

# Incline Plane Lab

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PHYS 253

Section 253D, 6:00 PM Tuesday

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## Abstract

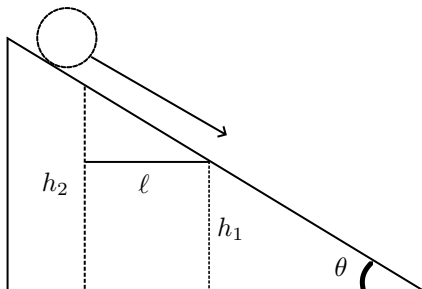
This study aims to determine the acceleration due to gravity ( $g$ ) by analyzing a cart's motion down an inclined track at three different angles. Using a motion detector and LoggerPro software, the data for the position and velocity were collected for nine trials (three per angle). Data analysis for this lab involves creating scatter plots and trendlines to compute experimental values for  $g$ . The results are then compared to the accepted  $g$  value ( $9.81 \text{ m/s}^2$ ) to evaluate the experiment's accuracy through percent error calculations.

## 1 Theory

From our everyday observations, we know that a ball will accelerate downhill if nothing stops it. Now, we aim to quantify this acceleration. To start, we must determine the slope of the incline by measuring its height at two points,  $h_1$  and  $h_2$ , set apart by a distance,  $\ell$ . Referring to Figure 1, these measurements help us identify two smaller triangles within a larger one, sharing the same angle,  $\theta$ , with the horizontal. Using basic trigonometry, we can calculate this angle from our measurement

$$\tan(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{h_2 - h_1}{\ell} \implies \theta = \tan^{-1}\left(\frac{h_2 - h_1}{\ell}\right) \quad (1)$$

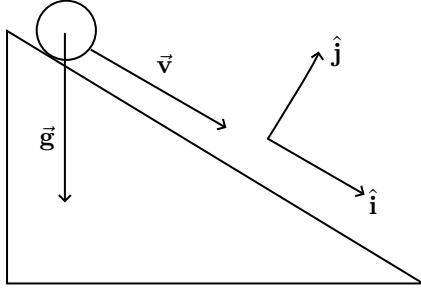
This measurement then enables us to correlate the acceleration we observe with  $g$ , the gravitational acceleration value.



**Figure 1:** Shows the similarity between the smaller and larger triangle, as well as labeling the measurements required for finding the angle  $\theta$

**Remark.** We have the flexibility to set up our coordinate systems in a manner that simplifies describing the ball's motion, positioning the x-direction parallel to the incline does just this (Figure 2). We can then consider the kinematic equations for position and velocity.

$$v(t) = v_0 + at$$
$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2.$$



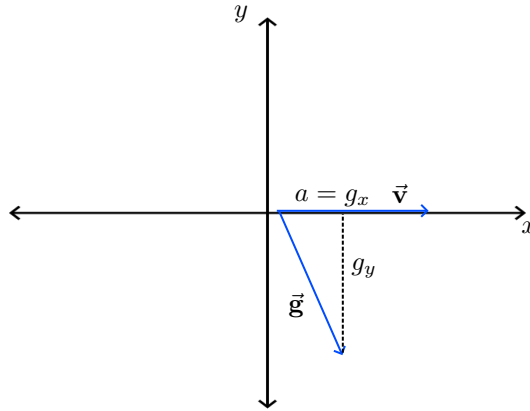
**Figure 2:** Shows how we define our coordinate access

Given the ability to define our coordinate system in such a way to aid in our computations, we set the ball's starting position at  $x = 0$ . With the ball beginning from a stationary state, its initial velocity,  $v_0$ , is also zero. Consequently, this simplification leads to the equation

$$v(t) = at \quad (2)$$

Next, we account for gravity acting directly downwards, at an angle to the plane. The component of gravity along the x-axis is responsible for the acceleration down the plane, which we define within our coordinate system as (figure 3)

$$a = g \sin(\theta) \quad (3)$$



**Figure 3:** Shows the relationship between  $g$  and  $g_x$ , as defined by our coordinate axis

We now substitute this value for  $a$  into equation 2.

