**Comprehensive Compendium:**
Calculus II


**Nathan Warner**


Northern Illinois University


Computer Science
Northern Illinois University

August 28,2023
United States

## Contents

# 1 Calc II

## 1.1 Chapter 1 Key Equations

- **Mean Value Theorem For Integrals**: If $f(x)$ is continuous over an interval [a,b], then there is at least one point $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a} \int f(x)\ dx.$$

- **Integrals resulting in inverse trig functions**

1.

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{|a|} + C.$$

2.

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C.$$

3.

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{|a|} \sec^{-1} \frac{|x|}{a} + C.$$

## 1.2 Chapter 2 Key Terms / Ideas

- **Finding limits of integration for region between two functions**: Usually, we want our limits of integration to be the points where the functions intersect
- A **"complex region"** between curves usually refers to an area that is not easily described by a single, continuous function over the interval of interest.
- **compound regions** are regions bounded by the graphs of functions that cross one another
- **Cross-section:** The intersection of a plane and a solid object.
- a **cylinder** is a three-dimensional shape that has two parallel, congruent bases connected by a curved surface. The bases are usually circles, but they can be other shapes as well
- The line segment connecting the centers of the two bases is called the **"axis" of the cylinder.**
- **Slicing method:** A method of calculating the volume of a solid that involves cutting the solid into pieces, estimating the volume of each piece, then adding these estimates to arrive at an estimate of the total volume; as the number of slices goes to infinity, this estimate becomes an integral that gives the exact value of the volume.
  1. Examine the solid and determine the shape of a cross-section of the solid. It is often helpful to draw a picture if one is not provided.
  2. Determine a formula for the area of the cross-section.
  3. Integrate the area formula over the appropriate interval to get the volume.
- **Solid of revolution:** A solid generated by revolving a region in a plane around a line in that plane.
- **Disk method:** A special case of the slicing method used with solids of revolution when the slices are disks.
- A **Washer (Annuli)** is a disk with holes in the center.
- **Washer method:** A special case of the slicing method used with solids of revolution when the slices are washers.
- **Method of cylindrical shells:** A method of calculating the volume of a solid of revolution by dividing the solid into nested cylindrical shells; this method is different from the methods of disks or washers in that we integrate with respect to the opposite variable.
- **Arc length:** The arc length of a curve can be thought of as the distance a person would travel along the path of the curve.
- **Catenary:** A curve in the shape of the function $y = a\cosh(x/a)$ is a catenary; a cable of uniform density suspended between two supports assumes the shape of a catenary.
- **Center of mass:** The point at which the total mass of the system could be concentrated without changing the moment.
- **Centroid:** The centroid of a region is the geometric center of the region; laminas are often represented by regions in the plane; if the lamina has a constant density, the center of mass of the lamina depends only on the shape of the corresponding planar region; in this case, the center of mass of the lamina corresponds to the centroid of the representative region.
- **Density function:** A density function describes how mass is distributed throughout an object; it can be a linear density, expressed in terms of mass per unit length; an area density, expressed in terms of mass per unit area; or a volume density, expressed in terms of mass per unit volume; weight-density is also used to describe weight (rather than mass) per unit volume.
- **Doubling time:** If a quantity grows exponentially, the doubling time is the amount of time it takes the quantity to double, and is given by $\frac{\ln 2}{k}$.

- **Exponential decay:** Systems that exhibit exponential decay follow a model of the form $y = y_0 e^{-kt}$.
- **Exponential growth:** Systems that exhibit exponential growth follow a model of the form $y = y_0 e^{kt}$.
- **Frustum:** A portion of a cone; a frustum is constructed by cutting the cone with a plane parallel to the base.
- **Half-life:** If a quantity decays exponentially, the half-life is the amount of time it takes the quantity to be reduced by half. It is given by $\frac{\ln 2}{k}$.
- **Hooke's Law:** This law states that the force required to compress (or elongate) a spring is proportional to the distance the spring has been compressed (or stretched) from equilibrium; in other words, $F = kx$, where $k$ is a constant.
- **Hydrostatic pressure:** The pressure exerted by water on a submerged object.
- **Lamina:** A thin sheet of material; laminas are thin enough that, for mathematical purposes, they can be treated as if they are two-dimensional.
- **Moment:** If $n$ masses are arranged on a number line, the moment of the system with respect to the origin is given by $M = \sum_{i=1}^{n} m_i x_i$; if, instead, we consider a region in the plane, bounded above by a function $f(x)$ over an interval $[a, b]$, then the moments of the region with respect to the $x$- and $y$-axes are given by $M_x = \rho \int_a^b \frac{[f(x)]^2}{2} dx$ and $M_y = \rho \int_a^b x f(x) dx$, respectively.
- **Surface area:** The surface area of a solid is the total area of the outer layer of the object; for objects such as cubes or bricks, the surface area of the object is the sum of the areas of all of its faces.
- **Symmetry principle:** The symmetry principle states that if a region $R$ is symmetric about a line $l$, then the centroid of $R$ lies on $l$.
- **Theorem of Pappus for volume:** This theorem states that the volume of a solid of revolution formed by revolving a region around an external axis is equal to the area of the region multiplied by the distance traveled by the centroid of the region.
- **Work:** The amount of energy it takes to move an object; in physics, when a force is constant, work is expressed as the product of force and distance.

