Calculus 2
Problem Set 1.

Nathan Warner

Computer Science
Northern Illinois University
August 27, 2023
United States

1.a Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(6)$

$g(0)$

$$g(0) = \int_0^0 f(t)\ dt$$
$$= 0 \quad \text{(By integral with the same bounds equals zero)}.$$

$g(1)$

$$g(1) = \int_0^1 f(t)\ dt$$
$$= lw$$
$$= 1 \cdot 2$$
$$= 2.$$

$g(2)$

$$g(2) = \int_0^2 ft\ dt$$
$$= \left(lw\right) + \left(\frac{1}{2}bh\right)$$
$$= \left(2 \cdot 2\right) + \left(\frac{1}{2}(1)(2)\right)$$
$$= 4 + 1$$
$$= 5.$$

$g(3)$

$$g(3) = \int_0^3 f(t)\ dt$$
$$= g(2) + \frac{1}{2}bh$$
$$= 5 + \frac{1}{2}(1)(4)$$
$$= 5 + 2$$
$$= 7.$$

$g(6)$

$$g(6) = \int_0^6 ft\ dt$$
$$= g(3) - \left(\int_3^6 ft\ dt\right)$$
$$= 7 - \left(\int_3^5 f(t)\ dt + \int_5^6 f(t)\ dt\right)$$
$$= 7 - \left(\frac{1}{2}(2)(2) + 1 \cdot 2\right)$$
$$= 7 - \left(2 + 2\right)$$
$$= 7 - 4$$
$$= 3.$$

1.b g has a maximum value at $g(3)$

1.c g is increasing on the interval $(0, 3)$

1.d Rough sketch of g:

      Figure:
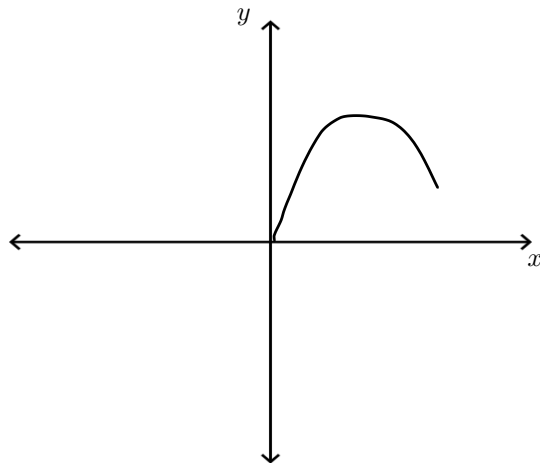
2. Use part 1 of the Fundamental Theorem of Calculus to find the derivative of the functions

Remark. Part 1: $\int_a^x f(t)\, dt = f(x),\ a \leqslant x \leqslant b$

2.a

$$If:\ g(x) = \int_3^x \sqrt{9 - t^2}\, dt$$
$$Then:\ g'(x) = \frac{d}{dx} \int_3^x \sqrt{9 - t^2}\, dt$$
$$= \sqrt{9 - x^2}.$$

2.b

$$If:\ y = \int_0^{\ln x} e^t\, dt$$
$$Then:\ y' = \frac{d}{dx} \int_0^{\ln x} e^t\, dt$$
$$= e^{\ln x} \cdot \frac{d}{dx} \ln x$$
$$= e^{\ln x} \cdot \frac{1}{x}$$
$$= \frac{e^{\ln x}}{x}.$$

3. Use part 2 of the Fundamental Theorem of Calculus to evaluate the integrals.

3.a $\int_1^4 \frac{2-x^{\frac{1}{2}}}{x^2} \, dx$

$$\int_1^4 \frac{2 - x^{\frac{1}{2}}}{x^2} \, dx$$

$$= \int_1^4 \frac{2}{x^2} - \frac{x^{\frac{1}{2}}}{x^2} \, dx$$

$$= \int_1^4 2x^{-2} - x^{-\frac{3}{2}} \, dx$$

$$= -2x^{-1} + 2x^{-\frac{1}{2}} \Big]_1^4$$

$$= \left(-2(4)^{-1} + 2(4)^{-\frac{1}{2}} \right) - \left(-2(1)^{-1} + 2(1)^{-\frac{1}{2}} \right)$$