$$v(t) = g \sin(\theta)t \quad (4)$$

When conducting measurements of position and velocity during the experiment, plotting the data should reveal a linear relationship between velocity and time, akin to the standard linear equation  $y = mx + b$ , where  $m$  represents the slope of the line. By analogy, with

$y = mx + b$ ,  $t$  serves as our independent variable, making the slope ( $m$ ) proportional to  $g$  through the relationship

$$m = g \sin(\theta) \Rightarrow g = \frac{m}{\sin(\theta)} \quad (5)$$

We can then evaluate our experimental  $g$  value against the accepted value of  $9.81 \text{ m/s}^2$  by calculating the percent error as follows:

$$\%Error = \left| \frac{\text{Accepted} - \text{Measured}}{\text{Accepted}} \right| \times 100\% \quad (6)$$

## 2 Data

The measurements taken for each trial are displayed in the following table

| Trial no. | $H_1$  | $H_2$  | $\ell$ | $\theta$    |
|-----------|--------|--------|--------|-------------|
| 1         | 13.5cm | 17cm   | 30cm   | $7.6^\circ$ |
| 2         | 12.5cm | 16cm   | 31cm   | $6.4^\circ$ |
| 3         | 9.5cm  | 12.5cm | 31cm   | $5.5^\circ$ |

**Table 1:** Shows the measurements and angle  $\theta$  for each trial. Recall that each trial will have three iterations

### 2.1 Colab data: Functions for graph creations

```
[16] import numpy as np

import matplotlib.pyplot as plt

def pvt(time, position):
    plt.scatter(time, position, label='Data points')

    slope, y_intercept = np.polyfit(time, position, 1)

    plt.plot(time, slope * time + y_intercept, color='red', label=f'Linear Equation: y = {slope:.2f}x + {y_intercept:.2f}')

    # Label the axes
    plt.xlabel('Time (s)')
    plt.ylabel('Position (m)')
    plt.grid(True)
    plt.legend()
    # Show the plot
    plt.show()

[21] # Function for
def tvv(time, velocity):
    plt.scatter(time, velocity, label='Data points')

    slope, y_intercept = np.polyfit(time, velocity, 1)

    plt.plot(time, slope * time + y_intercept, color='red', label=f'Linear Equation: y = {slope:.2f}x + {y_intercept:.2f}')

    # Label the axes
    plt.xlabel('Time (s)')
    plt.ylabel('velocity (m/s)')
    plt.grid(True)
    plt.legend()
    # Show the plot
    plt.show()
```

**Figure 4:** Shows the functions used to create the graphs

## 2.2 Colab data: Trial 1

```
# Trial 1 data as numpy arrays

time_1 = np.array([0.999, 1.0323, 1.0656, 1.0989, 1.1322, 1.1655, 1.1988,
1.2321, 1.2654, 1.2987, 1.332, 1.3653, 1.3986, 1.4319])

position_1 = np.array([0.18179, 0.195853, 0.217119, 0.2406145, 0.2663395,
0.293265, 0.3212195, 0.351232, 0.3819305, 0.416059, 0.449673, 0.485345, 0.5222175, 0.557718])

velocity_1 = np.array([0.2711768644, 0.47739746, 0.6253140641, 0.7187678303,
0.7813421547, 0.8263845095, 0.8714054054, 0.916576514, 0.9669836086, 1.009386512,
1.031167355, 1.030008571, 0.9416191191, 0.6762790916])

time_2 = np.array([0.5994, 0.6327, 0.666, 0.6993, 0.7326, 0.7659, 0.7992
, 0.8325, 0.8658, 0.8991, 0.9324, 0.9657, 0.999, 1.0323, 1.0656, 1.0989])

position_2 = np.array([0.713783, 0.74774, 0.776209, 0.8021055, 0.8288595,
0.8575, 0.8809955, 0.9094645, 0.930902, 0.938448,
0.9427355, 0.9571415, 0.9688035, 0.973091, 0.968632, 0.9770355])

velocity_2 = np.array([0.9726058559, 0.9183576076, 0.8474786662, 0.8155262763,
0.8120928428, 0.7813636136, 0.740913476, 0.6542407407,
0.4654019019, 0.3097386136, 0.2942237863, 0.2977215966,
0.2078729354, 0.08572854104, 0.07291754254, 0.0939044044])

time_3 = np.array([1.0323, 1.0656, 1.0989, 1.1322, 1.1655, 1.1988, 1.2321,
1.2654, 1.2987, 1.332, 1.3653, 1.3986])

position_3 = np.array([0.2070005, 0.231182, 0.2577645, 0.285719, 0.3157315,
0.346773, 0.381759, 0.416059, 0.4515595, 0.4892895, 0.5287345, 0.5702375])

velocity_3 = np.array([0.6061941316, 0.728853541, 0.8095821446, 0.8701393268,
0.9228210711, 0.9809963088, 1.027540791, 1.060222785,
1.06166416, 1.15906275, 1.20914796, 1.23987719])
```

