Nate Warner CS 360 Spring 2025

Assignment 2 - Due: Fri, Jan 31

1. Convert the following binary numbers to their decimal representations:

   a. 11

   b. 1101

   c. 111011

   d. 0101

   e. 1101011

We have

(a) $11_2 = 1 \cdot 2^0 + 1 \cdot 2^1 = 3_{10}$

(b) $1101_2 = 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 = 13_{10}$

(c) $111011_2 = 1 \cdot 2^0 + 1 \cdot 2^1 + 0 + 1 \cdot 2^3 + 1 \cdot 2^4 + 1 \cdot 2^5 = 59_{10}$

(d) $0101_2 = 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 0 \cdot 2^3 = 5_{10}$

(e) $1101011_2 = 1 \cdot 2^0 + 1 \cdot 2^1 + 0 + 1 \cdot 2^3 + 0 + 1 \cdot 2^5 + 1 \cdot 2^6 = 107_{10}$

2. Convert the following hexadecimal numbers to their decimal representations

   a. 11

   b. $A1$

   c. CEF

   d. $BA9$

   e. $C89$

We have

(a) $11_{16} = 1 \cdot 16^0 + 1 \cdot 16^1 = 17_{10}$

(b) $A1_{16} = 1 \cdot 16^0 + 10 \cdot 16^1 = 161_{10}$

(c) $CEF_{16} = 15 \cdot 16^0 + 14 \cdot 16^1 + 12 \cdot 16^2 = 3311_{10}$

(d) $BA9_{16} = 9 \cdot 16^0 + 10 \cdot 16^1 + 11 \cdot 16^2 = 2985_{10}$

(e) $C89_{16} = 9 \cdot 16^0 + 8 \cdot 16^1 + 12 \cdot 16^2 = 3209$

3. Convert the following decimal numbers to both their hexadecimal and binary representations

a. 11

b. 4000

c. 42

d. 4095

a.) We first convert 11_{10} to its base two representation using the division algorithm. If n is an integer in its decimal representation, we divide n by two to get its quotient and remainder, we then express the remainder in base two representation and set $n = q$, where q is the quotient. We stop this procedure once we hit $q = 0$. We form the binary representation by working down the expressions, adding each remainder to the left of the existing representation.

$$\begin{aligned} 11 &= 2(5) + 1 : \; 1_{10} = 1_2 \\ 5 &= 2(2) + 1 : \; 2_{10} = 1_2 \\ 2 &= 2(1) + 0 : \; 0_{10} = 0_2 \\ 1 &= 2(0) + 1 : \; 1_{10} = 1_2 \end{aligned}$$

Thus, $11_{10} = 1011_2$. A similar algorithm converts 11_{10} to its hexadecimal representation

$$11 = 16(0) + 11 : \; 11_{10} = B_{16}$$

Thus, $11_{10} = B_{16}$

b.) In binary, we have

$$\begin{aligned} 4000 &= 2(2000) + 0 : \; 0_{10} = 0_2 \\ 2000 &= 2(1000) + 0 : \; 0_{10} = 0_2 \\ 1000 &= 2(500) + 0 : \; 0_{10} = 0_2 \\ 500 &= 2(250) + 0 : \; 0_{10} = 0_2 \\ 250 &= 2(125) + 0 : \; 0_{10} = 0_2 \\ 125 &= 2(62) + 1 : \; 1_{10} = 1_2 \\ 62 &= 2(31) + 0 : \; 0_{10} = 0_2 \\ 31 &= 2(15) + 1 : \; 1_{10} = 1_2 \\ 15 &= 2(7) + 1 : \; 1_{10} = 1_2 \\ 7 &= 2(3) + 1 : \; 1_{10} = 1_2 \\ 3 &= 2(1) + 1 : \; 1_{10} = 1_2 \\ 1 &= 2(0) + 1 : \; 1_{10} = 1_2 \end{aligned}$$

Thus, $4000_{10} = 111110100000_2$. For further conversions, we will omit part of the remainder conversion and simply state its representation. For example, $62 = 2(31) + 0 : \; 0_{10} = 0_2$ should simply be stated as $62 = 2(31) + 0 : 0_2$.