$$= \left(-\frac{2}{4} + \frac{2}{\sqrt{4}} \right) - \left(-2 + 2 \right)^{\,0}$$

$$= -\frac{1}{2} + 1$$

$$= \frac{1}{2}.$$

3.b
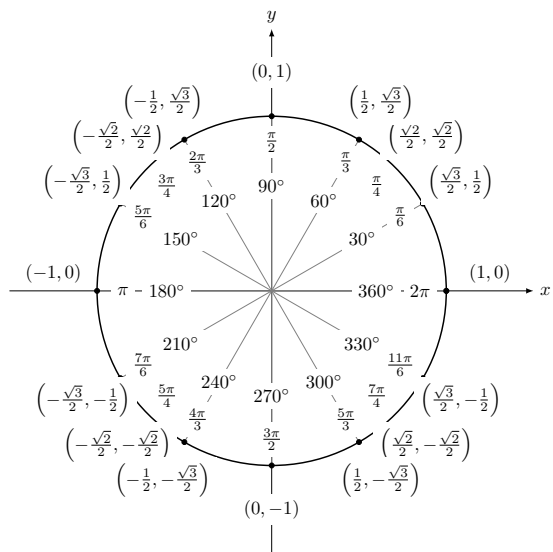
$$\int_0^\pi \left(\sin x - 3\sqrt{x} \right) dx$$

$$= \int_0^\pi \left(\sin x - 3x^{\frac{1}{2}} \right) dx$$

$$= -\cos x - 2x^{\frac{3}{2}} \Big]_0^\pi$$

$$= \left(-\cos \pi - 2(\pi)^{\frac{3}{2}} \right) - \left(-\cos(0) - 2(0)^{\frac{3}{2}} \right)$$

$$= \left(-(-1) - 2\pi^{\frac{1}{2}} \right) - \left(-1 \right)$$

$$= 1 - 2\pi^{\frac{3}{2}} + 1$$
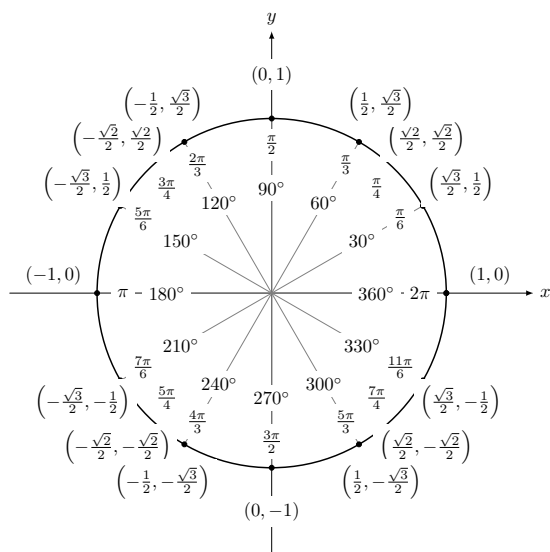
$$= 2 - 2\pi^{\frac{3}{2}}.$$

4. Evaluate the following integrals

4.a

$$\int x^{\frac{1}{2}}(x^2 + 5x + 2) \; dx$$

$$= \int x^{\frac{1}{2}+2} + 5x^{1+\frac{1}{2}} + 2x^{\frac{1}{2}} \; dx$$

$$= \int x^{\frac{5}{2}} + 5x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \; dx$$