Figure 5: Shows the data points in trial one iterations

## 2.3 Colab data: Trial 2

```
# Trial 2 data

time_2_1 = np.array([1.0656, 1.0989, 1.1322, 1.1655, 1.1988, 1.2321, 1.2654,
1.2987, 1.332, 1.3653, 1.3986, 1.4319, 1.4652])

position_2_1 = np.array([0.17836, 0.180418, 0.1895075, 0.2094015, 0.230839,
0.253477, 0.278173, 0.303898, 0.3304805, 0.358092,
0.389648, 0.4200035, 0.451388])

velocity_2_1 = np.array([0.07452696446, 0.2130445445, 0.4032996747, 0.5593063063,
0.6451206832, 0.7049268018, 0.7512137763, 0.7878013013,
0.8260411662, 0.8769632758, 0.9154391892, 0.9387436186, 0.9745586211])

time_2_2 = np.array([0.999, 1.0323, 1.0656, 1.0989, 1.1322, 1.1655, 1.1988,
1.2321, 1.2654, 1.2987, 1.332, 1.3653, 1.3986])

position_2_2 = np.array([0.2214065, 0.2437015, 0.267197, 0.292579, 0.319333,
0.3466015, 0.375242, 0.4073125, 0.4388685, 0.471282,
0.505239, 0.540568, 0.5770975])

velocity_2_2 = np.array([0.6213012387, 0.6844978729, 0.7337247247, 0.7766426426,
0.8132087087, 0.8514700325, 0.9020917167, 0.9421341341,
0.9679063438, 1.0009746, 1.035523524, 1.060287162, 1.064772085])

time_2_3 = np.array([0.9657, 0.999, 1.0323, 1.0656, 1.0989, 1.1322, 1.1655,
1.1988, 1.2321, 1.2654, 1.2987, 1.332, 1.3653, 1.3986])

position_2_3 = np.array([0.228781, 0.2515905, 0.2756005, 0.3009825, 0.327908,
0.355691, 0.3867325, 0.417088, 0.448987, 0.482258,
0.516558, 0.55223, 0.5891025, 0.6278615])

velocity_2_3 = np.array([0.6454425676, 0.7002916667, 0.7437460586, 0.7855910285,
0.82719995, 0.8749461336, 0.9133576702, 0.9421126752,
0.9785929054, 1.014944382, 1.052197135, 1.090823261,
1.126852853, 1.146530718])
```