In hex, we have

$$\begin{aligned} 4000 &= 16(250) + 0 : \; 0_{16} \\ 250 &= 16(15) + 10 : \; A_{16} \\ 15 &= 16(0) + 15 : \; F_{16} \end{aligned}$$

Thus, $4000_{10} = FA0_{16}$

c.) Binary:

$$\begin{aligned}
42 &= 2(21) + 0 : \ 0_2 \\
21 &= 2(10) + 1 : \ 1_2 \\
10 &= 2(5) + 0 : \ 0_2 \\
5 &= 2(2) + 1 : \ 1_2 \\
2 &= 2(1) + 0 : \ 0_2 \\
1 &= 2(0) + 1 : \ 1_2
\end{aligned}$$

Thus, $42_{10} = 101010_{10}$. For hex,

$$\begin{aligned}
42 &= 16(2) + 10 : \ A_{16} \\
2 &= 16(0) + 2 : \ 2_{16}
\end{aligned}$$

Thus, $42_{10} = 2A_{16}$

d.) Binary:

$$\begin{aligned}
4095 &= 2(2047) + 1 : \ 1_2 \\
2047 &= 2(1023) + 1 : \ 1_2 \\
1023 &= 2(511) + 1 : \ 1_2 \\
511 &= 2(255) + 1 : \ 1_2 \\
255 &= 2(127) + 1 : \ 1_2 \\
127 &= 2(63) + 1 : \ 1_2 \\
63 &= 2(31) + 1 : \ 1_2 \\
31 &= 2(15) + 1 : \ 1_2 \\
15 &= 2(7) + 1 : \ 1_2 \\
7 &= 2(3) + 1 : \ 1_2 \\
3 &= 2(1) + 1 : \ 1_2 \\
1 &= 2(0) + 1 : \ 1_2
\end{aligned}$$

Thus, $4095_{10} = 111111111111_2$. For hex,

$$\begin{aligned}
4095 &= 16(255) + 15 : \ F_{16} \\
255 &= 16(15) + 15 : \ F_{16} \\
15 &= 16(0) + 15 : \ F_{16}
\end{aligned}$$

Thus, $4095_{10} = FFF_{16}$

4. Do the following binary arithmetic giving the answer in binary

   a. $10110 + 01101$

   b. $11001 + 00101$

   c. $10110 - 01101$

a.)

$$\begin{array}{cccccc} 1 & 1 & 1 & & & \\ & 1 & 0 & 1 & 1 & 0 \\ + & 0 & 1 & 1 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 & 1 & 1 \end{array}$$

b.)

$$\begin{array}{cccccc} & & & & 1 & \\ & 1 & 1 & 0 & 0 & 1 \\ + & 0 & 0 & 1 & 0 & 1 \\ \hline & 1 & 1 & 1 & 1 & 0 \end{array}$$

c.)

$$\begin{array}{cccccc} & 0 & 2 & & 0 & 2 \\ & \cancel{1} & \cancel{0} & 1 & \cancel{1} & \cancel{0} \\ - & 0 & 1 & 1 & 0 & 1 \\ \hline & 0 & 1 & 0 & 0 & 1 \end{array}$$

5. Do the following hexadecimal arithmetic giving the answer in hexadecimal

   (a) $82CD + 1982$

   (b) $E2C + A31$

   (c) $FB28 - 3254$

   (d) $E2C - A31$

a.)

$$\begin{array}{ccccc} & & 1 & & \\ & 8 & 2 & C & D \\ + & 1 & 9 & 8 & 2 \\ \hline & 9 & C & 4 & F \end{array}$$

b.)

