**PSET 11 - Due: Thursday, August 1**

1. Determine whether or not each of the following is a valid pair of hypotheses.
   (a) $H_0 : \bar{x} = 5$ versus $H_A : \bar{x} < 5$
   (b) $H_0 : p = 0.70$ versus $H_A : p \neq 0.70$
   (c) $H_0 : \mu = 5$ versus $H_A : \mu \geqslant 5$
   (d) $H_0 : p = 0.30$ versus $H_A : p = 0.50$
   (e) $H_0 : \mu = 5$ versus $H_A : \mu < 5$

a.) Invalid, hypothesis testing typically involves population parameters, such as $\mu$, not a sample statistic.

b.) Valid

c.) Invalid, $H_a$ should not involve equality

d.) Invalid $H_a$ should not involve equality

e.) Valid

2. Determine whether or not each of the following statements is correct.

   (a) The value of the test statistic lies in the rejection region. Therefore, there is sufficient evidence to suggest the alternative hypothesis is true.

   (b) The value of the test statistic does not lie in the rejection region. Therefore, there is evidence to suggest the null hypothesis is true.

   (c) The value of the test statistic does not lie in the rejection region. Therefore, there is insufficient evidence to suggest the alternative hypothesis is true.

a.) Correct, if the test statistic lies in the rejection region, we have evidence to reject the null hypothesis and claim the alternative hypothesis is true.

b.) Incorrect, if the evidence is not enough to reject the null hypothesis, this does not mean the null hypothesis is true, only there is insufficient evidence to support it.

c.) Correct, if the value of the test statistic does not lie in the rejection region, we do not reject the null hypothesis and do not claim the alternative is true.

**IMPORTANT** - Problems 3, 4, and 5 ask you to perform a test of hypotheses. You should (i) explicitly give both hypotheses, (ii) calculate the test statistic, (iii) give the rejection region, (iv) give a decision, and (v) write a conclusion that relates your decision to the context of the problem.

3. Minor surgery on horses under field conditions requires a reliable short-term anesthetic producing good muscle relaxation, minimal cardiovascular and respiratory changes, and a quick, smooth recovery with minimal aftereffects so that horses can be left unattended. A research article reports that, for a sample of 73 horses to which ketamine was administered under certain conditions, the sample average lying-down time was 18.9 minutes. Does this data suggest that the true mean lying-down time is less than 20 minutes? Assume that lying-down time varies according to a normal distribution with population standard deviation 8.6 minutes. Test using the level of significance $\alpha = 0.10$.

We have $n = 73$, $\bar{x} = 18.9$, $\sigma = 8.6$, and $\alpha = 0.1$. Furthermore,

$$H_0: \ \mu = 20$$
$$H_a: \ \mu < 20.$$

For a level of significance $\alpha = 0.1$ and a left-tailed test, we have $-Z_{0.1} = -1.28$. Thus, the rejection region is $z < -1.28$.

With this, we calculate the test statistic

$$\begin{aligned} Z &= \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{18.9 - 20}{\frac{8.6}{\sqrt{73}}} = -1.0928. \end{aligned}$$

Since $z \not< -z_{0.1}$, we do not reject the null hypothesis. The evidence does not support the claim that the true mean lying-down time is less than 20 minutes.

4. White blood cells are the body's natural defense mechanism against disease and infection. The mean white blood cell count in healthy adults, measured as part of a CBC (complete blood count), is approximately 7.5 ($\times 10^3/\mu$L). A company developing a new drug to treat arthritis pain must check for any side effects. A random sample of 21 patients using the new drug had a mean white blood cell count of 8.0 ($\times 10^3/\mu$L). Assume the distribution of white blood cell counts is normal and $\sigma = 1.1$ ($\times 10^3/\mu$L). Conduct a hypothesis test to determine whether there is any change in the mean white blood cell count due to the arthritis drug. Test using $\alpha = 0.05$.

We have $n = 21$, $\bar{x} = 8000$, $\sigma = 1100$, and $\alpha = 0.05$. Furthermore,

$$H_0 : \ \mu = 7500$$
$$H_a : \ \mu \neq 7500.$$

For a two-tailed test with level of significance $\alpha = 0.05$, we have $-Z_{0.025} = -1.96$ and $Z_{0.025} = 1.96$. Thus, the region of rejection is $|Z| > 1.96$. The test statistic is

$$\begin{aligned} Z &= \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{8000 - 7500}{\frac{1100}{\sqrt{21}}} = 2.083. \end{aligned}$$

Since $Z = 2.083 > 1.96$, we reject the null hypothesis in favor of the alternate. Thus, there is evidence to support the claim of the mean white blood cell count changing due to the arthritis drug.

5. The biological dessert in the Gulf of Mexico called the Dead Zone is a region in which there is very little or no oxygen. Most marine life in the Dead Zone dies or leaves the region. The area of this region varies and is affected by agriculture, fertilizer runoff, and weather. The long-term mean area of the Dead Zone is 5960 square miles. As a result of recent flooding in the Midwest and subsequent runoff from the Mississippi River, researchers believe that the Dead Zone area will increase. A random sample of 50 days was obtained and the sample mean area of the Dead Zone was 6759 $\text{mi}^2$ with a sample standard deviation of 1850 $\text{mi}^2$. Does the sample provide enough evidence to confirm the researchers' belief? Test using $\alpha = 0.025$.

We have $n = 50$, $\bar{x} = 6759$, $\sigma = 1850$, and $\alpha = 0.025$. Furthermore,

$$\begin{aligned} H_0 &: \ \mu = 5960 \\ H_a &: \ \mu > 5960. \end{aligned}$$

For a right-tailed test with level of significance $\alpha = 0.025$, we have $Z_{0.025} = 1.96$. Thus, the area of rejection is $Z > 1.96$.

The test statistic is given by

$$\begin{aligned} Z &= \frac{6759 - 5960}{\frac{1850}{\sqrt{50}}} \\ &= 3.0539. \end{aligned}$$

Since $Z = 3.0539 > 1.96$, we reject the null hypothesis. There is evidence to suggest the mean area of the Dead Zone has increased due to recent flooding and subsequent runoff

6. For each of the following find (i) the p value of the hypothesis test and (ii) compare the p value to the given level of significance and give a decision regarding the test of hypotheses.

(a) $H_0 : \mu = 10$ versus $H_A : \mu < 10$, $Z = -1.72$, $\alpha = 0.05$

(b) $H_0 : \mu = 50$ versus $H_A : \mu > 50$, $Z = 1.15$, $\alpha = 0.10$

(c) $H_0 : \mu = 100$ versus $H_A : \mu \neq 100$, $Z = 1.75$, $\alpha = 0.05$

a.) The p-value for this left-tailed test is

$$\Phi(-1.72) = 0.0427.$$

Since $p - value < \alpha$, reject the null hypothesis. There is evidence to suggest $\mu < 10$

b.) The p-value for this right-tailed test is

$$1 - \Phi(1.15) = 0.1251.$$

Since $p - value > \alpha$, do not reject the null hypothesis. There is Insufficient evidence to suggest $\mu > 50$

c.) The p-value for this two-tailed test is

$$2(1 - \Phi(1.75)) = 0.0802.$$

Since $p - value > \alpha$, do not reject the null hypothesis. There is insufficient evidence to suffest $\mu \neq 100$