1.a Evaluate $g(0),\ g(1),\ g(2),\ g(3),\ g(6)$

$g(0)$

$$
\begin{aligned}
g(0) &= \int_0^0 f(t)\ dt \\
&= 0 \quad \text{(By integral with the same bounds equals zero).}
\end{aligned}
$$

$g(1)$

$$
\begin{aligned}
g(1) &= \int_0^1 f(t)\ dt \\
&= lw \\
&= 1 \cdot 2 \\
&= 2.
\end{aligned}
$$

$g(2)$

$$
\begin{aligned}
g(2) &= \int_0^2 ft\ dt \\
&= \left(lw\right) + \left(\frac{1}{2}bh\right) \\
&= \left(2 \cdot 2\right) + \left(\frac{1}{2}\left(1\right)\left(2\right)\right) \\
&= 4 + 1 \\
&= 5.
\end{aligned}
$$

$g(3)$

$$
\begin{aligned}
g(3) &= \int_0^3 f(t)\ dt \\
&= g(2) + \frac{1}{2}bh \\
&= 5 + \frac{1}{2}(1)(4) \\
&= 5 + 2 \\
&= 7.
\end{aligned}
$$

$g(6)$

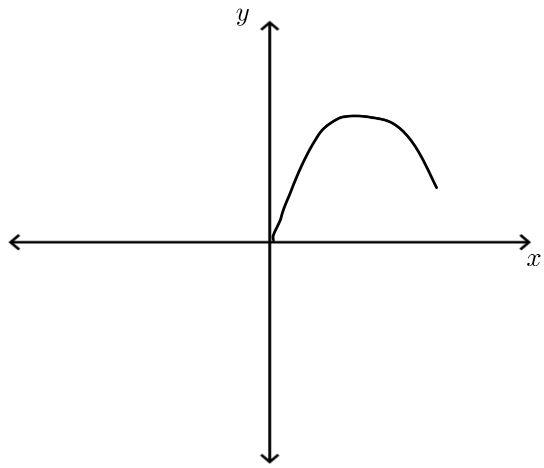
$$
\begin{aligned}
g(6) &= \int_0^6 ft\ dt \\
&= g(3) - \left(\int_3^6 ft\ dt\right) \\
&= 7 - \left(\int_3^5 f(t)\ dt\ + \int_5^6 f(t)\ dt\right) \\
&= 7 - \left(\frac{1}{2}(2)(2) + 1 \cdot 2\right) \\
&= 7 - \left(2 + 2\right) \\
&= 7 - 4 \\
&= 3.
\end{aligned}
$$

1.b g has a maximum value at $g(3)$

1.c g is increasing on the interval $(0, 3)$

1.d Rough sketch of g:

Figure:



2. Use part 1 of the Fundamental Theorem of Calculus to find the derivative of the functions

Remark. Part 1: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x),\ a \leqslant x \leqslant b$

2.a

$$\begin{aligned} If:\ g(x) &= \int_3^x \sqrt{9-t^2}\ dt \\ Then:\ g'(x) &= \frac{d}{dx}\int_3^x \sqrt{9-t^2}\ dt \\ &= \sqrt{9-x^2}. \end{aligned}$$

2.b

$$\begin{aligned} If:\ y &= \int_0^{\ln x} e^t\ dt \\ Then:\ y' &= \frac{d}{dx}\int_0^{\ln x} e^t\ dt \\ &= e^{\ln x}\cdot\frac{d}{dx}\ln x \\ &= e^{\ln x}\cdot\frac{1}{x} \\ &= \frac{e^{\ln x}}{x}. \end{aligned}$$

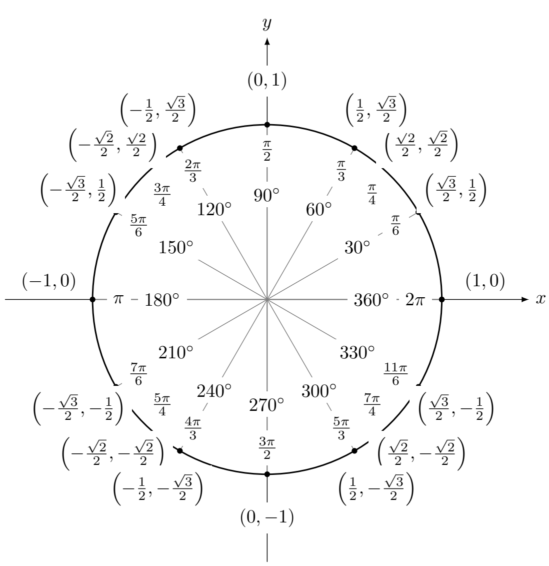
3. Use part 2 of the Fundamental Theorem of Calculus to evaluate the integrals.

