Math 170: Quiz 3

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1. Using the graph of the function f , find the following. Circle your final answers.

1.a: Local Min: $f(2) = 3$

1.b: Local Max $f(4) = 5$

1.c: Absolute Min: None

1.d: Absolute Max: $f(0) = 5$ and $f(4) = 5$

3. The height in feet of a given body about the surface of the earth at time t seconds is given.

$$y = 1000 + 160t - 16t^2, \; 3 \leqslant t \leqslant 6.$$

Find:

a.) The maximum and minimum height of the body

b.) The maximum and minimum velocity v of the body, and

c.) The maximum and minimum speed s of the body

During the given time interval.

Closed Interval: [3,6]

3.a:

y':

$$y' = 160t - 32t.$$

Set $y' = 0$

$$\begin{aligned} 160 - 32t &= 0 \\ -32t &= -160 \\ t &= \frac{-160}{-32} \\ t &= 5. \end{aligned}$$

Since $5 \in [3, 6]$, 5 is a critical value

y' DNE

There are no values of t that make this function undefined

Plug critical values into y:

$$y(5) = 1000 + 160(5) - 16(5)^2$$
$$\boxed{= 1400}.$$

Find $y(a)$ and $y(b)$:

$$y(3) = 1000 + 160(3) - 16(3)^2$$
$$\boxed{= 1336}.$$

$$y(6) = 1000 + 160(6) - 16(6)^2$$
$$\boxed{= 1384}.$$

Therefore:

Max Height: f(5) = 1400 ft Min Height: f(3) = 1336 ft.

3.b:

Find v(t):

$$v(t) = 160 - 32t.$$

$v'(t)$:

$$v'(t) = -32.$$

Set $v'(t) = 0$:

$$-32 = 0.$$

Since $-32 \notin [3, 6]$, -32 is not a critical value

Find $v(a)$ and $v(b)$:

$$\begin{aligned} v(3) &= 160 - 32(3) \\ &\boxed{= 64\ ft\setminus s}. \end{aligned}$$

$$\begin{aligned} v(6) &= 160 - 32(6) \\ &\boxed{= -32\ ft\setminus s}. \end{aligned}$$

Therefore:

Max Velocity: v(3) = 64 ft$\setminus$ s

Min Velocity: v(6) = -32 ft$\setminus$s

3.c:

Since $s(t) = |v(t)|$, we can take the absolute value of our values from 3.b and retrieve that maximum and minimum speeds:

$$\begin{aligned} s(3) &= |160 - 32(3)| \\ &\boxed{= 64\ ft\setminus s}. \end{aligned}$$

$$\begin{aligned} s(6) &= |160 - 32(6)| \\ &\boxed{= 32\ ft\setminus s}. \end{aligned}$$

Therefore:

Max Speed: s(3) = 64 $ft\setminus s$

Min Speed: s(6) = 32 $ft\setminus s$

4. Find the limit using l'Hospital's Rule, if the rule is appropriate.

4.a:

$$\lim_{x\to 1} \frac{3x^2+2x-5}{x^4+3x^2-4}.$$

$$\lim_{x\to 1} 3x^2+2x-5 = 0 \ \ \textit{and} \ \ \lim_{x\to 1} x^4+3x^2-4 = 0.$$

Since we have an indeterminate form of the type $\frac{0}{0}$, we can use L'Hospital's Rule to evaluate the limit.

So:

$$\begin{aligned} L'H &= \lim_{x\to 1} \frac{6x+2}{4x^3+6x} \\ &= \frac{8}{10} \end{aligned}$$

$$= \frac{4}{5}$$

4.b:

$$\lim_{x\to\infty} \frac{e^{2+\ln x}}{3x+4}.$$

$$\lim_{x\to\infty} e^{2+\ln x} = \infty \ \textit{and} \ \lim_{x\to\infty} 3x+4 = \infty.$$

Since we have an indeterminate form of the type $\frac{\infty}{\infty}$, We must use L'Hospital's Rule.

$$\begin{aligned} L'H &= \lim_{x\to\infty} \frac{e^{2+\ln x}\cdot\frac{1}{x}}{3} \\ L'H &= \lim_{x\to\infty} \frac{\frac{e^{2+\ln x}}{x}}{3} \\ L'H &= \lim_{x\to\infty} \frac{e^{2+\ln x}}{3x} \end{aligned}.$$

At it's current state, we are stuck in a loop of applying L'Hospital's Rule to no avail, therefore we must try and simpliy the equation

$$\begin{aligned} L'H &= \lim_{x\to\infty} \frac{e^{2+\ln x}}{3x} \\ L'H &= \lim_{x\to\infty} \frac{e^2\cdot e^{\ln x}}{3x} \\ L'H &= \lim_{x\to\infty} \frac{e^2\cdot x}{3x} \\ L'H &= \lim_{x\to\infty} \frac{e^2}{3} \end{aligned}.$$

$$= \frac{e^2}{3}$$

4.c:

$$\lim_{x\to\pi} \frac{\sin^2 x}{(x-\pi)^2}.$$

$$\lim_{x\to\pi} \sin^2 x = 0 \textit{ and } \lim_{x\to\pi} (x-\pi)^2 = 0.$$

Since we have an indeterminate form of the type $\frac{0}{0}$, we can use L'Hospital's Rule

$$L'H = \lim_{x\to\pi} \frac{2\sin x \cos x}{2(x-\pi)(1)}$$
$$L'H = \lim_{x\to\pi} \frac{2\sin x \cos x}{2x-2\pi)}$$
$$.$$

$$\lim_{x\to\pi} 2\sin(x)\cos(x) = 0 \textit{ and } \lim_{x\to\pi} 2x - 2\pi = 0.$$

We still have the indeterminate form of the type $\frac{0}{0}$, so once again, we must use L'Hospital's Rule

$$L'H = \lim_{x\to\pi} \frac{2[(\sin x)(-\sin x) + (\cos x)(\cos x)]}{2}$$
$$L'H = \lim_{x\to\pi} \frac{2[-\sin^2 x + \cos^2 x]}{2}$$
$$L'H = \lim_{x\to\pi} \frac{-2\sin^2 x + 2\cos^2 x}{2}.$$

From here if we evaluate:

$$\lim_{x\to\pi} -2\sin^2 x + \cos^2 x.$$

We get:

$$\begin{aligned} -2\sin^2 \pi + 2\cos^2 \pi \\ = -2(0)^2 + 2(-1)^2 \\ = 2. \end{aligned}$$

Therefore we have

$$\frac{2}{2}$$
$$.$$

$$1$$