$$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + C.$$

4.b

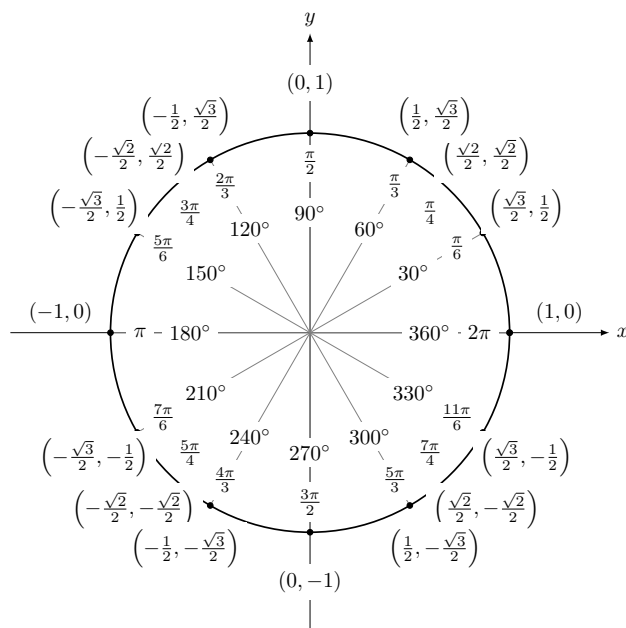$$\int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^3} \right) \; dx$$

$$= \int_1^2 x^{-2} - x^{-3} \; dx$$

$$= -x^{-1} + \frac{1}{2}x^{-2} \Big]_1^2$$

$$= \left(-(2)^{-1} + \frac{1}{2}(2)^{-2} \right) - \left(-(1)^{-1} + \frac{1}{2}(1)^{-2} \right)$$

$$= \left(-\frac{1}{2} + \frac{1}{8} \right) - \left(-1 + \frac{1}{2} \right)$$

$$= -\frac{3}{8} - \left(-\frac{1}{2} \right)$$

$$= -\frac{3}{8} + \frac{1}{2}$$

$$= \frac{1}{8}.$$

4.c

$$\int_0^\pi (2\sin x - 3\sec^2 x) \; dx$$

$$= -2\cos x - 3\tan x \Big]_0^\pi$$

$$= \left(-2\cos \pi - 3\tan \pi \right) - \left(-2\cos 0 - 3\tan 0 \right)$$

$$= \left(-2(-1) - 3(0) \right) - \left(-2(1) - 3(0) \right)$$

$$= 2 - (-2)$$

$$= 4.$$

4.d

$$\int_0^{\frac{\pi}{4}} \sec x \tan x \ dx$$

$$= \sec x \Big]_0^{\frac{\pi}{4}}$$

$$= \sec \frac{\pi}{4} - \sec 0$$

$$= \frac{1}{\frac{\sqrt{2}}{2}} - 1$$

$$= \frac{2\sqrt{2}}{2} - 1$$

$$= \sqrt{2} - 1.$$

5. Find displacement and distance traveled of $t^2 - 3t - 18$, $0 \leqslant t \leqslant 6$

Remark. Displacement: $\int_a^b v(t) \ dt$ and Distance Traveled: $\int_a^b |v(t)| \ dt$

Displacement:

$$\int_0^6 t^2 - 3t - 18 \ dt$$

$$= \frac{1}{3}t^3 - \frac{3}{2}t^2 - 18t \Big]_0^6$$

$$= \left(\frac{1}{3}(6)^3 - \frac{3}{2}(6)^2 - 18(6) \right) - \left(\frac{1}{3}(0)^3 - \frac{3}{2}(0)^2 - 18(0) \right)$$

$$= 72 - 54 - 108$$

$$= -90.$$

Distance Traveled:

$$\int_0^6 |t^2 - 3t - 18| \ dt.$$

Find where the function turns negative

$$t^2 - 3t - 18 = 0$$
$$(t + 3)(t - 6)$$
$$t = -3^{[1]}, 6$$

Rewrite Piecewise

$$v(t) = \begin{cases} t^2 - 3t - 18 & \text{if } t > 6 \\ -(t^2 - 3t - 18) & \text{if } t \leqslant 6 \end{cases} \tag{1}$$

---

[1] not a solution

Thus:

$$\int_0^6 -(t^2 - 3t - 18) \; dt$$

$$= \int_0^6 -t^2 + 3t + 18 \; dt$$

$$= -\frac{1}{3}t^3 + \frac{3}{2}t^2 + 18t \bigg]_0^6$$

$$= \left(-\frac{1}{3}(6)^3 + \frac{3}{2}(6)^2 + 18(6) \right) - \left(-\frac{1}{3}(0)^3 + \frac{3}{2}(0)^2 + 18(0) \right)$$

$$= 90.$$