Figure 6: Shows the data points in trial two iterations

## 2.4 Colab data: Trial 3

```
# Trail 3 data

time_3_1 = np.array([1.0656, 1.0989, 1.1322, 1.1655, 1.1988, 1.2321,
                    1.2654, 1.2987, 1.332, 1.3653, 1.3986, 1.4319, 1.4652])

position_3_1 = np.array([0.2385565, 0.2581075, 0.278173, 0.2992675,
                        0.321048, 0.343686, 0.367696, 0.394107,
                        0.419489, 0.4459, 0.4731685, 0.5012945, 0.5301065])

velocity_3_1 = np.array([0.563126001, 0.5933831331, 0.6184686562,
                        0.6447987988, 0.6724593969, 0.7057207833,
                        0.7445185811, 0.7693895145, 0.7848614239,
                        0.8075650025, 0.8317277903, 0.8564914289,
                        0.8835297172])

time_3_2 = np.array([0.9324, 0.9657, 0.999, 1.0323, 1.0656, 1.0989,
                    1.1322, 1.1655, 1.1988, 1.2321, 1.2654, 1.2987, 1.332])

position_3_2 = np.array([0.2224355, 0.240786, 0.2601655, 0.2807455,
                        0.3016685, 0.3236205, 0.34643, 0.370097,
                        0.396851, 0.4224045, 0.4488155, 0.4759125,
                        0.5043815])

velocity_3_2 = np.array([0.5259161662, 0.5654221096, 0.5967307307, 0.6219450075,
                        0.6467086461, 0.6750773899, 0.7061285035, 0.7458704955,
                        0.773295045, 0.7872648273, 0.8084233609, 0.8344101602,
                        0.8610621872])

time_3_3 = np.array([1.4985, 1.5318, 1.5651, 1.5984, 1.6317, 1.665,
                    1.6983, 1.7316, 1.7649, 1.7982, 1.8315, 1.8648, 1.8981])

position_3_3 = np.array([0.197911, 0.2148895, 0.232897, 0.2522765, 0.2721705,
                        0.292922, 0.3143595, 0.337169, 0.360493, 0.386561,
                        0.4117715, 0.4381825, 0.464765])

velocity_3_3 = np.array([0.4263036787, 0.5041138639, 0.5536840591, 0.5865806431,
                        0.6110223974, 0.6367516892, 0.6662362988, 0.697502002,
                        0.7337461837, 0.7610419795, 0.7782949825, 0.7986166166,
                        0.821942505])
```

**Figure 7:** Shows the data points in trial three iterations



### 3 Results

#### 3.1 Colab graphs and computations: Trial 1

##### 3.1.1 Iteration 1 Graphs

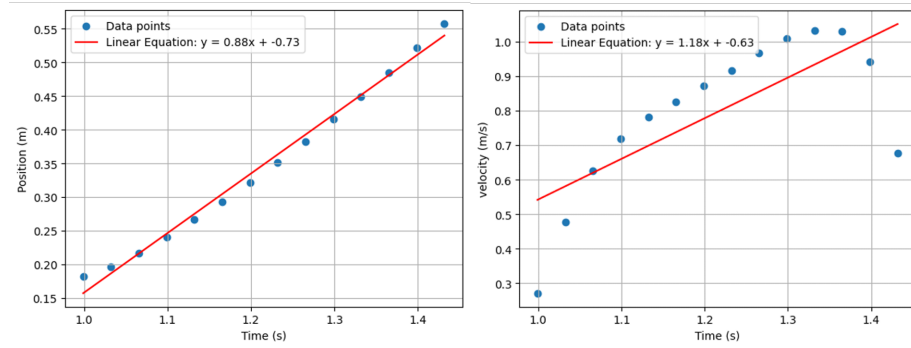


Figure 8:

##### 3.1.2 Iteration 1 Equations

The linear equation for the position vs time graph is given by

$$y = 0.88x - 0.73.$$

The linear equation for the velocity vs time graph is given by

$$y = 1.18x - 0.63.$$

##### 3.1.3 Iteration 2 Graphs

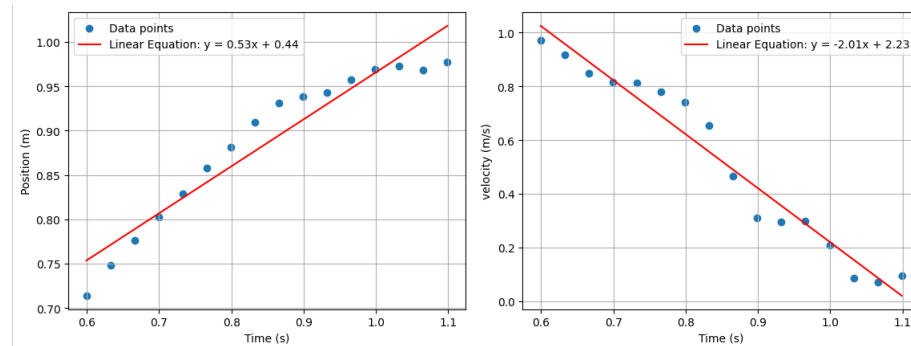


Figure 9:

### 3.1.4 Iteration 2 Equations

The linear equation for the position vs time graph is given by

$$y = 0.53x + 0.44.$$

The linear equation for the velocity vs time graph is given by

$$y = -2.01x + 2.23.$$

### 3.1.5 Iteration 3 Graphs

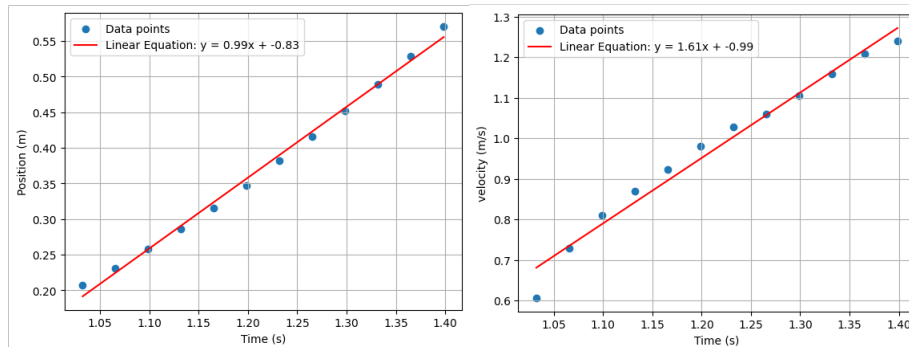


Figure 10:

### 3.1.6 Iteration 3 Equations

The linear equation for the position vs time graph is given by

$$y = .99x - 0.83.$$

The linear equation for the velocity vs time graph is given by

$$y = 1.61x - .99.$$

## 3.2 Colab graphs and computations: Trial 2

### 3.2.1 Iteration 1 Graphs

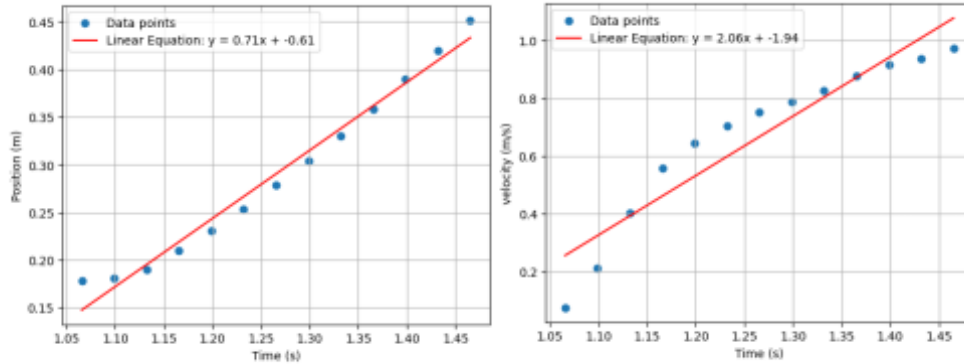


Figure 11:

### 3.2.2 Iteration 1 Equations

The linear equation for the position vs time graph is given by

$$y = 0.71x - 0.61.$$

The linear equation for the velocity vs time graph is given by

$$y = 2.06x - 1.94.$$

### 3.2.3 Iteration 2 Graphs

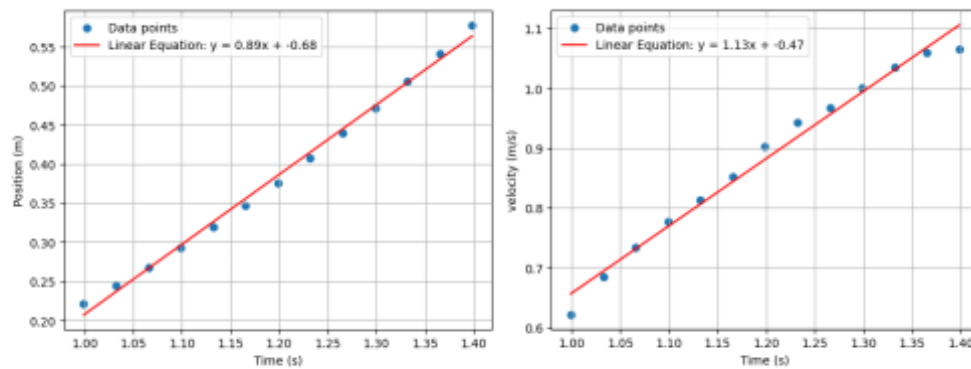


Figure 12:

### 3.2.4 Iteration 2 Equations

The linear equation for the position vs time graph is given by

$$y = 0.89x - 0.68.$$

The linear equation for the velocity vs time graph is given by

$$y = 1.13x - 0.47.$$

### 3.2.5 Iteration 3 Graphs

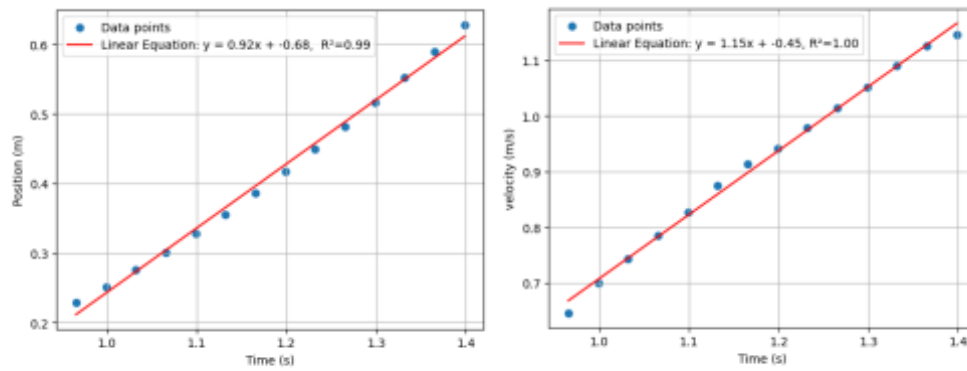


Figure 13:

### 3.2.6 Iteration 3 Equations

The linear equation for the position vs time graph is given by

$$y = 0.92x - 0.68.$$

With  $R^2 = 1$

The linear equation for the velocity vs time graph is given by

$$y = 1.15x - 0.45.$$

With  $R^2 = 1$

## 3.3 Colab graphs and computations: Trial 3

### 3.3.1 Iteration 1 Graphs

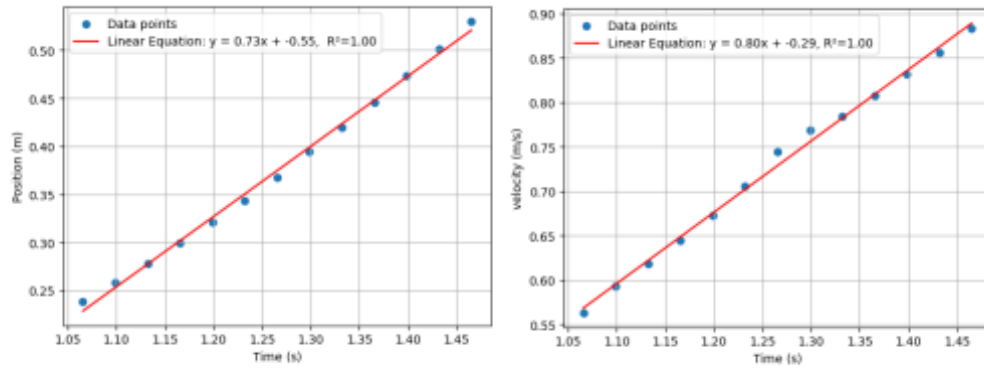


Figure 14:

### 3.3.2 Iteration 1 Equations

The linear equation for the position vs time graph is given by

$$y = 0.73x - 0.55.$$

With  $R^2 = 1$

The linear equation for the velocity vs time graph is given by

$$y = 0.80x - 0.29.$$

With  $R^2 = 1$

### 3.3.3 Iteration 2 Graphs

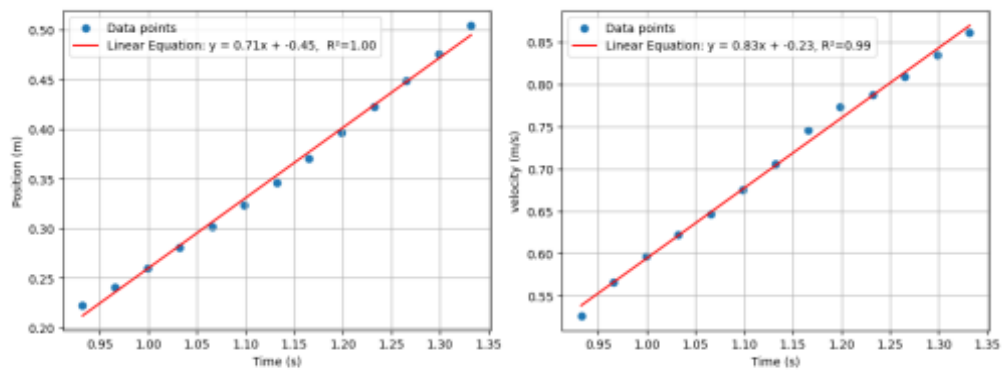


Figure 15:

### 3.3.4 Iteration 2 Equations

The linear equation for the position vs time graph is given by

$$y = 0.71x - 0.45.$$

With  $R^2 = 1$

The linear equation for the velocity vs time graph is given by

$$y = 0.83x - 0.23.$$

With  $R^2 = 0.99$

### 3.3.5 Iteration 3 Graphs

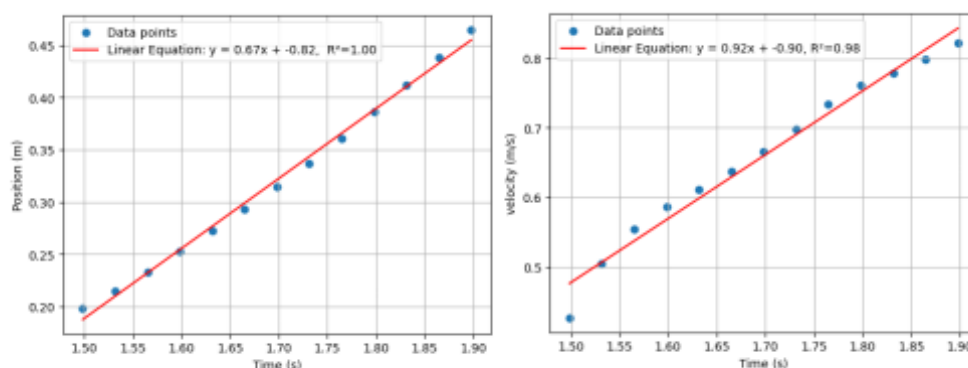


Figure 16:

### 3.3.6 Iteration 3 Equations

The linear equation for the position vs time graph is given by

$$y = 0.67x - 0.82.$$

With  $R^2 = 1$

The linear equation for the velocity vs time graph is given by

$$y = 0.92x - 0.90.$$

With  $R^2 = 0.98$

## 3.4 Experimental values for $g$

The computation for the experimental values of  $g$  shall be shown using trial one  $\theta = 7.6^\circ$ , and iteration one  $m = 0.88$ . Using equation 5, we see

$$\begin{aligned} g &= \frac{m}{\sin(\theta)} = \frac{1.18}{\sin(7.6^\circ)} \\ &= 8.92m/s^2. \end{aligned}$$

