Discrete Structures
Graph Theory

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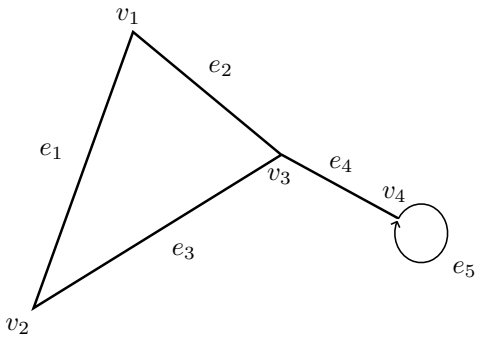
1 Graphs

Definition 1. A graph G consists of two finite sets: a nonempty set $V(G)$ of vertices and a set $E(G)$ of edges, where each edge is associated with a set consisting of either one or two vertices called its endpoints. Formally, a graph is defined as an ordered pair $G = (V, E)$, where V is the set of vertices and E is the set of edges

$$G = (V, E)$$
$$V = \{v_1, v_2, v_3, ..., v_n\}$$
$$E = \{e_1, e_2, e_3, ..., e_m\}.$$



$$V = \{v_1, v_2, v_3, v_4\}$$
$$E = \{e_1, e_2, e_3, e_4, e_5\}.$$

We can also represent the edges by only stating the vertices which connect the edges

Edges	Endpoints
e_1	$\{v_1, v_2\}$
e_2	$\{v_1, v_3\}$
e_3	$\{v_2, v_3\}$
e_4	$\{v_3, v_4\}$
e_5	$\{v_4\}$

2 Subgraphs

Definition 2. **Graph H is said to be a subgraph of a graph H iff every vertex in H is also a vertex in G, every edge in H is also an edge in G, and every edge in H has the same endpoints as it has in G.**