Remark. Part 2: $\int_a^b\ f(x)\ dx = F(b) - F(a)$ where $F'(x) = f(x)$

3.a $\int_1^4\ \frac{2-x^{\frac{1}{2}}}{x^2}\ dx$

$$
\begin{aligned}
&\int_1^4\ \frac{2-x^{\frac{1}{2}}}{x^2}\ dx \\
&= \int_1^4\ \frac{2}{x^2} - \frac{x^{\frac{1}{2}}}{x^2}\ dx \\
&= \int_1^4\ 2x^{-2} - x^{-\frac{3}{2}}\ dx \\
&= -2x^{-1} + 2x^{-\frac{1}{2}}\Big]_1^4 \\
&= \Big(-2(4)^{-1} + 2(4)^{-\frac{1}{2}}\Big) - \Big(-2(1)^{-1} + 2(1)^{-\frac{1}{2}}\Big) \\
&= \left(-\frac{2}{4} + \frac{2}{\sqrt{4}}\right) - \cancelto{0}{\Big(-2+2\Big)} \\
&= -\frac{1}{2} + 1 \\
&= \frac{1}{2}.
\end{aligned}
$$

3.b



$$
\begin{aligned}
&\int_0^\pi\ (\sin x - 3\sqrt{x})\ dx \\
&= \int_0^\pi\ (\sin x - 3x^{\frac{1}{2}})\ dx \\
&= -\cos x - 2x^{\frac{3}{2}}\Big]_0^\pi \\
&= \Big(-\cos\pi - 2(\pi)^{\frac{3}{2}}\Big) - \Big(-\cos(0) - 2(0)^{\frac{3}{2}}\Big) \\
&= \Big(-(-1) - 2\pi^{\frac{1}{2}}\Big) - \Big(-1\Big) \\
&= 1 - 2\pi^{\frac{3}{2}} + 1 \\
&= 2 - 2\pi^{\frac{3}{2}}.
\end{aligned}
$$

4. Evaluate the following integrals

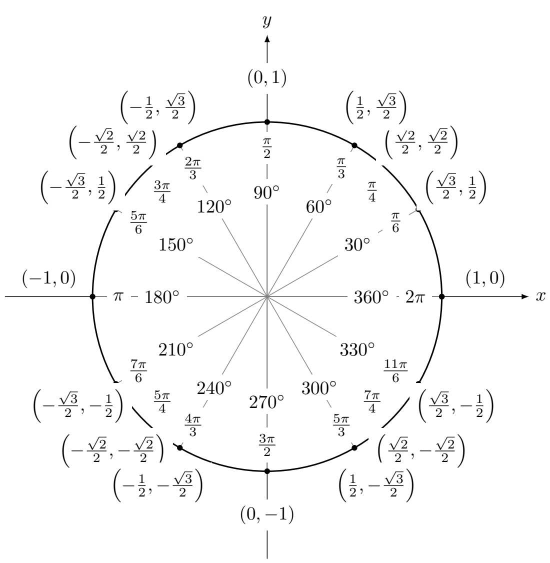
4.a

$$\begin{aligned}
&\int x^{\frac{1}{2}}(x^2+5x+2)\ dx \\
&= \int x^{\frac{1}{2}+2}+5x^{1+\frac{1}{2}}+2x^{\frac{1}{2}}\ dx \\
&= \int x^{\frac{5}{2}}+5x^{\frac{3}{2}}+2x^{\frac{1}{2}}\ dx \\
&= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{5}{2}}+\frac{4}{3}x^{\frac{3}{2}}+C.
\end{aligned}$$