$$\begin{array}{cccc} 1 & & & \\ & E & 2 & C \\ + & A & 3 & 1 \\ \hline 1 & 8 & 5 & D \end{array}$$

c.)

$$\begin{array}{ccccc} & & A & 18 & \\ & F & \cancel{B} & \cancel{2} & 8 \\ - & 3 & 2 & 5 & 4 \\ \hline & C & 8 & D & 4 \end{array}$$

d.)

$$\begin{array}{cccc} & D & 18 & C \\ & \cancel{E} & \cancel{2} & C \\ - & A & 3 & 1 \\ \hline & 3 & F & B \end{array}$$

6. Do the following arithmetic as if these were five-bit signed representations and indicate if overflow occurs and, if so, why

(a) $10110 + 01101$

(b) $11001 + 00101$

(c) $10110 - 01101$

(d) $11111 - 01011$

a.)

$$\begin{array}{cccccc} \boxed{1} & \boxed{1} & 1 & & & \\ & 1 & 0 & 1 & 1 & 0 \\ + & 0 & 1 & 1 & 0 & 1 \\ \hline & 0 & 0 & 0 & 1 & 1 \end{array}$$

Since the carry into the sign bit and the carry out of the sign bit (the boxed numbers) match, there is no overflow and the result is valid.

b.)

$$\begin{array}{cccccc} \boxed{0} & \boxed{0} & & & & \\ & 1 & 1 & 0 & 0 & 1 \\ + & 0 & 0 & 1 & 0 & 1 \\ \hline & 1 & 1 & 1 & 1 & 0 \end{array}$$

Since the boxed carries match, no overflow.

c.) We convert the subtrahend to its two's complement and add. 01101 has two's complement 10011. Thus,

$$\begin{array}{cccccc} \boxed{1} & \boxed{0} & 1 & 1 & & \\ & 1 & 0 & 1 & 1 & 0 \\ + & 1 & 0 & 0 & 1 & 1 \\ \hline & 0 & 1 & 0 & 0 & 1 \end{array}$$

Since the boxed carries match, we have overflow.

d.) We first convert the subtrahend to its two's complement, then add. 01011 has two's complement 10101. Thus,

$$\begin{array}{cccccc} \boxed{1} & \boxed{1} & 1 & 1 & 1 & \\ & 1 & 1 & 1 & 1 & 1 \\ + & 1 & 0 & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 & 0 \end{array}$$

Since the boxed carries match, no overflow.

7. Assume that

$$\text{Register 0 contains } 0007F144$$
$$\text{Register 1 contains } 00000028$$
$$\text{Register 7 contains } EC088840$$

If they are valid, calculate the absolute $D(X, B)$ addresses for the representations below. If they are not valid, explain why

(a) $56(, 1)$

(b) $0(0, 1, 7)$

(c) $6(7, 0)$

(d) $11(1, 7)$

Note: Remember that addresses are 24 bits long, NOT 32.)

Remark. When register zero is encountered, we ignore it in the calculations.

a.) First, we convert 56_{10} to its hexadecimal representation.

$$56 = 16(3) + 8 : \ 8_{16}$$
$$3 = 16(0) + 3 : \ 3_{16}$$

Thus, $56_{10} = 38_{16}$. Next, we add this to the contents of $R1$. Thus, we have $00000028 + 38 = 60$

Thus, the absolute address of $56(, 1)$ is 000060

b.) Not valid, notation has no meaning

c.) We add $6_{10} = 6_{16}$ to the contents of register 7. Thus, we have $EC088840+6 = EC088846$

Therefore, the absolute address $6(7, 0)$ is 088846

d.) First we add the contents of $R1$ and $R7$. We have $EC088840 + 00000028 = EC088868$. Then, we add $11_{10} = B_{16}$. Thus, $EC088868 + B = EC088873$

Therefore, the absolute address of $11(1, 7)$ is 088873