligned} 4\cot\theta\csc\theta &= 0 \\ \frac{\cos\theta}{\sin^2\theta} &= 0 \\ \cos\theta &= 0 \\ \theta &= \cos^{-1} 0 \\ \theta &= \frac{\pi}{2} + k\pi,\ k \in \mathbb{R}. \end{aligned}$$

Now we need to determine the behavior of the graph as θ approaches 0 and π

$$\begin{aligned} \lim_{\theta\to 0} 4\frac{\cos\theta}{\sin^2 0} &= \infty \\ \lim_{\theta\to \pi} 4\frac{\cos\theta}{\sin^2 0} &= -\infty. \end{aligned}$$

From this information, the polar curve can be sketched

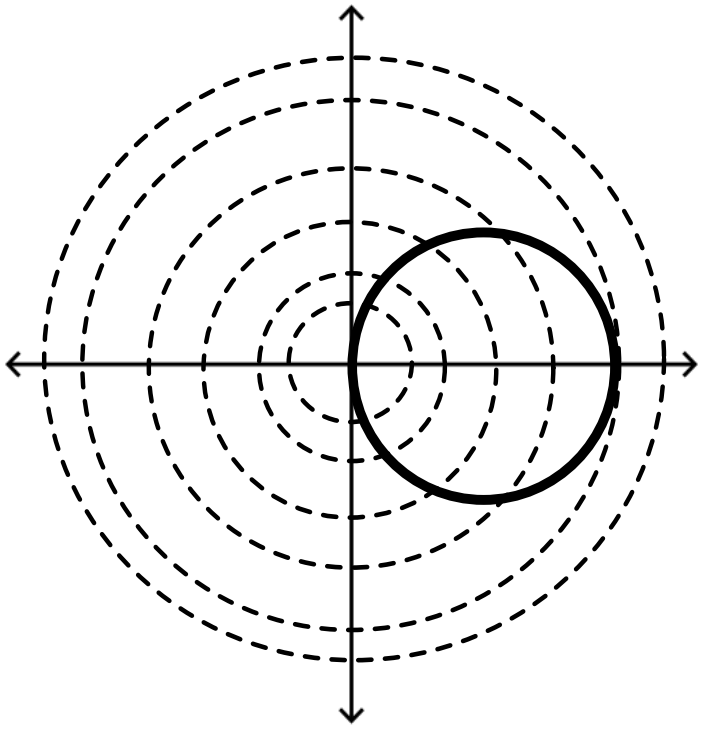


2. Convert the polar equation $r = 6\cos\theta$ to rectangular form and sketch its graph.

Converting to rectangular form we get.

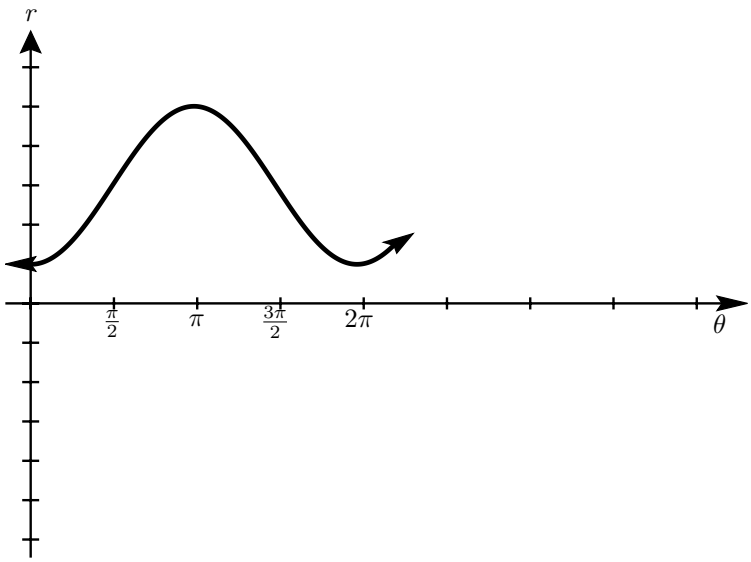
$$\begin{aligned} r^2 &= 6r\cos\theta \\ x^2 + y^2 &= 6x \\ x^2 - 6x + y^2 &= 0 \\ (x-3)^2 + y^2 &= 3^2. \end{aligned}$$

We see that this is the equation of a circle, with center (3,0) and radius $r = 3$. Thus we have



3. Sketch the curve $r = 3 - 2\cos\theta$ by first sketching the graph of r as a function of θ in Cartesian coordinates.

Sketching this curve in the rectangular system,



From this we can sketch the polar curve (Very Rough sketch)