Note that we are using  $m$  from the velocity vs time graphs

The values for the remaining trials are found in the following table

| Trial & Iteration | $\theta$    | $m$  | $g$   |
|-------------------|-------------|------|-------|
| 1, 1              | $7.6^\circ$ | 1.18 | 8.92  |
| 1, 2              | $7.6^\circ$ | 2.01 | 15.2  |
| 1, 3              | $7.6^\circ$ | 1.61 | 12.17 |
| 2, 1              | $6.4^\circ$ | 2.06 | 18.48 |
| 2, 2              | $6.4^\circ$ | 1.13 | 10.14 |
| 2, 3              | $6.4^\circ$ | 1.15 | 10.23 |
| 3, 1              | $5.5^\circ$ | 0.80 | 8.35  |
| 3, 2              | $5.5^\circ$ | 0.83 | 8.7   |
| 3, 3              | $5.5^\circ$ | 0.92 | 9.6   |

**Table 2:** Shows the experimental values of  $g$  among each trial and iteration

### 3.5 Percent Error

We can then evaluate our experimental  $g$  value against the accepted value of  $9.81 \text{ m/s}^2$  by calculating the percent error. We demonstrate this with trial one iteration one. We compare our experimental value  $g = 8.92 \text{ m/s}^2$  with the accepted value  $9.8 \text{ m/s}^2$

$$\begin{aligned}\text{Percent Error} &= \left| \frac{9.8 \text{ m/s}^2 - 8.92 \text{ m/s}^2}{9.8 \text{ m/s}^2} \right| \times 100\% \\ &= 9\%.\end{aligned}$$

The value for the remaining trials are found in the following table

| Trial & Iteration | $\theta$    | $m$  | $g$   | % Error |
|-------------------|-------------|------|-------|---------|
| 1, 1              | $7.6^\circ$ | 1.18 | 8.92  | 9%      |
| 1, 2              | $7.6^\circ$ | 2.01 | 15.2  | 55.10%  |
| 1, 3              | $7.6^\circ$ | 1.61 | 12.17 | 24.18%  |
| 2, 1              | $6.4^\circ$ | 2.06 | 18.48 | 88.57%  |
| 2, 2              | $6.4^\circ$ | 1.13 | 10.14 | 3.47%   |
| 2, 3              | $6.4^\circ$ | 1.15 | 10.23 | 4.39%   |
| 3, 1              | $5.5^\circ$ | 0.80 | 8.35  | 14.80%  |
| 3, 2              | $5.5^\circ$ | 0.83 | 8.7   | 11.22%  |
| 3, 3              | $5.5^\circ$ | 0.92 | 9.6   | 2.04%   |

**Table 3:** Shows the percent error among experimental values of  $g$

## 4 Discussion

The investigation into measuring the acceleration due to gravity,  $g$ , revealed a range of experimental values, some that are close to the accepted  $9.81 \text{ m/s}^2$  and others far from it, with percent errors spanning from 2.04% to 88.57%. This discrepancy shows the sensitivity of our measurements due to various factors, likely the precision in determining the angle,  $\theta$ .

Furthermore, Friction and air resistance were not accounted for in our theoretical model, which played a role in the experimental outcomes. The assumption of a frictionless environment greatly simplifies the theoretical model but doesn't align with realistic conditions.

## 5 Conclusion

This lab explored the experimental determination of the acceleration due to gravity,  $g$ , providing a application of theoretical concepts discussed in our physics course. The process of measuring  $g$  across various inclines not only deepened my understanding of gravitational acceleration but also showed the importance of precision in physics. Despite the challenges, the experiment was a valuable exercise in applying theoretical knowledge to real-world scenarios.

In conclusion, the lab effectively showed the connection between theory and practice, demonstrating the principles of gravitational acceleration. The variations observed between the experimental and accepted values of  $g$  shows the complexities of physical measurements.