4.b

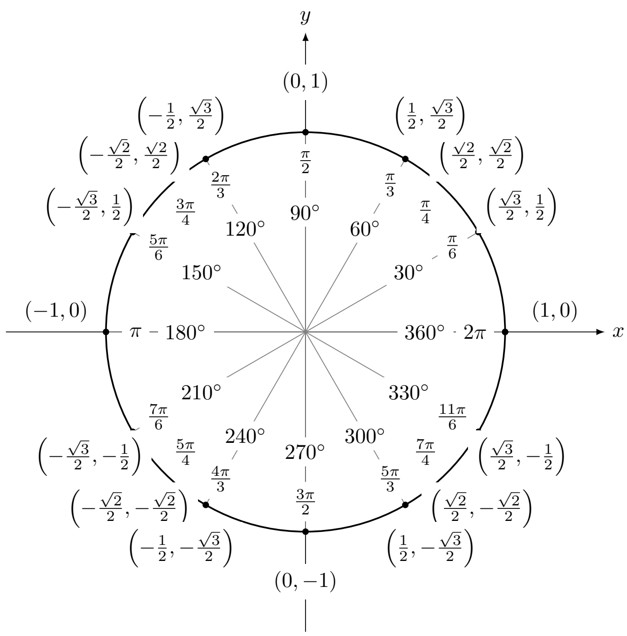
$$\begin{aligned}
&\int_1^2 \left(\frac{1}{x^2}-\frac{1}{x^3}\right) dx \\
&= \int_1^2 x^{-2}-x^{-3}\ dx \\
&= -x^{-1}+\frac{1}{2}x^{-2}\Big]_1^2 \\
&= \left(-(2)^{-1}+\frac{1}{2}(2)^{-2}\right)-\left(-(1)^{-1}+\frac{1}{2}(1)^{-2}\right) \\
&= \left(-\frac{1}{2}+\frac{1}{8}\right)-\left(-1+\frac{1}{2}\right) \\
&= -\frac{3}{8}-\left(-\frac{1}{2}\right) \\
&= -\frac{3}{8}+\frac{1}{2} \\
&= \frac{1}{8}.
\end{aligned}$$

4.c



$$\begin{aligned}
&\int_0^{\pi} (2\sin x-3\sec^2 x)\ dx \\
&= -2\cos x-3\tan x\Big]_0^{\pi} \\
&= \left(-2\cos\pi-3\tan\pi\right)-\left(-2\cos 0-3\tan 0\right) \\
&= \left(-2(-1)-3(0)\right)-\left(-2(1)-3(0)\right) \\
&= 2-(-2) \\
&= 4.
\end{aligned}$$

4.d



$$\int_0^{\frac{\pi}{4}} \sec x \tan x \; dx$$
$$= \sec x \Big]_0^{\frac{\pi}{4}}$$
$$= \sec \frac{\pi}{4} - \sec 0$$
$$= \frac{1}{\frac{\sqrt{2}}{2}} - 1$$
$$= \frac{2\sqrt{2}}{2} - 1$$
$$= \sqrt{2} - 1.$$

5. Find displacement and distance traveled of $t^2 - 3t - 18$, $0 \leqslant t \leqslant 6$

Remark. Displacement: $\int_a^b \; v(t) \; dt$ and Distance Traveled: $\int_a^b \; |v(t)| \; dt$

Displacement:

$$\int_0^6 \; t^2 - 3t - 18 \; dt$$
$$= \frac{1}{3}t^3 - \frac{3}{2}t^2 - 18t \Big]_0^6$$
$$= \left(\frac{1}{3}(6)^3 - \frac{3}{2}(6)^2 - 18(6)\right) - \cancelto{0}{\left(\frac{1}{3}(0)^3 - \frac{3}{2}(0)^2 - 18(0)\right)}$$
$$= 72 - 54 - 108$$
$$= -90.$$

Distance Traveled:

$$\int_0^6 \; |t^2 - 3t - 18| \; dt.$$

Find where the function turns negative

$$t^2 - 3t - 18 = 0$$
$$(t+3)(t-6)$$
$$t = \cancel{-3}$$[1]$$, 6$$

Rewrite Piecewise

$$v(t) = \begin{cases} t^2 - 3t - 18 & \text{if } t > 6 \\ -(t^2 - 3t - 18) & \text{if } t \leqslant 6 \end{cases} \tag{1}$$

[1] not a solution

Thus:

$$
\begin{aligned}
&\int_0^6 -(t^2 - 3t - 18)\ dt \\
&= \int_0^6 -t^2 + 3t + 18\ dt \\
&= -\frac{1}{3}t^3 + \frac{3}{2}t^2 + 18t\Big]_0^6 \\
&= \left(-\frac{1}{3}(6)^3 + \frac{3}{2}(6)^2 + 18(6)\right) - \cancelto{0}{\left(-\frac{1}{3}(0)^3 + \frac{3}{2}(0)^2 + 18(0)\right)} \\
&= 90.
\end{aligned}
$$