

PSET 6 - Due: Sunday, July 14

1. Let X = the number of typos per page in the rough draft of a particular book. Suppose that X follows a Poisson distribution and that, on average, it has one typo every four pages so that $\mu = 0.25$ typos/page. Use the probability mass function to find each of the following.

- (a) Find the chance that a randomly selected page has no typos.
- (b) Find the chance that a randomly selected page has at most one typo.
- (c) Suppose that three pages are selected independently of each other. Find the chance that none of them have any typos.

Remark. A discrete random variable X is said to have a *Poisson distribution* with parameter μ ($\mu > 0$) if the pmf of X is

$$p(x; \mu) = \frac{e^{-\mu} \cdot \mu^x}{x!} \quad \text{for } x = 0, 1, 2, 3, \dots$$

a.)

$$\begin{aligned} p(0; 0.25) &= \frac{e^{-0.25} \cdot 0.25^0}{0!} \\ &= \frac{1}{e^{0.25}} = 0.7788. \end{aligned}$$

b.)

$$\begin{aligned} P(X \leq 1) &= \sum_{y=0}^1 p(y; \mu) \\ &= p(0; 0.25) + p(1; 0.25) \\ &= 0.7788 + \frac{e^{-0.25} \cdot 0.25^1}{1!} \\ &= 0.7788 + 0.1947 = 0.9735. \end{aligned}$$

c.)

$$P(X = 0)^3 = 0.7788^3 = 0.4724.$$

2. Suppose that X = the number of small aircraft arriving per hour at a particular airport can be modeled by a Poisson distribution with $\mu = 4.0$. Use the table of cumulative Poisson probabilities to find each of the following.

- (a) Find the chance that fewer than six small aircraft will arrive.
- (b) Find the chance that more than two small aircraft will arrive.
- (c) Find the standard deviation σ .
- (d) Find $P(\mu - \sigma < X < \mu + \sigma)$.

a.)

$$P(X < 6) = P(X \leq 5) = 0.785.$$

b.)

$$P(X > 2) = 1 - P(X \leq 2) = 1 - 0.238 = 0.762.$$

Remark. If X has a Poisson distribution with parameter μ , then $E(X) = V(X) = \mu$.

c.)

$$\sigma = \sqrt{V(X)} = \sqrt{\mu} = \sqrt{4} = 2.$$

d.)

$$\begin{aligned} P(\mu - \sigma < X < \mu + \sigma) &= P(2 < x < 6) = P(X \leq 5) - P(X \leq 2) \\ &= 0.785 - 0.238 = 0.547. \end{aligned}$$