### 1.3 Chapter 2 Key Equations

- **Area between two curves, integrating on the x-axis**

$$A = \int_a^b [f(x) - g(x)]\, dx \tag{1}$$

Where $f(x) \geqslant g(x)$

$$A = \int_a^b \ [g(x) - f(x)] \ dx.$$

for $g(x) \geqslant f(x)$

- **Area between two curves, integrating on the y-axis**

$$A = \int_c^d [u(y) - v(y)]\, dy \tag{2}$$

- **Areas of compound regions**

$$\int_a^b \ |f(x) - g(x)| \ dx.$$

- **Area of complex regions**

$$\int_a^b \ f(x) \ dx + \int_b^c \ g(x) \ dx.$$

- **Slicing Method**

$$V(s) = \sum_{i=1}^{n} \ A(x_i^*) \ \Delta x = \int_a^b \ A(x) \ dx.$$

- **Disk Method along the x-axis**

$$V = \int_a^b \pi [f(x)]^2 \, dx \tag{3}$$

- **Disk Method along the y-axis**

$$V = \int_c^d \pi [g(y)]^2 \, dy \tag{4}$$

- **Washer Method along the x-axis**

$$V = \int_a^b \pi [(f(x))^2 - (g(x))^2] \, dx \tag{5}$$

- **Washer Method along the y-axis**

$$V = \int_c^d \pi [(u(y))^2 - (v(y))^2] \, dy \tag{6}$$

- **Radius if revolved around other line (Washer Method)**

$$\begin{aligned} If: \ x &= -k \\ Then: \ r &= Function + k. \end{aligned}$$

$$\begin{aligned} If: \ x &= k \\ Then: \ r &= k - Function. \end{aligned}$$

- **Method of Cylindrical Shells (x-axis)**

$$V = \int_a^b 2\pi x f(x)\, dx \tag{7}$$

- **Method of Cylindrical Shells (y-axis)**

$$V = \int_c^d 2\pi y g(y)\, dy \tag{8}$$

- **Region revolved around other line (method of cylindrical shells):**

$$If: \; x = -k$$
$$Then: \; V = \int_a^b \; 2\pi(x+k)(f(x)) \; dx.$$

$$If: \; x = k$$
$$Then: \; V = \int_a^b \; 2\pi(k-x)(f(x)) \; dx.$$

- **A Region of Revolution Bounded by the Graphs of Two Functions (method cylindrical shells)**

$$V = \int_a^b \; 2\pi x \left[f(x) - g(x)\right] \; dx.$$

- **Arc Length of a Function of x**

$$\text{Arc Length} = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx \tag{9}$$

- **Arc Length of a Function of y**

$$\text{Arc Length} = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy \tag{10}$$

- **Surface Area of a Function of x**

$$\text{Surface Area} = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\, dx \tag{11}$$

- **Mass of a one-dimensional object**

$$m = \int_a^b \rho(x)\, dx \tag{12}$$

- **Mass of a circular object**

$$m = \int_0^r 2\pi x \rho(x)\, dx \tag{13}$$

- **Work done on an object**

$$W = \int_a^b F(x)\, dx \tag{14}$$

- **Hydrostatic force on a plate**

$$F = \int_a^b \rho w(x) s(x)\, dx \tag{15}$$

- **Mass of a lamina**

$$m = \rho \int_a^b f(x)\, dx \tag{16}$$

- **Moments of a lamina**

$$M_x = \rho \int_a^b \frac{[f(x)]^2}{2}\, dx, \quad M_y = \rho \int_a^b x f(x)\, dx \tag{17}$$

- **Center of mass of a lamina**

$$\bar{x} = \frac{M_y}{m}, \text{ and } \bar{y} = \frac{M_x}{m} \tag{18}$$

- **Natural logarithm function**

$$\ln x = \int_1^x \frac{1}{t}\, dt\ Z \tag{19}$$

- **Exponential function**

$$y = e^x, \quad \ln y = \ln(e^x) = x\ Z \tag{20}$$

For the following exercises, draw the region bounded by the curves. Then, find the volume when the region is rotated around the y-axis.

$$y = 4 - x, y = x, x = 0.$$