Discrete Structures
Combinatorics

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Contents

1 Factorials

Definition 1:

The **Factorial** for a positive integer n, is the product of all the positive integers that are less than or equal to n

$$n! = n \cdot (n-1) \cdots 3 \cdot 2 \cdot 1.$$

2 The fundamental counting principal (Basic counting principal)

Definition 2:

The **fundamental counting principal** says that if there are a ways of doing event 1, b ways of doing event 2, and c ways of doing event 3, then the total number of outcomes are (when no do events are dependent on each other)...

$$a \cdot b \cdot c.$$

Note: If there is dependency between the events, we must use addition

3 Permutations

Definition 3:

A **permutation** of a set is an arrangement of its members into a sequence or linear order. Order matters.

When dealing wit permutations, there are two cases that become important. They are:

- With repetition
- Without repetition

Theorem 1: Permutation with repetition

$$n^r.$$

Where n is the number of choices and r is the repetition.

Example. Suppose we want to know how many possible permutations can be made for a 4 digit lock using the digits 0-9. We would have:

$$\begin{aligned} &10 \times 10 \times 10 \times 10 \\ &= 10,000 \text{ Possible permutations.} \end{aligned}$$

Using the theorem:

$$\begin{aligned} &10^4 \\ &= 10,000 \text{ Possible permutations.} \end{aligned}$$

Theorem 2: Permutations without repetition

$$n!.$$

Where n is the number of choices

Example. Suppose we have 5 balls in a bag, and every time we pick a ball from the bag we do not replace it. Then we can compute the number of permutations by:

$$\begin{aligned} &5 \times 4 \times 3 \times 2 \times 1 \\ &= 120. \end{aligned}$$

Using the theorem:

$$\begin{aligned} &5! \\ &= 120. \end{aligned}$$

But what if r, the number of repetitions, is less than the choices? In this case we can use the following theorem:

Theorem 3

$$\frac{n!}{(n-r)!}.$$

Denoted by:

$$P(n, k).$$

4 Combinations

Definition 4:

A **combination** of a set is an arrangement of its members into a sequence or linear order. Order does not matter

Like permutations, we have two possibilities:

- When repetition is allowed
- When repetition is not allowed

Theorem 4: Combinations when repetition is not allowed

$$\frac{n!}{k!(n-k)!}.$$

Denoted by:

$$C(n,k) \quad \text{or} \quad \binom{n}{k}.$$

Theorem 5: Combinations when repetition is allowed

$$\frac{(k+n-1)!}{k!(n-1)!}.$$

5 Pigeonhole principle

Definition 5:

If n items are put into m containers with $n > m$, then at least one container must contain more than one item

6 Pascals Triangle

$n = 0$								1						
$n = 1$							1		1					
$n = 2$						1		2		1				
$n = 3$					1		3		3		1			
$n = 4$				1		4		6		4		1		
$n = 5$			1		5		10		10		5		1	
$n = 6$		1		6		15		20		15		6		1